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Structural analysis with nonlinear behavior: The importance of going beyond the line

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Abstract. This work discusses the importance of numerical methods in structural analysis. It deals specifically with the analysis of frame structures where geometric and material nonlinearity is taken into account. The importance of considering these effects is highlighted with examples of catastrophic events related to instability phenomena. A computational program developed by the authors with educational purposes is presented and used to perform the analysis of a simple frame.

Keywords: structural analysis, numerical methods, geometric nonlinearity, material nonlinearity.

1 Introduction

One of the most important activities that must be mastered by every civil engineer is the structural analysis. It consists of modeling a structure, evaluating its behavior (i.e. solve the equations), and interpreting the results. All these tasks are performed with the aid of a computer program. In the modeling stage, many simplifications are assumed in order to represent the real structure as a mathematical and computational model. A very common modeling simplification for structures whose members are slender, such as beams and columns, is that their behavior is described by one-dimensional elements. These are called frame structural models. A usual simplification for the behavior of frame elements, and for any other element type, is the assumption of linear behavior. It is with this idealization that the behavior of structures is usually taught in undergraduate engineering courses. The reason is the simplicity that it brings to the aforementioned tasks of structural analysis. In this way, the equilibrium and constitutive equations of structural elements, which represent the equation of lines, are assembled into a linear system that can be easily solved without relying on more complex numerical methods. However, this assumption is not always valid and may lead to several risks when designing structures. Therefore, engineers sometimes have to resort to less simplified idealizations that consider nonlinearity in the behavior of structural models in order to design safe structures.

The nonlinearity may originate from different sources, such as the geometry of the problem, the material behavior, the boundary constraints, etc. Structural analysis with these considerations involves several choices from the analyst and relies heavily on numerical methods. One of the choices is the nonlinear formulation to be used for structural elements, as the nonlinearity provides much more formulating options compared to linear assumptions. Because of that, the authors of this work have been investigating different nonlinear formulations for frame elements [1-4]. Another selection is the numerical method to solve the nonlinear system of equations, and the parameters used by these methods, which often need to be carefully calibrated. These methods are necessary because, differently from a linear analysis, an analytical solution of the system of nonlinear equations is impossible to find. Therefore, numerical algorithms are used to obtain the curves of equilibrium configurations of the structure, or to determine critical points of these curves that indicate dangerous behaviors. These algorithms for numerical solution have also been studied by the authors [5-7]. Finally, the interpretation of the results also becomes more challenging in a nonlinear analysis. Therefore, it is important that programs are user-friendly and have good graphical resources. In this sense, the authors have developed educational software to encourage and help undergraduate students in the use of numerical methods applied to structural engineering. These programs include nonlinear effects from the geometry [5,7-12], material behavior [4] and semi-rigid-connections [13-15], also considering dynamic effects [16-23].

2 Sources of nonlinearity

Two of the main sources of nonlinearity will be briefly described, namely **geometric nonlinearity** and **material nonlinearity**.

The geometric nonlinearity arises when the displacements, rotations and/or deformations of the structural elements are not small enough to be neglected (as they are in a linear analysis), so they need to be taken into account when formulating the equilibrium equations. Take the column of Figure 1 as an example. In the case of a linear behavior (Fig. 1a), the bending moment reaction "*M*" does not depend on the vertical force at the tip of the column. This is because "*Pv*" is assumed to be pointing towards the base of the column (horizontal displacement is negligible). In the case of a geometrically nonlinear analysis (Fig. 1b), the horizontal displacement of the tip is taken into account and the vertical force has a lever arm relative to the base, thus contributing to the bending moment reaction. The mutual dependence of the external / internal forces of elements and the geometry makes the equilibrium equations to become nonlinear.

The material nonlinearity concerns the relationship between stresses and strains. This relationship is formulated according to a constitutive model. A generic example of a constitutive curve is depicted in Fig. 2, where ε and σ stand, respectively, for the strain and stress subjected by the material. It is noted that the relationship is approximately linear for small strains/stresses. However, for large strains/stresses the linear assumption is not valid anymore and the constitutive equation introduces a nonlinearity to the formulation.

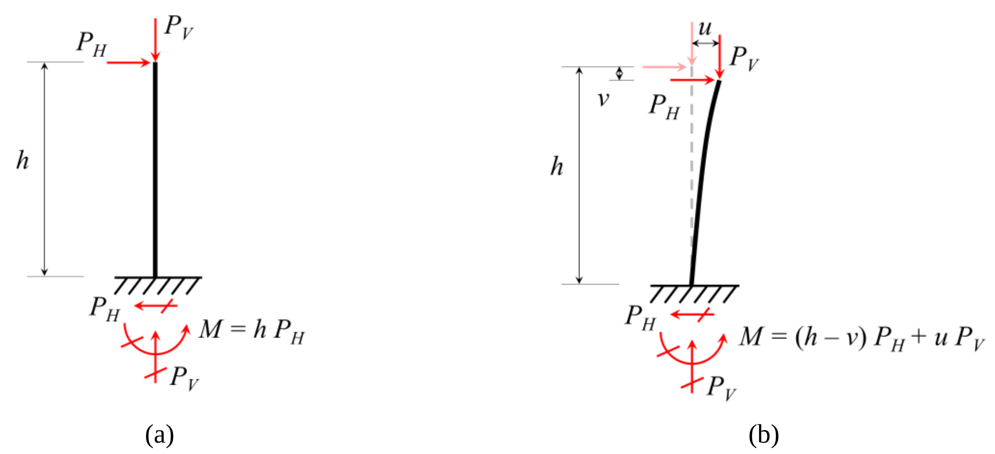


Figure 1. Column analyzed with (a) linear behavior and (b) geometrically nonlinear behavior

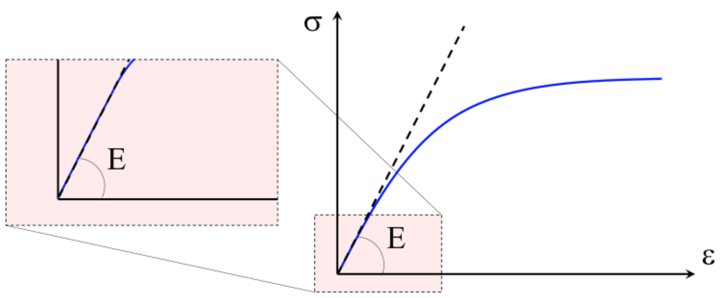
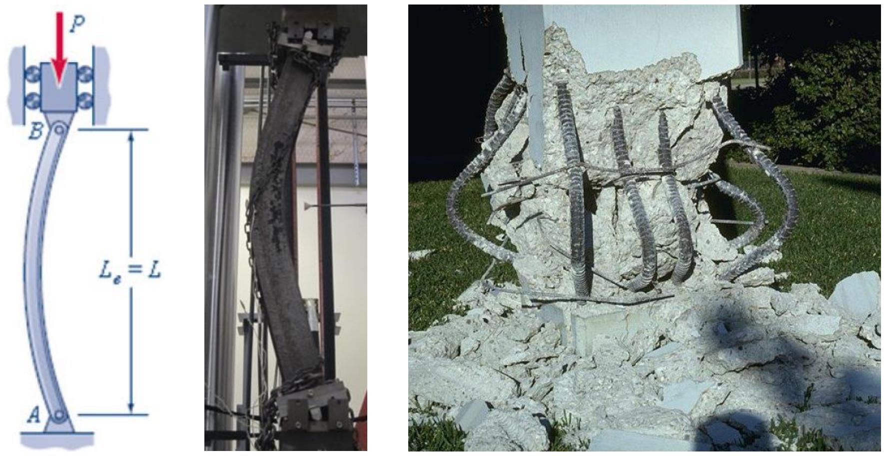


Figure 2. Generic example of a constitutive model

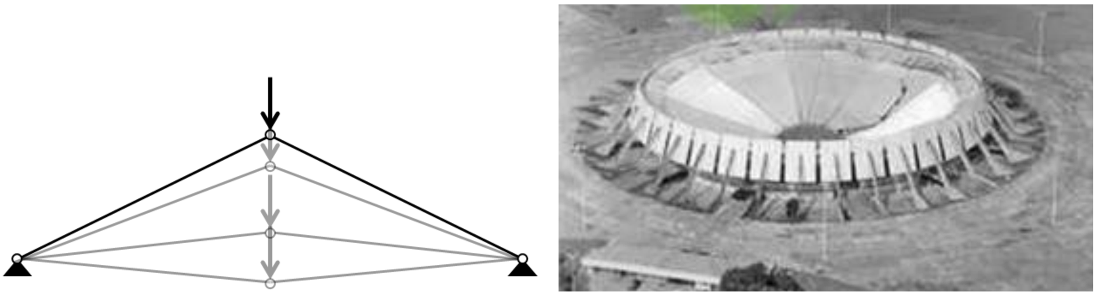
3 Risks associated to nonlinear behavior of structures

Engineers must be aware of the consequences of their assumptions during the analysis of structures. Failing to properly consider the different sources of nonlinearity can lead to catastrophic events, mostly related to instability phenomena. Figures 3 to 6 show examples of these phenomena. Geometrically nonlinear analyses can indicate buckling of columns (Fig. 3), snap-through of shallow structures (Fig. 4), and also dynamic instabilities (Fig. 4). The material nonlinearity can be considered to model the formation of plastic hinges (Fig. 6).



(a) (b) (c)

Figure 3. Buckling of columns:
(a) schematic representation, (b) experimental test [24], (c) collapse of a reinforced concrete column



(a) (b)

Figure 4. Snap-through of shallow structures:
(a) schematic representation of snap-through, (b) Nilson Nelson arena (Brasília – DF, 1991)

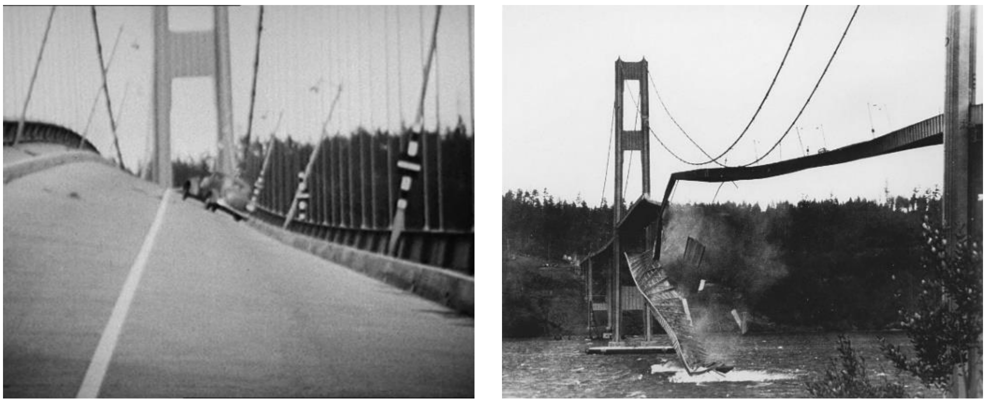
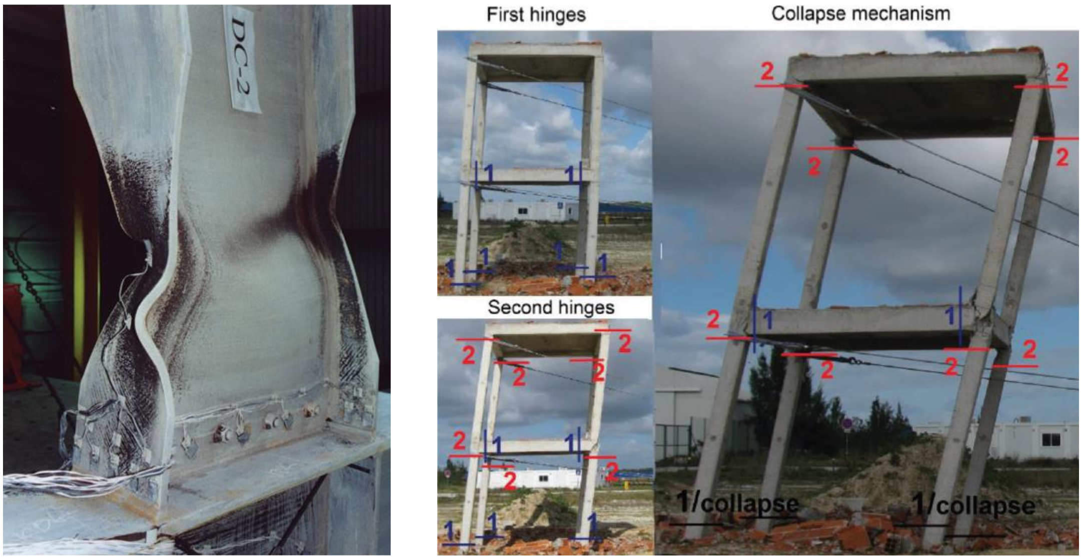


Figure 5. Dynamic instability: resonance in Tacoma Narrows Bridge (Tacoma – WA, 1940)



(a) (b)

Figure 6. Plastic hinge formation: (a) steel beam, (b) concrete frame [25]

4 Learning tools

To help engineering students, and also professionals, to deal with the complexity of nonlinear structural analyses, educational software provide a user-friendly environment with graphical resources to perform these simulations. An example of a widely used program developed by the authors where nonlinear analysis features of two-dimensional frames have been recently incorporated is FTOOL [5,10,12], whose interface is shown in Fig. 7.

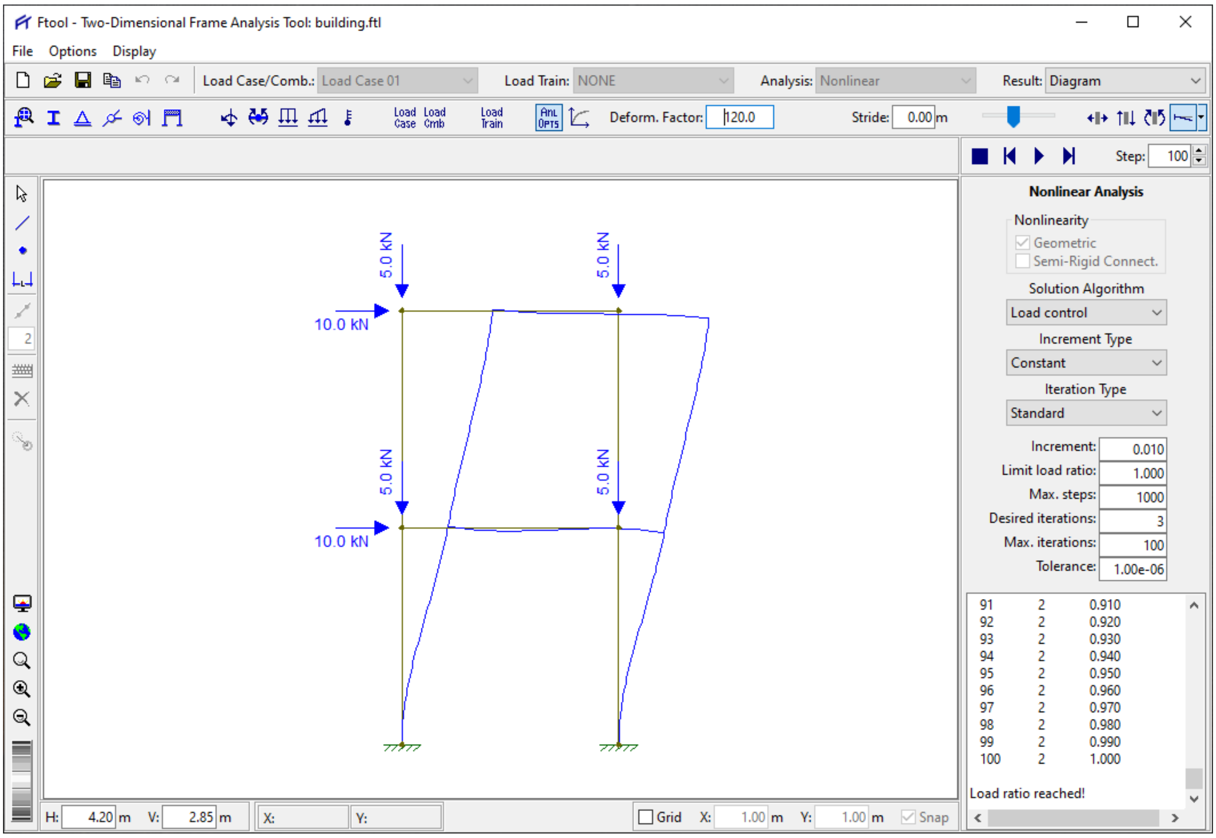


Figure 7. Interface of FTOOL: an educational software for linear and nonlinear analysis of 2D frame models

5 Practical example

The solution of the nonlinear system of equations that describe the equilibrium and constitutive behavior of the structural model must be done numerically, as no analytical solution is possible. The goal is usually to obtain the equilibrium configuration of the structure, expressed by the displacement of specific points, for different levels of applied load. The numerical methods used for this purpose are based on an incremental approach, where the load and/or displacements are incremented to calculate the next equilibrium configuration. The displacement computed for each equilibrium configuration is plotted against the respective load. The generated curve is called equilibrium path. It allows the analyst to predict the displacements of the structure with more accuracy. In addition, through the inspection of the equilibrium paths, it is possible to determine, for example, the critical load level that causes instability phenomena. This is indicated by critical points in the equilibrium paths, typically represented by a peak or a valley in the chart.

Figure 8 shows the analysis results for the frame model presented in the screenshot of Fig. 7. The result of the linear analysis is compared with the equilibrium path of a nonlinear analysis considering both geometric and material nonlinearities. The linear assumptions are very accurate for modeling the problem up to a load ratio of 0.1. At this point, the behavior of the structure presents a critical point where load increments are not supported and would lead to the collapse of the structure. This is only possible to be concluded from a nonlinear analysis, as the linear result suggests that the structure is resistant to any load level.

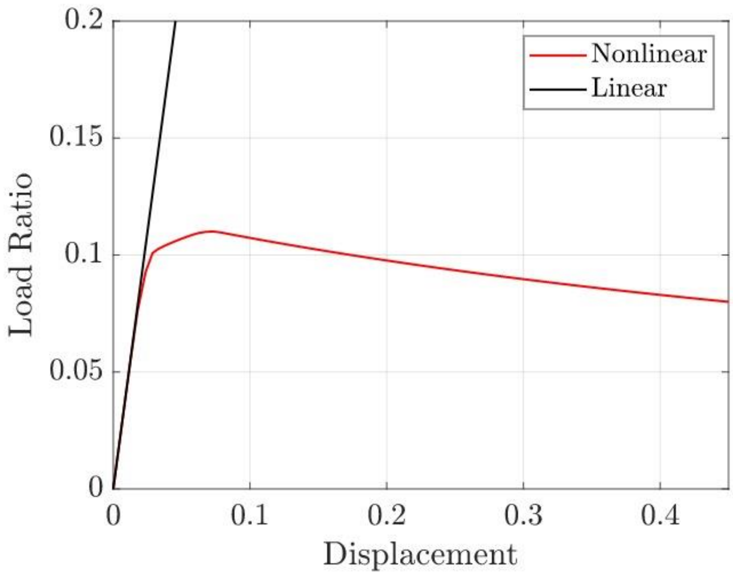


Figure 8. Comparison between linear and nonlinear behavior of a two-story frame model

6 Conclusions

The effects of the nonlinear behavior of frame models were shown to be crucial for designing safe structures. This type of structural analysis strongly rely on numerical methods to be performed. However, these methods are rarely covered in undergraduate engineering courses. Therefore, user-friendly computer programs are very important for learning this subject and obtain clear results that are easy to interpret.

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References

[1] M.A.C. Rodrigues, R.B. Burgos, R.L. Rangel, and L.F. Martha, "An Integrated Formulation to Predict Pre and Post-Critical Behavior of Frames". *Joint 42nd Ibero-Latin-American Congress on Computational Methods in Engineering (XLII CILAMCE) and III Pan-American Congress on Computational Mechanics (PANACM), Rio de Janeiro, RJ*, 2021.
[2] M.A.C. Rodrigues, P.H.A. Guimarães, R.B. Burgos, R.L. Rangel, and L.F. Martha, "Influence evaluation of high-order terms in the strain tensor for a complete geometric nonlinear analysis with Timoshenko element". 43rd *Ibero-Latin-American Congress on Computational Methods in Engineering (XLIII CILAMCE), Foz do Iguaçú, PR*, 2022.
[3] R.B. Burgos, L.F. Martha, M.A.C. Rodrigues, and R.L. Rangel, "Modelling of the P-δ effect using interpolating functions". *Joint 42nd Ibero-Latin-American Congress on Computational Methods in Engineering (XLII CILAMCE) and III Pan-American Congress on Computational Mechanics (PANACM), Rio de Janeiro, RJ*, 2021.
[4] D.B. Cavalcanti, R.L. Rangel, and L.F. Martha, "Nonlinear analysis of inelastic frames considering a corotational approach and plasticity by layers: a discussion about computational implementation". 43rd *Ibero-Latin-American Congress on Computational Methods in Engineering (XLIII CILAMCE), Foz do Iguaçú, PR*, 2022.
[5] R.L. Rangel. *Educational Tool for Structural Analysis of Plane Frame Models with Geometric Nonlinearity*. M.Sc. dissertation, Pontifical Catholic University of Rio de Janeiro, 2019.
[6] D.B. Cavalcanti, R.L. Rangel, and L.F. Martha, "Numerical experiments to assess the performance of different formulations and solution algorithms for geometrically nonlinear analysis of two-dimensional frames". 43rd *Ibero-Latin-American Congress on Computational Methods in Engineering (XLIII CILAMCE), Foz do Iguaçú, PR*, 2022.
[7] R.L. Rangel and L.F. Martha, "Programa para Análise Geometricamente Não Linear de Pórticos por Meio de um Controle Extensivo do Usuário". *40th Ibero-Latin-American Congress on Computational Methods in Engineering (XL CILAMCE), Natal, RN*, 2019.
[8] R.L. Rangel and L.F. Martha, "Educational Tool for the Analysis of Structures with Geometric Nonlinearity". *39th Ibero-Latin-American Congress on Computational Methods in Engineering (XXXIX CILAMCE), Paris/Compiègne, France, pp. 472-473*, 2018.
[9] R.L. Rangel and L.F. Martha, "Implementation of a User-Controlled Structural Analysis Module with Geometric Nonlinearity". *25th International Congress of Mechanical Engineering (COBEM), Uberlândia, MG*, 2019.
[10] L.F. Martha and R.L. Rangel, "FTOOL: Three decades of success as an educational program for structural analysis". 43rd *Ibero-Latin-American Congress on Computational Methods in Engineering (XLIII CILAMCE), Foz do Iguaçú, PR*, 2022.
[11] R.L. Rangel, P.C. Lopes, and L.F. Martha, "A graphical tool for structural analysis of linear element models with an OOP approach". *Workshop on Computational Modeling and Numerical Analysis (WCMNA), Petrópolis, Brazil*, 2019.
[12] R.L. Rangel and L.F. Martha, "Ftool 5.0: Nonlinear, stability and natural vibration analyses". 41st *Ibero-Latin-American Congress on Computational Methods in Engineering (XLI CILAMCE), Foz do Iguaçú, PR*, 2020.
[13] C.L. Dias, R.L. Rangel, and L.F. Martha, "Nonlinear analysis in FTOOL with semi-rigid connections: a partial development". 41st *Ibero-Latin-American Congress on Computational Methods in Engineering (XLI CILAMCE), Foz do Iguaçú, PR*, 2020.
[14] C.L. Dias, R.L. Rangel, and L.F. Martha, "Educational tool for analysis of steel frames with semi-rigid connections". *Joint 42nd Ibero-Latin-American Congress on Computational Methods in Engineering (XLII CILAMCE) and III Pan-American Congress on Computational Mechanics (PANACM), Rio de Janeiro, RJ*, 2021.
[15] I.R. Marques, P.C. Lopes, R.L. Rangel, and L.F. Martha, "Implementação de conexão semirrígida em modelos reticulados no contexto da programação orientada a objetos". *40th Ibero-Latin-American Congress on Computational Methods in Engineering (XL CILAMCE), Natal, RN*, 2019.
[16] P.C. Lopes, R.L. Rangel, and L.F. Martha, "Development of a graphical application for the dynamic analysis of linear elements structural models". *25th International Congress of Mechanical Engineering (COBEM), Uberlândia, MG*, 2019.
[17] P.C. Lopes, R.L. Rangel, and L.F. Martha, "Extensão de programa gráfico para análise de vibrações de modelos estruturais reticulados". *40th Ibero-Latin-American Congress on Computational Methods in Engineering (XL CILAMCE), Natal, RN*, 2019.
[18] C.H.B. Resende, R.L. Rangel, P.C.F. Lopes, L.F. Rangel, "An OOP architecture for the implementation of dynamic and nonlinear effects in structural analysis". *14th World Congress in Computational Mechanics (WCCM) and ECCOMAS Congress*, Paris, France, pp. 2634, 2020
[19] R.L. Rangel. *Development of a graphic program for structural analysis of linear element models*. B.Sc. dissertation, Pontifical Catholic University of Rio de Janeiro, 2016.
[20] P.C. Lopes, R.L. Rangel, and L.F. Martha, "An interactive user interface for a structural analysis software using computer graphics techniques in MATLAB". *Computer Applications in Engineering Education*, vol. 29, n. 6, pp. 1505–1525, 2021.
[21] R.L. Rangel and L.F. Martha, "LESM—An object-oriented MATLAB program for structural analysis of linear element models". *Computer Applications in Engineering Education*, vol. 27, n. 3, pp. 553–571, 2019.
[22] P.C. Lopes, R.L. Rangel, and L.F. Martha, "An interactive-graphics tool for modeling and analyzing structures in MATLAB". 3*9th Ibero-Latin-American Congress on Computational Methods in Engineering (XXXIX CILAMCE), Paris/Compiègne, France, pp. 472-473*, 2018.
[23] R.J.L. Assunção, R.L. Rangel, P.C. Lopes, A.C.J. Heitor, and L.F. Martha, "Implementação da norma brasileira de aço em um programa MATLAB orientado a objetos". 41st *Ibero-Latin-American Congress on Computational Methods in Engineering (XLI CILAMCE), Foz do Iguaçú, PR*, 2020.
[24] Ferguson Structural Engineering Laboratory, "Time-dependent buckling of steel columns exposed to elevated temperatures due to fire": http://fsel.engr.utexas.edu/research/spotlight/293-time-dependentbuckling.
[25] J. Pacheco, J. Brito, J. Ferreira, and D. Soares, "Destructive horizontal load tests of full-scale recycled-aggregate concrete structures". *ACI Structural Journal*, vol. 112, n. 6, 2015.