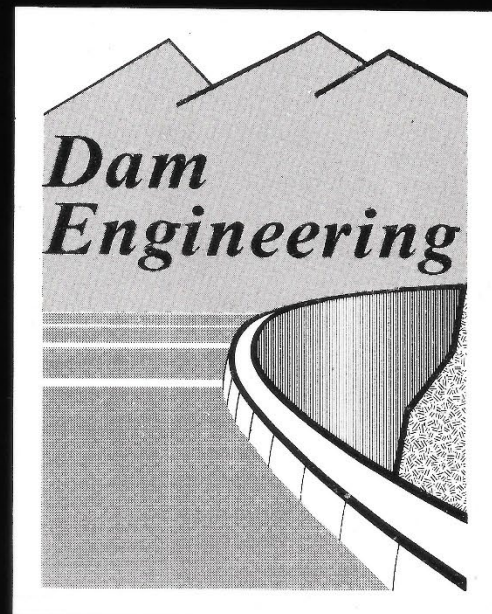


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CONTENTS

- Poroplastic analysis of concrete dams and their foundations** 165
By B. Fauchet, O. Coussy, A. Carrère and B. Tardieu
- Numerical simulation of crack initiation and propagation in an arch dam** 193
By L.F. Martha, J. Llorca, A.R. Ingraffea and M. Elices
- Backanalysis of Paltinu arch dam** 215
By D. Stematiu and Al. Constantinescu
- Numerical analysis for the remedial works of Kölnbrein dam** 235
By P. Oberhuber, P. Sterling and G. Zenz
- Stress analysis of non-circular arch dams with variable thickness** 253
By Zhu Bofang, Rao Bin and Jia Jinsheng
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Numerical simulation of crack initiation and propagation in an arch dam

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SUMMARY

A three-dimensional numerical simulation of crack initiation and propagation in a concrete double curvature arch dam is presented¹. The analysis is carried out in the context of an integrated design system created to study crack evolution in three-dimensional bodies. Criteria for crack initiation and propagation are based upon linear elastic fracture mechanics (LEFM). The features of the system are described and the applicability of LEFM to concrete arch dams is discussed.

Introduction

Engineers have long been confronted with the problem of evaluating the influence of cracks upon the behaviour of structures. In particular, cracking of large plain concrete structures, such as arch dams, has been of special interest. Fracture of an arch dam could lead to a catastrophic accident, with an uncontrolled emptying of the reservoir. This process can be very fast, because of the brittle nature of concrete, and it is likely that once the fracture has started, there is no time to react

¹This is an expanded version of a paper which first appeared in *Applications of Fracture Mechanics to Dam Engineering* (Ed. F. H. Wittmann), ETH Zurich, pp. 82-88 (1990).

and prevent further propagation of the crack. There are several sources of cracking of an arch dam, such as earthquakes, temperature variation, or concrete shrinkage. However, in general the fracturing mechanism itself is controlled solely by the state of stress and strain in the concrete mass. This state depends essentially on thermal effects, on many material parameters (which are normally not known in detail), and, most importantly, on the dam/foundation interaction. Arch dams are particularly sensitive to geometrical imperfections related to the dam/foundation interaction. This is one of the main sources of crack formation and fracture developments in these structures, and despite the application of careful geological studies and design processes, it is very difficult to eliminate completely arch dam cracking. Traditionally, dam calculations involve simplified assumptions, such as the fictitious separation of the dam structure into horizontally layered arches and vertical cantilevers. Recently, to decrease the degree of uncertainty in the evaluation of crack formation and fracture evolution and to improve the safety of these structures, design processes are beginning to incorporate more advanced analytical procedures, such as the finite element method and fracture mechanics methodologies (Linsbauer *et al.* I and II, 1989).

The theoretical foundations for fracture mechanics analysis have been known for more than twenty years. LEFM shows that once the crack has begun to propagate, different criteria can be used to study the direction of crack propagation in accordance with the values of the stress intensity factors K_I , K_{II} , and K_{III} at each point of the crack front (Erdogan and Sih, 1963; Hussain *et al.*, 1974; and Sih, 1974). Previous studies have shown the applicability of these theories in studying crack initiation and propagation in a gravity dam (Chappell and Ingraffea, 1981; Llorca *et al.*, 1987) and in a concrete arch dam (Linsbauer *et al.* I and II, 1989), both using a simplified two-dimensional methodology.

However, the application of LEFM to three-dimensional structures, such as arch dams, has been delayed by the practical difficulties associated with three-dimensional problems: no closed-form solutions exist for the stress intensity factors for real structures and they have to be calculated numerically. Furthermore, depending on the model or flaw geometry, problem specification and modelling may be difficult and time consuming. A useful analysis tool must be capable of modelling an existing flaw in three dimensions, predicting its growth, updating model geometry to simulate its propagation, and performing a new analysis. This task requires an integrated analysis system based on an interactive graphics interface, which has only recently been developed (Wawrzynek *et al.*, 1990; Martha *et al.*, 1990). The fundamental advantage of this new tool is that it is designed specifically to simulate problems with changing geometry in three dimensions. In addition, it takes full advantage of the latest advances in computer hardware, data structures, interactive graphics, and scientific visualization. Based on this analysis system, a numerical simulation of crack initiation and propagation in a double curvature arch dam is presented in this paper.

Initially, the theoretical foundations for the use of LEFM to study fracture in large concrete structures are discussed. Crack initiation and propagation criteria are presented in this discussion; the main characteristics of the numerical analysis tool are described. Finally, the capabilities of this system to simulate three-dimensional fracturing are demonstrated by means of the dam example. The cause of crack initiation is assumed to be the weakening of one of the abutments which support the dam. A finite element simulation of the dam and the foundation shows that tensile stresses develop in the up-stream surface of the dam when the mechanical properties of one of the abutments drop. Then a boundary element analysis is carried out to ascertain the features of the crack propagation.

1 Theoretical foundations

1.1 Applicability of fracture mechanics to the study of dam fracture

The classic design methodology of dams involves the use of the 'no-tension' approach to describe the concrete behaviour. This model assumes that concrete is not able to withstand tensile stresses, and it was first formulated for the stress analysis of rocks (Zienkiewicz *et al.*, 1968). It was argued that by neglecting the tensile strength of the material, a lower bound of failure load is obtained, and

the 'no-tension' model results are conservative when they are used in the structural design of dams. However, it has been shown recently (Bažant, 1990) that if a certain critical dam size is exceeded, the exact no-tension solution of any cracked dam gives a larger maximum load than LEFM for any given value of the concrete fracture toughness. This is because the elastic energy stored in the regions subjected to tensile stresses can be released at the crack tip, helping the crack to propagate. If this energy is neglected in the analysis (as occurs in a 'no-tension' model), it has to be supplied by the work of the external forces, giving rise to larger failure loads.

Fracture mechanics is now a well established discipline to study fracture in many engineering materials, such as metals, ceramics, composites, rocks, and, of course, concrete. For very large mass concrete structures, such as dams, it is commonly accepted that fracture mechanics in its simplest form (LEFM) can be used. This hypothesis implies that the zone where the fracture processes take place is small when compared with all the characteristic dimensions of the structure, including the crack length. The fracture process zone length is controlled by the material characteristic length, l_{ch} , given by (Hillerborg *et al.*, 1976)

$$l_{ch} = \frac{G_F E}{\sigma_t^2} \quad (1)$$

where G_F is the concrete fracture energy, E the Young's modulus, and σ_t the tensile strength. Brühwiler (1988) has measured the fracture properties of the concrete from several dams, obtaining l_{ch} values between 1100 mm and 1700 mm. These values are fairly large for concrete, as could be anticipated by taking into account the poor tensile strength and the large aggregate size of dam concrete. Typically, a ratio of ten between the smallest structural characteristic dimension² and l_{ch} is necessary to get accurate values of the failure load when using LEFM (Petersson, 1981). It is evident that this condition cannot be fulfilled in the early stages of crack propagation, when the crack size is indeed small, and the use of LEFM has to be limited to the study of large gravity dams which are already cracked, precisely the situations in which the classic 'no-tension' approach leads to unsafe results.

The comments presented in the previous paragraph seem to indicate that the fracture analysis of arch dams should be carried out by using non-linear fracture mechanics (NLFM). This conclusion is distressing because NLFM for concrete is currently under development and has not yet reached a mature stage, as in the case of metals. A good solution to this problem involves the use of concepts such as the R -curve or the effective crack length. Both are simplified versions of NLFM which can be used within the framework of LEFM, giving accurate results within the range of crack lengths and structure sizes where LEFM tends to overestimate the failure load. In the R -curve approach, the concrete crack growth resistance (R) increases with the crack length, a , until the fracture process zone is fully developed. Beyond this point, R is equal to G_F , in accordance with the postulates of LEFM. The relationship between R and a can be obtained for any given material and structure by using NLFM, or approximated through various simplified models (Broek, 1986; Bažant, 1986; Planas *et al.*, 1989). Although the R -curve is geometry dependent, the influence of the geometry is expected to be weak at the early stages of crack propagation in a dam because the crack size is very small when compared with the dam dimensions. The effective crack model (Karihaloo and Nallathambi, 1989) is similar to the R -curve, but instead of modifying the crack growth resistance, an effective crack length, a_e , is defined for each a . Then a_e and G_F are employed to calculate the failure load, providing a useful tool for the design engineer.

As mentioned in the introduction, the practical use of LEFM to study fracture of dams has been delayed because of the lack of numerical tools capable of modelling the crack evolution in a three-dimensional structure. These tools are now available, and this paper is a good example. Approximations of the concrete fracture behaviour, such as the R -curve, can be used within the framework of LEFM to predict the evolution of a crack in an arch dam.

²It is worth noting that the crack length a is one of the structural characteristic dimensions.

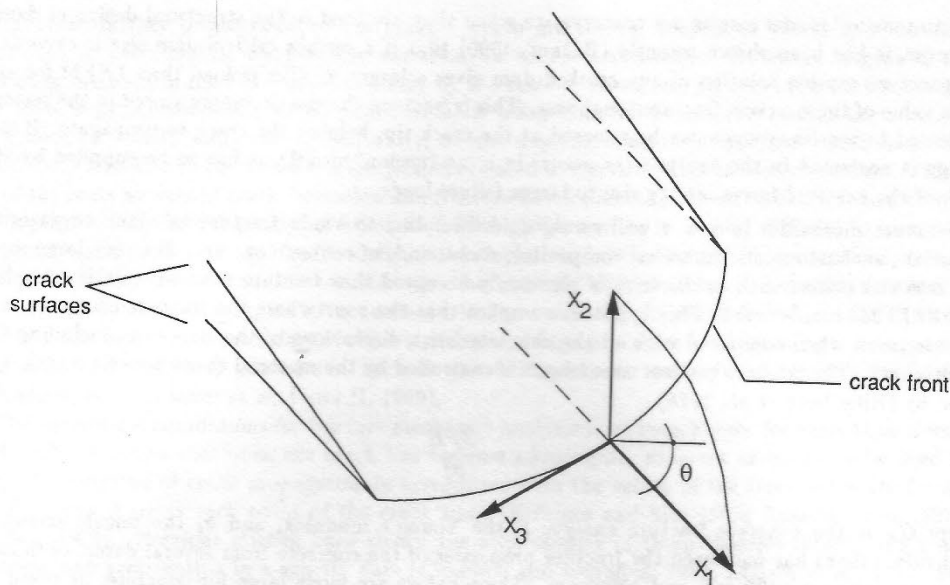


Figure 1. Local coordinate system at the crack front

1.2 Direction of crack propagation

Fracture mechanics is able to ascertain the direction of crack propagation as well as the failure load. This capability is as important as the determination of the failure load because it allows a discrimination between cracks which are dangerous for the dam stability, and can lead to catastrophic failures or water leakages, and those which are not. The determination of the crack shape evolution has to be carried out incrementally. In addition, the geometry of the crack front line has to be discretized into an arbitrary number of points. At a particular stage of crack propagation, the new position of the crack front is determined from the actual crack front position. A local system of coordinates is defined at each node of the crack front in the following way: x_3 is tangential to the geometric description of the crack front, x_2 is the direction normal to the crack surface at that point, and x_1 is perpendicular to x_2 and x_3 (Figure 1).

It is assumed that the crack propagates at each point within the plane perpendicular to the crack front, defined by x_1 and x_2 . The angle θ that defines the direction of crack propagation in this plane can be computed through three different criteria. The first one, named the maximum circumferential (hoop) stress theory (Erdogan and Shih, 1963), establishes that the crack will propagate for the θ value where the circumferential stress σ_θ is maximum. The angle can be obtained by solving the equation,

$$K_I \sin \theta + K_{II} (3 \cos \theta - 1) = 0 \quad (2)$$

The minimum strain energy density theory establishes that the crack will propagate in the direction along which the strain energy density is minimum (Shih, 1974). The function to minimize is

$$S(\theta) = a_{11}K_I^2 + 2a_{12}K_I K_{II} + a_{22}K_{II}^2 + a_{33}K_{III}^2 \quad (3)$$

where

$$a_{11} = \frac{1}{16\mu} (1 + \cos \theta)(\kappa - \cos \theta) \quad (4a)$$

$$a_{12} = \frac{1}{16\mu} \sin \theta (2 \cos \theta - \kappa + 1) \quad (4b)$$

$$a_{22} = \frac{1}{16\mu} \left[(1 - \cos \theta)(\kappa + 1) + (1 + \cos \theta)(3 \cos \theta - 1) \right] \quad (4c)$$

$$a_{33} = \frac{1}{4\mu} \quad (4d)$$

where μ stands for the shear modulus, $\kappa = 3 - 4\nu$ (plane strain conditions are assumed along the crack front), and ν is the Poisson's coefficient.

Finally, the maximum energy release rate theory states that the crack extension will take place in the direction of maximum energy release rate (Hussain *et al.*, 1974). In the case of a plane specimen subjected to tension and in-plane shear (mode I and mode II), the value of θ is obtained by maximization of the function

$$G(\theta) = \frac{4}{E} \left[\frac{1}{3 + \cos^2 \theta} \right]^2 \left[\frac{1 - \theta/\pi}{1 + \theta/\pi} \right]^{\theta/\pi} \left[(1 + 3 \cos^2 \theta) K_I^2 + 8 \sin \theta \cos \theta K_I K_{II} + (9 - 5 \cos^2 \theta) K_{II}^2 \right] \quad (5)$$

where $G(\theta)$ stands for the energy release rate if the crack propagates in the direction defined by θ . Other expressions can be found in the literature (Bilby and Cardew, 1975; Swedlow, 1975; Wu, 1978) for different mixed-mode loading conditions, including mode I and mode III, or mode II and mode III.

The three criteria give very close results for plane specimens where $K_{III}=0$, and K_I and K_{II} are different from zero (see, for instance, two recent reviews: Taha and Swartz, 1989; Jenq and Shah, 1989). In these conditions, it has been shown that the crack propagates in such a way that K_{II} tends to be minimum and the crack is subjected mainly to mode I loading conditions. As the three criteria are equivalent for pure mode I, the crack paths obtained are very close.

However, there is no proof of their validity in the case of three-dimensional cracks, and particularly the influence of K_{III} on θ is not yet well known. The maximum hoop stress theory is purely two-dimensional and cannot take into account the effect of antiplane shear on θ . The minimum energy density criterion is three-dimensional, as follows from equations (3) and (4), but the contribution of mode III to the strain energy is constant and hence it does not influence the direction of crack propagation. Moreover, in the case of pure mode III, with K_I and K_{II} equal to zero, this theory fails to predict a particular direction for crack propagation. The most appropriate criterion from a mechanical point of view seems to be the energy release rate theory. Wu (1978) gives several reasons to support this statement. First, the maximum energy release rate theory is based on a fundamental principle in mechanics, namely the principle of minimum potential energy. Second, it is an obvious generalization of the Griffith's energy release rate concept, and third, it can cope with three-dimensional mixed-mode problems. However, several shortcomings of this model at its present stage have to be pointed out. The solutions available for the energy release rate under mixed-mode loading assume a traction-free condition on the crack surfaces, which may not be true in the case of dominant mode II or mode III situations. Also, it is often postulated that the crack will propagate in the plane normal to the crack front ($x_1 x_2$ in Figure 1), which is not true in the case of pure antiplane loading. As can be seen, the topic remains open and further research is necessary to ascertain the range of validity of the proposed criteria.

1.3 Crack extension length

In two-dimensional specimens, the stress intensity factors are constant along the crack front, and if surface effects are neglected, it can be assumed that the crack will propagate in the same amount along the crack front. This is not the case for three-dimensional cracks, where the stress intensity factors do change along the crack front. Then, two important questions arise: can one use a local criterion to predict the crack propagation? and how can one determine the crack propagation length along the crack front?

The stress intensity factor distribution along the crack front depends on the crack shape. Thus, the critical condition for crack propagation will be attained first at a particular point of the crack front, whereas the rest of the crack front has not reached this situation. It is unlikely that crack extension will take place at that moment, and the crack will probably propagate when a certain length of the crack front has become critical. Moreover, the stress intensity factors depend on the crack shape. If a zone of the crack front propagates, the stress intensity factors will increase all along the crack front but the increase rate will be larger in the non-propagating crack zone, which in turn can reach the instability condition. Thus, a local condition for crack propagation does not seem to be a good criterion and, on the other hand, it is reasonable to predict that the crack will propagate all along the crack front once a certain length of the crack front has reached a critical situation. Neither this length nor the amount of crack propagation at each point of the crack front is known, and some rough simplifications have to be made in order to give a solution. In this work it was assumed that the crack propagation takes place simultaneously along the crack front and that the increase in crack length at each point is proportional to the K_I^2 value at the point. This hypothesis is based on the fact that the crack shape evolves in such a way that K_I is dominant. Other criteria relying on K_I , K_{II} and K_{III} should give close results since K_I is preponderant.

1.4 Numerical determination of the stress intensity factors

The efficiency of the method relies on the precise and fast calculation of the stress intensity factors along the crack front at each step of the crack propagation. To this effect, the displacement correlation technique is employed, jointly with quarter-point singular elements at the crack front. The local coordinates system x_1 x_2 x_3 defined at each point of the crack front is used as a reference framework (Figure 1). The displacement field in the neighborhood of the crack front can be expanded in a series, with a high predominance of the singular stress field for points sufficiently close to the crack front. Assuming plain strain conditions, the relative displacements of the two points (here referred to as COD points) nearest to the crack front in the crack wake are given by (Sousa *et al.*, 1989),

$$COD = K_I \frac{4(1-\nu^2)}{E} \left[\frac{2r}{\pi} \right]^{1/2} \quad (6a)$$

$$CSD = K_{II} \frac{4(1-\nu^2)}{E} \left[\frac{2r}{\pi} \right]^{1/2} \quad (6b)$$

$$CTD = K_{III} \frac{4(1+\nu)}{E} \left[\frac{2r}{\pi} \right]^{1/2} \quad (6c)$$

where *COD*, *CSD* and *CTD* are respectively the opening, shearing and tearing opening displacements (relative displacements between both crack faces projected onto the local coordinates system), and *r* stands for the distance to the COD points (Figure 2). The distribution of stress intensity factors along the crack front is smoothed by fitting hermitian polynomials along a parametric description of the crack front line.

1.5 Numerical capabilities

The techniques for numerical analysis of arbitrary, three-dimensional cracks and simulation of crack propagation remain open issues. Until recently (Wawrzynek *et al.*, 1990; Martha *et al.*, 1990), there was no general analysis tool that could allow a designer to simulate flaw growth in arbitrary three-dimensional objects. The system described in these references is used in this work to perform a three-dimensional fracture simulation in an arch dam. This numerical analysis system is a superworkstation-based program that exploits the state of the art in high-performance computer graphics, direct manipulation user-interfaces, and data representation techniques, which provide the requirements for efficient interactive analysis. All the tasks of model definition, mesh generation, numerical analysis, and visualization (post-processing) are combined in one seamless package with

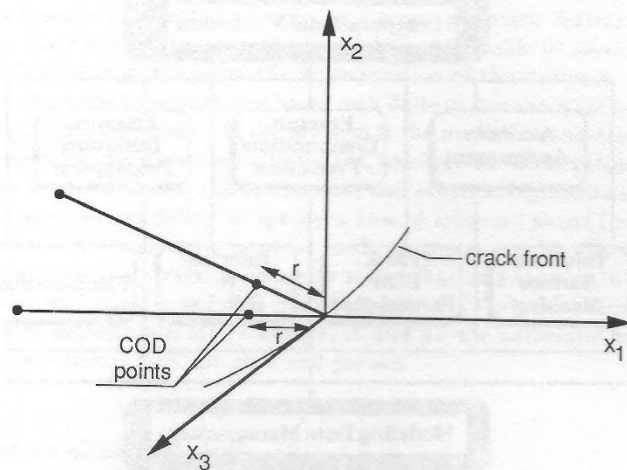


Figure 2. Stress intensity factor calculation at the crack front

one consistent user-interface (see figures in following sections). By hiding or automating much of the complexity inherent in a three-dimensional fracture simulation, the system allows the analyst to concentrate on the particular physical problem being studied, rather than on details of the simulation.

The analysis system always maintains a consistent, underlying, solid representation of any object being modeled. This system, however, is not a solid modeller, although many solid-model-like features and algorithms are available within it. These are necessary to support the underlying representation, but the system is not designed to be a tool for creating the initial geometric description of an object to be modeled. This system requires as input a solid model boundary representation. The representation consists of a collection of planar and curved polygonal patches which describe the outer skin of the object. The representation may include internal features such as interfaces and cracks. Planar polygonal patches are described by the location of their vertices. Curved patches are described by means of a grid of interpolating points located on the surface of the patch. The initial boundary representation is converted into a representation internal to the system. Internally, the geometry of the model is allowed to change during the course of the simulation, but no attempt is made to reincorporate the updated geometry into a representation in the solid modeller.

Figure 3 shows a schematic representation of the global software organization in the present analysis system. This is a rather simplistic block diagram which does not depict the complete network organization of the system. The purpose of this figure is to position the issues addressed in this paper within the overall system organization which in this context is organized into three main components: Display and User-interface, Computational Mechanics, and Modelling Data Management. The user-interface is an essential part of any modelling process. The processes of attribute assignment, numerical discretization, and crack

initiation/propagation rely heavily on the sophisticated interactive user-interface module of this system. The modelling data representation is also an essential component. The efficient and sophisticated data representation adopted in this system provides the necessary capabilities for an interactive manipulation of the model during a fracture simulation. This component is described elsewhere (Martha, 1989).

The computational mechanics component is the primary motivation for the development of this

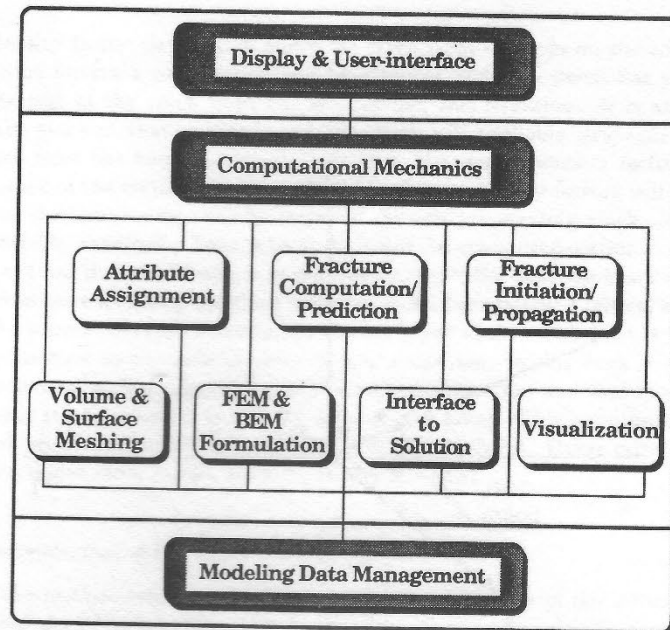


Figure 3. Software organization of numerical analysis system

system which is designed to grow indefinitely in the area of computational fracture mechanics. This involves several issues which are common to other areas in computational mechanics. Figure 3 shows the currently available modules in this component: Attribute Assignment, Volume/Surface Meshing, FEM/BEM Formulation, Interface to Solution, Fracture Computation/Prediction, Fracture Initiation/Propagation, and Scientific Visualization.

The part of Attribute Assignment to which this paper relates concerns the assignment of material properties, loading, and support conditions to the model. Here lies one of the main advantages of adopting an underlying solid modelling representation in a numerical analysis system: loads, support conditions, material properties, and other physical attributes are attached to geometric and topological entities, as opposed to elements and nodes of a numerical discretization. This means that the user need not reassign attributes during meshing and remeshing, or when a crack is introduced into the model and propagated. In the current implementation, several types of loads are incorporated: distributed surface loads, gravity loads, and hydrostatic pressure. In the particular case of dam analysis, hydrostatic pressure can be applied to both dam external faces and crack surfaces. This load is automatically updated with the crack propagation. This means that the hydrostatic load is automatically applied to the nodes on the crack faces taking into account the actual normal directions across the propagated crack surface. Current limitations relate to material assignment: only homogeneous material properties can be used.

The system has been designed to incorporate both finite and boundary element methods for the solution of stress analysis problems. The numerical discretization of the model takes place within its internal representation. Meshing is performed by a technique of progressive hierarchical refinement. In this environment, the user interactively subdivides an object into progressively simpler objects until a standard meshing algorithm can be used to mesh these objects. This relies on a sophisticated, topology-based data structure (Weiler, 1988; Martha, 1989) to support both linkage to the solid model and fast interaction. The numerical analysis of concrete arch dams is currently limited to linear-elastic analyses of the uncracked structure or of the cracked structure at each step of crack propagation.

The modular encapsulation of fracture mechanics theories and crack-growth models is one of the

primary objectives of the system. This permits the system to grow with new research in these areas by accommodating both existing and new theories and models. Crack modelling features such as flaw creation, representation, and modification resulting from propagation are based on this approach. In this environment, an initial configuration of flaws is defined by its geometric characteristics. Similarly, crack propagation is reduced to repeatedly predicting the new geometric description of the evolving crack front line. Any crack initiation/propagation model should be able to provide this information. Crack simulation is done with a true geometric representation of the structure via solid modelling. This is in contrast to the usual representation based on a finite or boundary element mesh which is a mathematical artifact. Since the essence of the problem is the prediction and tracking of a changing geometry, the crack modelling relies on the sophisticated, topology-based data structure of this system to support linkage to the solid model, fast interaction, and accurate representation of evolving flaw shapes. The system provides the ability to specify a flaw of arbitrary shape (including non-planar flaws), size, and orientation at arbitrary locations in the geometric model. The flaw is specified at the desired location in the actual structure geometry rather than at a location in the mesh. Another advantage is automatic remeshing to simulate crack propagation. Since crack modelling is integrated into the data structure organization of the system, it uses all the automatic and local remeshing capabilities for the simulation of flaw initiation and growth.

2 Example: Fracture of an arch dam

This section shows how the methodology presented can be used to study the evolution of crack in a concrete arch dam. To this purpose, a double curvature arch dam model has been used. The dimensions of the dam are indicated in Figure 4. First, a standard finite element analysis of the dam and the foundation is performed to ascertain the most likely places in the upstream dam surface for crack initiation. Then a study of crack evolution is carried out. The results presented here are tentative. The main goal of this paper is to present the state-of-the-art capabilities for full three-dimensional analysis of crack initiation and propagation in an arch dam.

2.1 Crack initiation

Three hundred and eighty 20-noded isoparametrical brick elements were used in the finite element discretization, giving a total of 2167 nodes. Both the dam and the foundation were modeled. In every cross-section of the finite element mesh, the foundation is modeled up to a depth equal to the height of the dam. All the materials were assumed to behave in a linear elastic fashion. Concrete properties were Young's modulus 40 GPa and density 2465 Kg/m³. The Young's modulus for the foundation was assumed to be an increasing function of the depth. It is known that differences between the mechanical properties of the left and right abutments can initiate cracking in arch dams. Two examples of this behaviour (Zeuzier dam in Switzerland and Glen Canyon dam in the United States) were reported at a recent meeting (Wittmann, 1990). Thus, the left abutment surface stiffness was 4 GPa whereas 7 GPa were assigned to the right abutment in the surface. In the bottom of the foundation, values of 13 GPa and 25 GPa were used for the right and left abutments, respectively. The forces considered in the analysis were the weight of the concrete and the hydrostatic pressure of the water in the reservoir.

The different stiffnesses of the abutments lead to an asymmetrical behaviour and tensile principal stresses develop in the upstream face, giving values in the range of 6 to 24 MPa in the symmetry plane of the dam, near the foundation. These stresses can overcome the tensile strength of the concrete and start a crack. Tensile stresses are also developed in the upper part of the upstream face, close to the abutments, but the nucleation of the crack is assumed to occur in the symmetry axis of the dam and near the foundation. A crack nucleated from that place is more dangerous for the structural stability of the dam than others that can develop from the upper part in the upstream side of the dam.

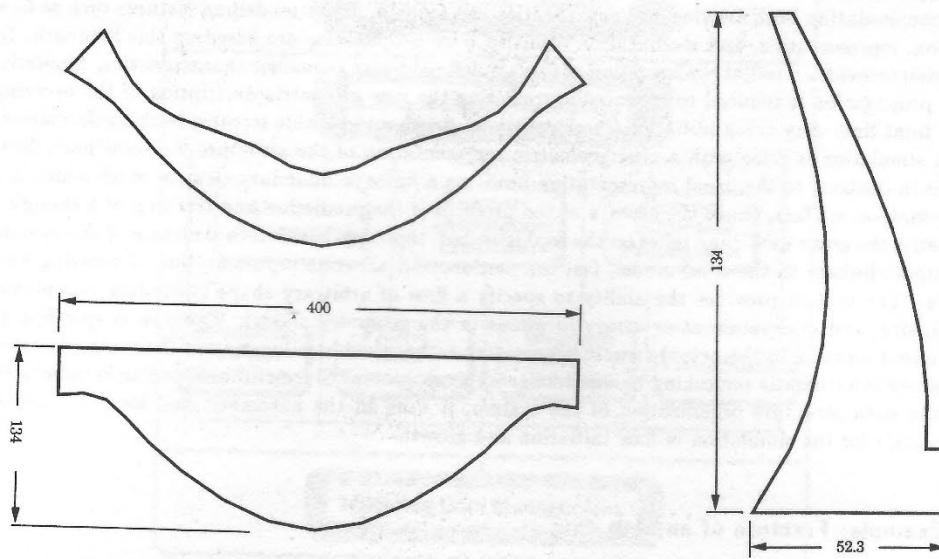


Figure 4. Geometry of the double curvature arch dam: (a) plan and elevation view, (b) central cross section (dimensions in meters)

2.2 Crack propagation

A simulation was performed of a surface crack arbitrarily nucleated on the dam's upstream face. The emphasis here is on the fracture modelling capabilities. The system capabilities illustrated in this analysis are:

- modelling the propagation of an arbitrary crack in a three-dimensional curved arch dam, while maintaining the true topological and geometric representation of the model;
- decomposing and meshing a complex three-dimensional, curved geometry; and,
- providing a history of stress intensity factors along the front of a propagating crack in an arch dam.

The simulation was carried out through the boundary element method. In this analysis, only the concrete dam has been modeled, the foundation being represented by fixity support conditions. This means that the different stiffnesses of the abutments are not taken into account in the propagation analysis. The initial boundary element mesh utilized linear elements, resulting in a total of 663 equations. Figure 5 shows a shaded view of the model and the initial boundary element mesh configuration. At the time this analysis was performed, the boundary element program could not consider gravity loads, and the only forcing effect considered in this analysis is the hydrostatic loading. Currently, body force capabilities have been added to the system (personal communication, Sousa, 1991). As mentioned previously, hydrostatic pressure acts on both crack faces at all stages of the fracture simulation. During crack propagation, each point along the crack front moves a distance which is proportional to the square of the mode I stress intensity factor at that location. In addition, each crack front point moves in the direction of the local maximum circumferential tensile stress ahead of the crack front. The absolute crack extension in each step was arbitrarily and interactively selected: the analyst selected the maximum crack extension for the step at the point of the largest mode I stress intensity factor, and the remainder of the crack front was automatically extended based on the chosen power (square) law.

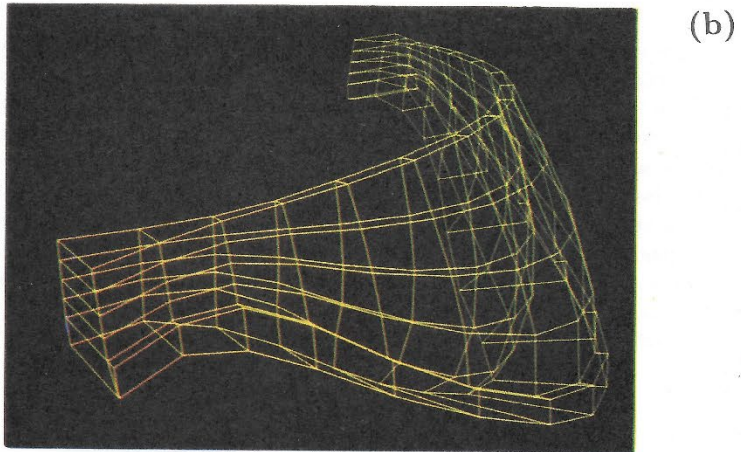
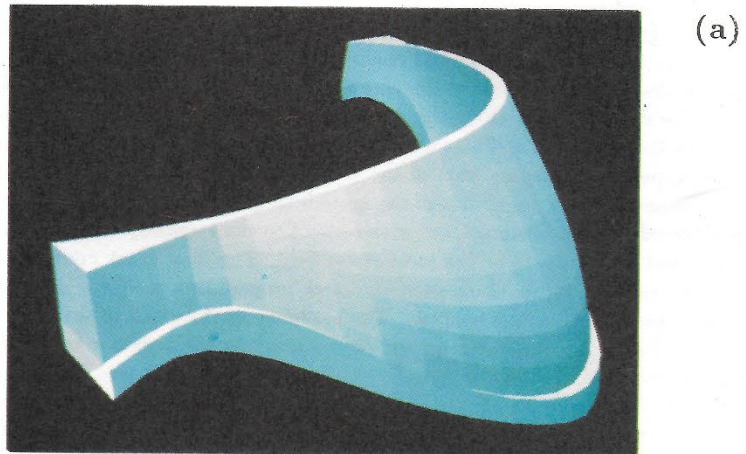


Figure 5. (a) Shaded view and (b) initial boundary element mesh configuration of the arch dam

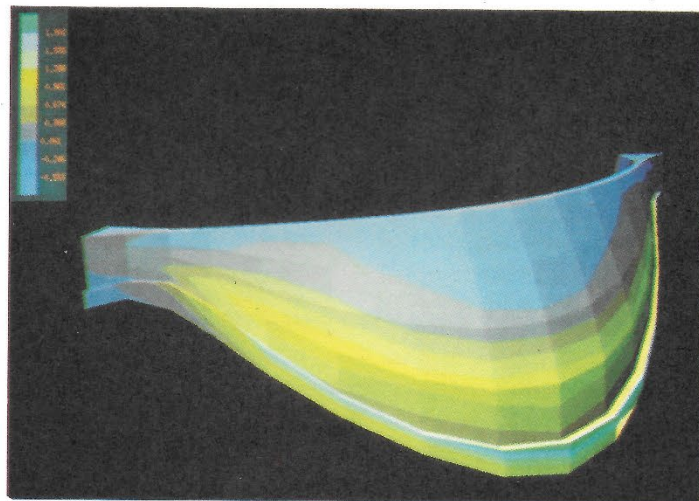


Figure 6. Contour of maximum principal stress response on the surface of the arch dam

Figure 6 shows a surface contour of the maximum principal stress on the upstream face of the dam. Yellow and green indicate tensile stresses whereas blue stands for compressive stresses. This figure corroborates the results of the finite element crack initiation study made in the previous section: both finite element and boundary element analyses show tensile maximum principal stresses at approximately the same position along the upstream face, where a crack was nucleated. However, as the differences in the abutment stiffnesses are not included in the boundary element analysis, tensile stresses do not appear in the upper part of the dam. The initial crack configuration is illustrated in Figure 7. To demonstrate the capability of the system to model an arbitrary crack configuration, the crack was positioned in a tilted orientation with respect to the geometry of the dam. The initial crack is therefore inclined with respect to the local stress field. The final crack configuration is illustrated in Figure 8. The evolution of the crack geometry with propagation is shown in Figure 9. Five propagation steps were performed in this analysis. The tendency of the crack to turn downwards is evident from this figure. Figure 10 illustrates details of the mesh in the region where the crack meets the upstream face of the dam. As mentioned above, the concrete weight was not taken into account in this analysis. If so, it is very likely that the crack trend to turn downwards was accentuated and the crack would not actually reach the downstream face of the dam. Figure 11 displays color contours of maximum principal stresses on the deformed dam surface for the first and last crack configurations. Yellow stands for tensile stresses (which develop around the crack front) whereas blue represents compressive stresses.

Finally, the history of stress intensity factors during the crack propagation is shown in Figure 12. It is worth noting the evolution of the mode II stress intensity factors. In the initial, arbitrarily-chosen crack configuration, the K_{II} values change sign along the crack front.

Because the crack front is allowed to turn, these factors reverse sign, showing a tendency to compensate for the artificially induced initial crack configuration. As the crack continues to propagate, the mode II stress intensity factors along the crack front decrease, becoming negligible after the fourth crack propagation step. The mode I stress intensity factor increases steadily along the crack front during crack propagation. As the crack reaches the downstream face, K_I tends to be maximum at the crack front points located in the upstream face of the dam, showing the tendency of the crack to spread along this face. This figure also points out that if quasi-static crack growth is going to occur, the concrete fracture resistance has to increase with crack length.

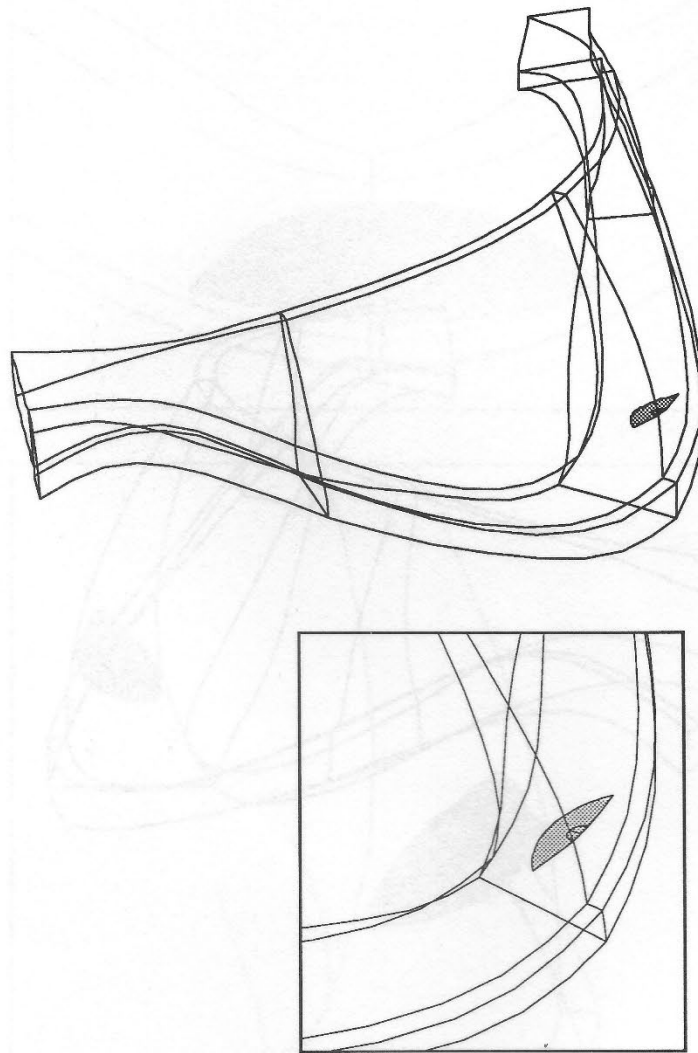


Figure 7. Initial crack configuration of the arch dam (crack face is shaded)

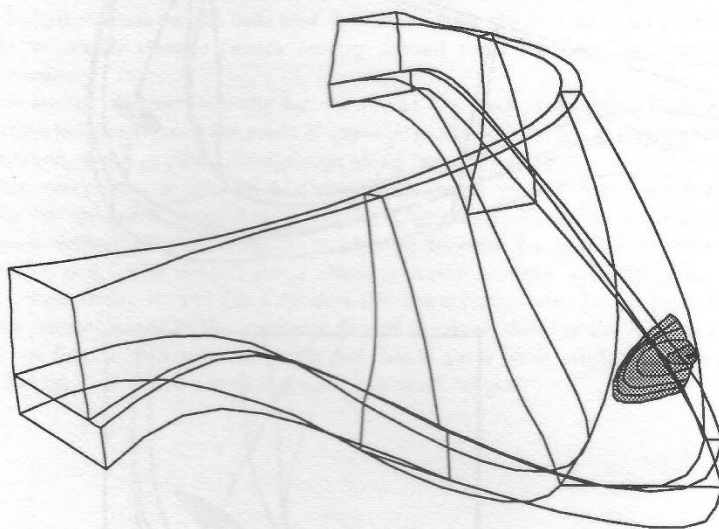


Figure 8. Final crack configuration (crack face is shaded)

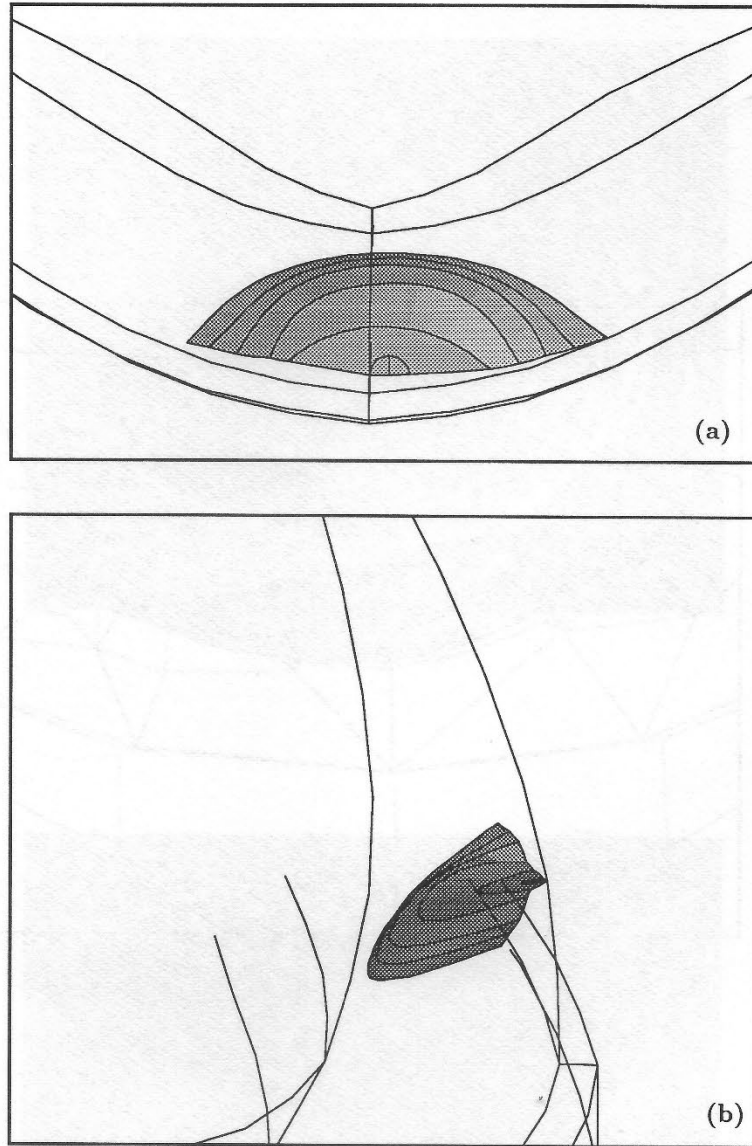


Figure 9. Crack geometry evolution in the arch dam (a) plan view (b) cross section

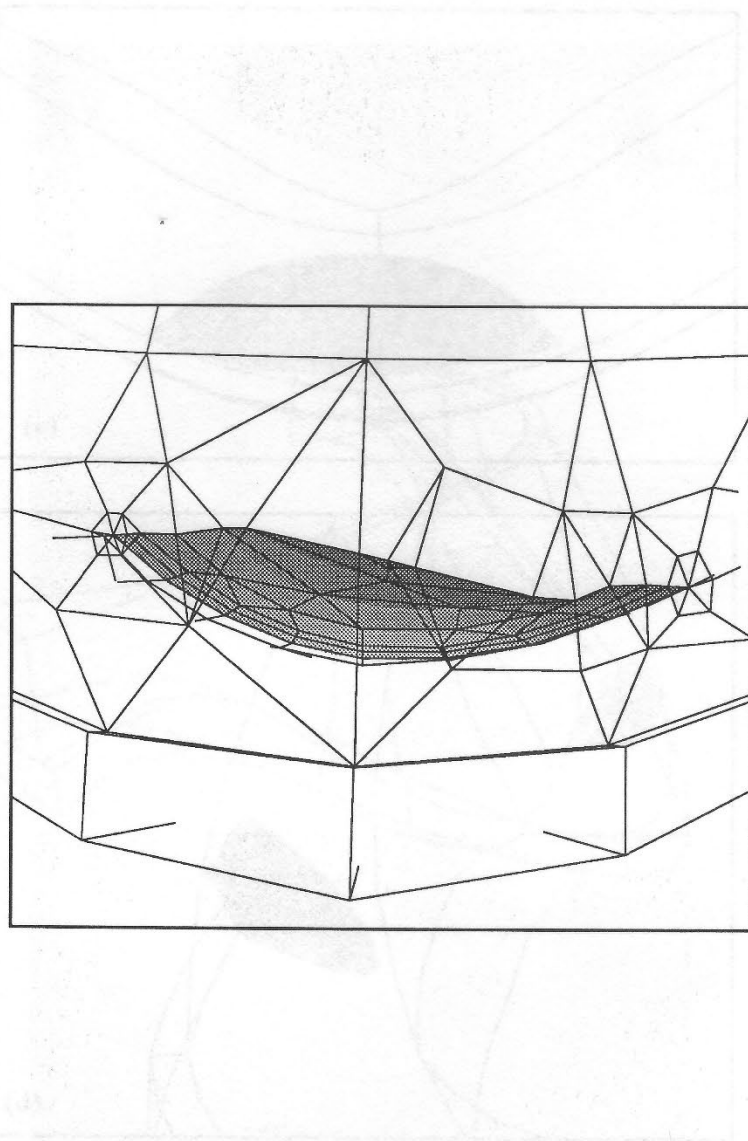


Figure 10. Final crack propagation step of the arch dam (detail of the boundary element mesh in the region where the crack face meets the upstream face of the dam)

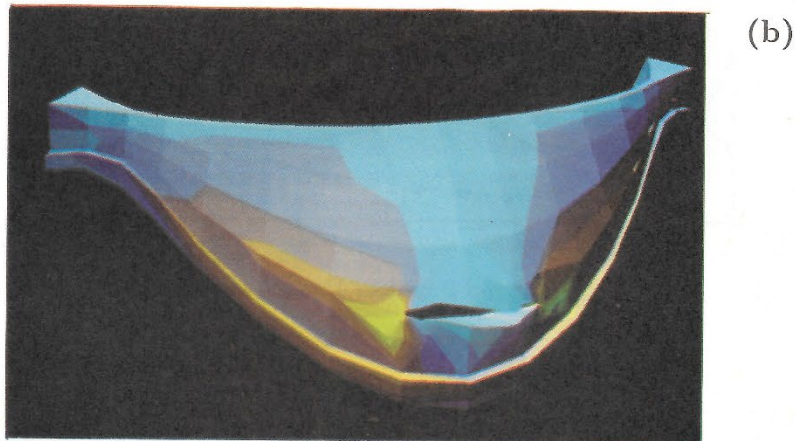
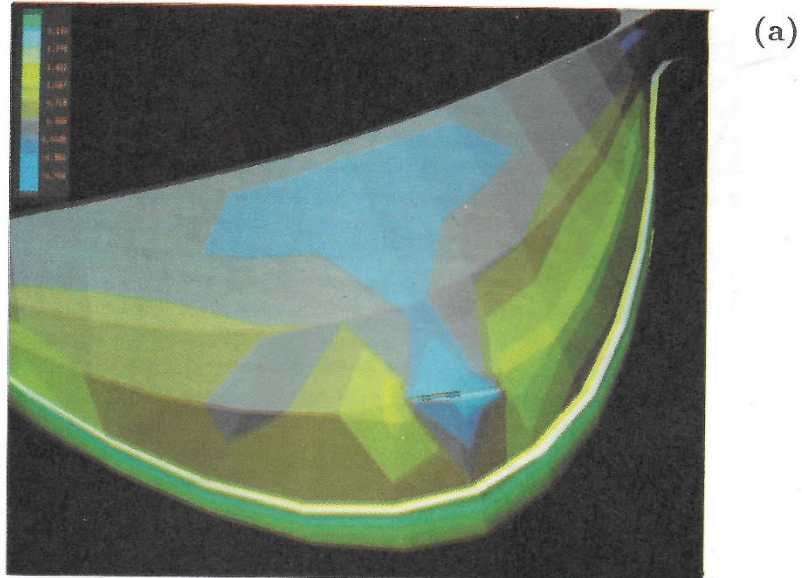


Figure 11. Contours of maximum principal stress response on the deformed configuration (magnification of 800 \times) for (a) the initial and (b) final crack propagation steps of the arch dam

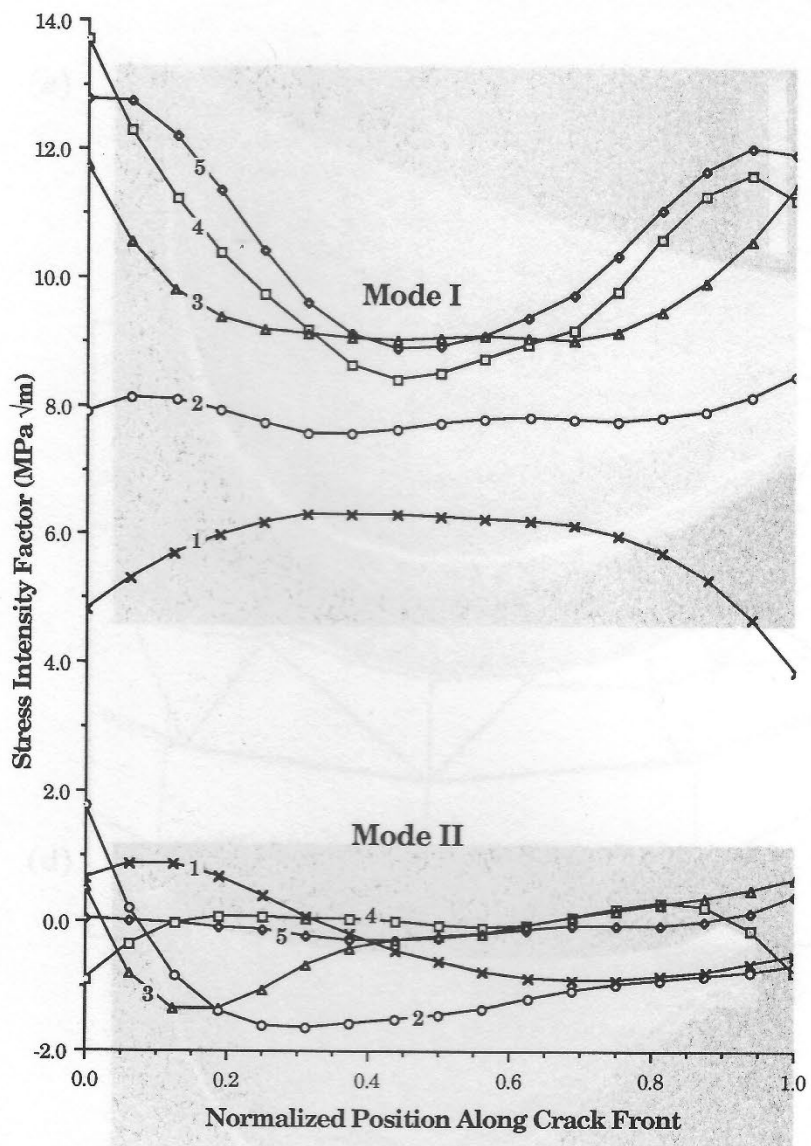


Figure 12. History of stress intensity factors for the arch dam

3 Concluding remarks

The dramatic development achieved in solid modelling and structural analysis makes it feasible to perform a full three-dimensional study of crack initiation and propagation of a double curvature arch dam. The new system presented here combines the state-of-the-art capabilities in high-performance computer graphics, direct manipulation user-interfaces and data representation techniques (necessary for an efficient interactive analysis) with LEFM criteria for predicting the crack evolution.

It is also clear that there is a long way to go before these techniques can be applied in the design and evaluation of dams. The weaknesses of the system are mainly on the side of structural analysis and fracture mechanics. A realistic structural model of an arch dam should be able to consider the construction joints and the anelastic behaviour of the foundation, as well as the effect of thermal stresses, and concrete dilatancy and shrinkage. There are no theoretical difficulties in including these topics in the system, and current developments (coupling between finite and boundary element method) are aimed at overcoming these limitations.

More difficult problems arise from the fracture mechanics viewpoint. The existing criteria for crack instability were developed for two-dimensional situations and have to be expanded to deal with three-dimensional structures. Expanded criteria should be contrasted experimentally by means of careful tests in materials to which LEFM is applicable. Similar work has to be done with regard to the direction of crack propagation at each point of the crack front. The maximum energy release rate criterion has a stronger theoretical foundation than either the maximum circumferential stress or the minimum strain energy density theory, but it is not supported by experimental results.

Finally, a third point of uncertainty lies in the use of LEFM for concrete arch dams in the early stages of crack propagation. Simplified models, such as the *R*-curve approach, are valid for pure mode I loading conditions, and its extension to mixed-mode has not yet been accomplished. It is likely that, as the crack tends to grow in the dominated mode I direction, these approximations can be extrapolated to mixed-mode without substantial changes for most situations, but again, experimental and numerical results should support this supposition. Taking into account the interest of the dam engineering community in getting more accurate information about the safety of new dams or those already in service, and the recent advances in the fields of computer graphics and concrete fracture mechanics, it is reasonable to expect that full three-dimensional analyses of crack evolution in concrete dams will become commonplace in the near future.

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