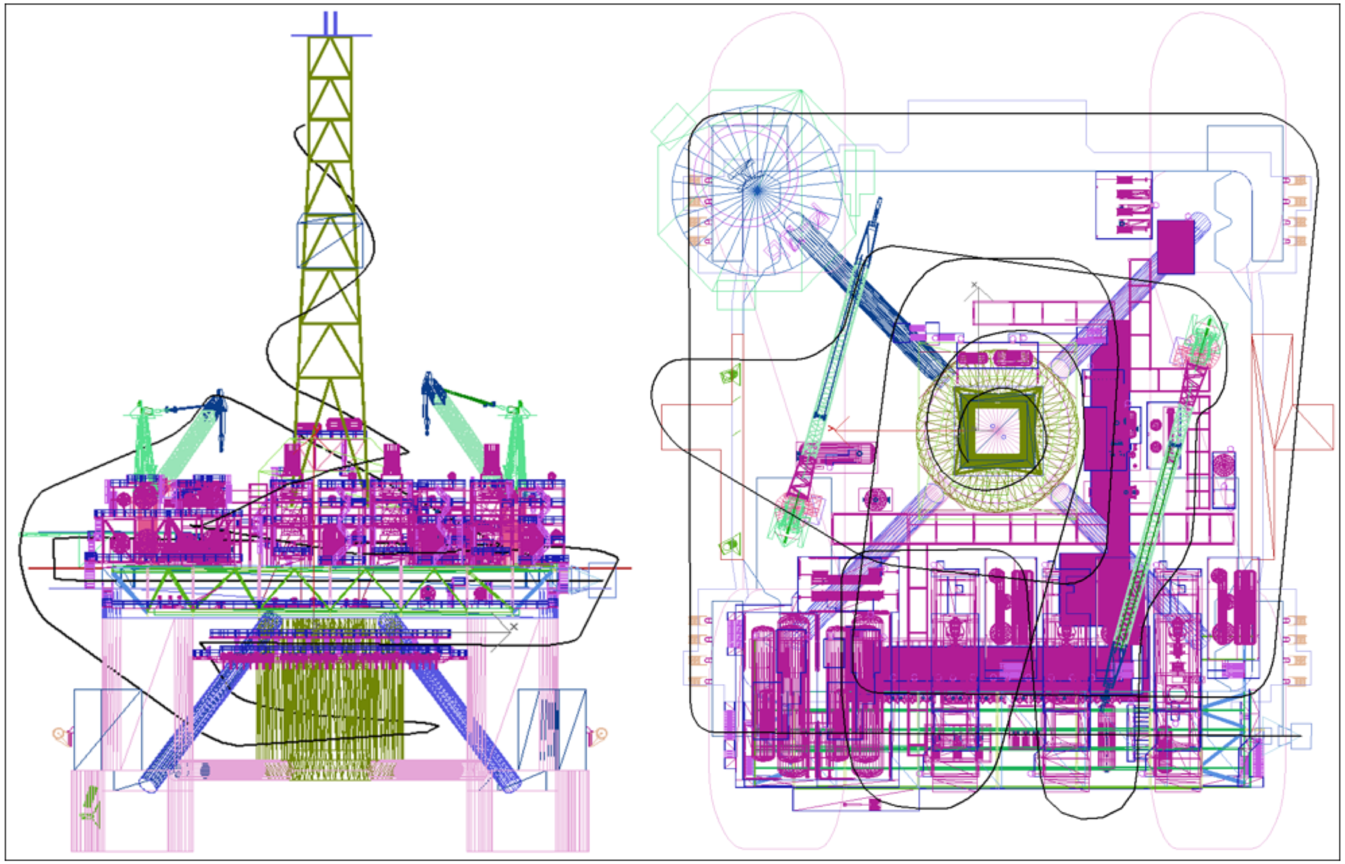
Instanciação de Cadeias Cinemáticas e Rotações no Espaço

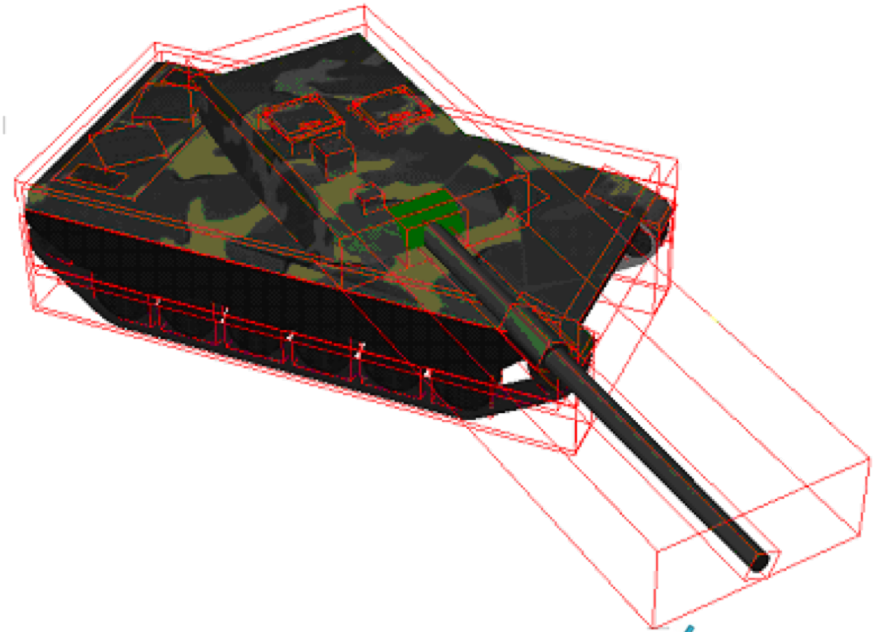
Coordenadas Locais e Globais

Quatérnios

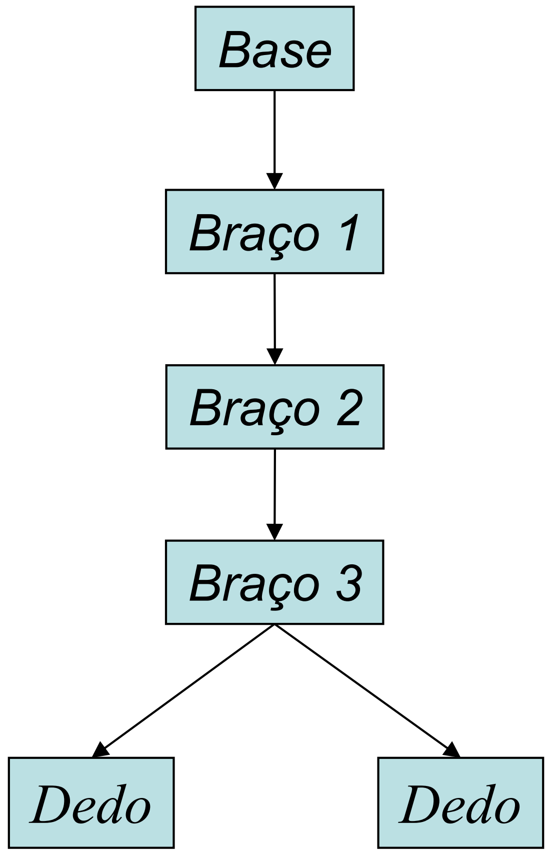
Motivação: representação de movimentos e formas

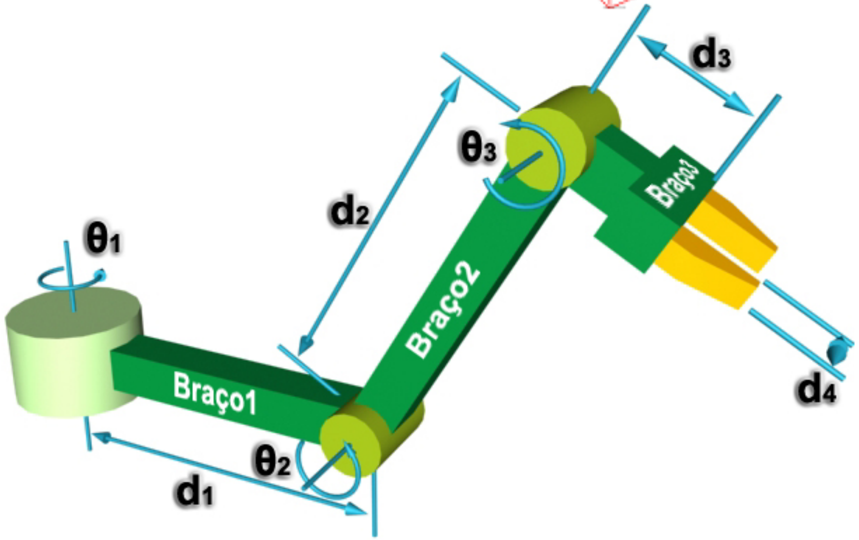


Objetos compostos hierarquicamente



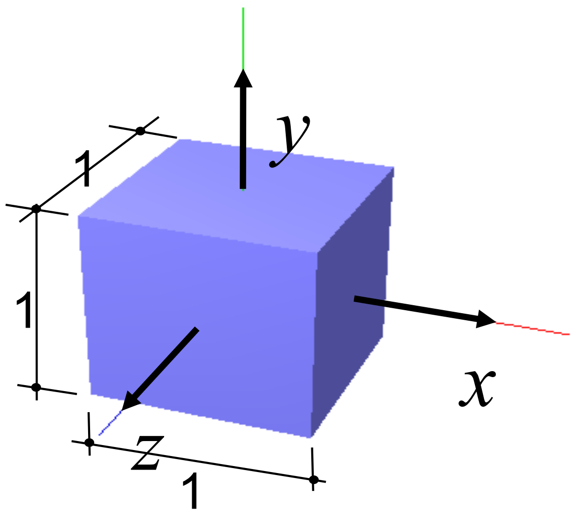
Hieraquia de movimentos

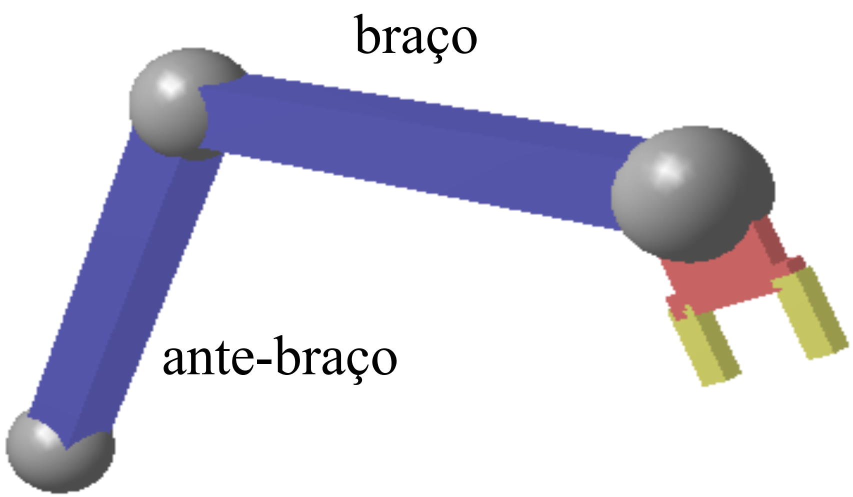




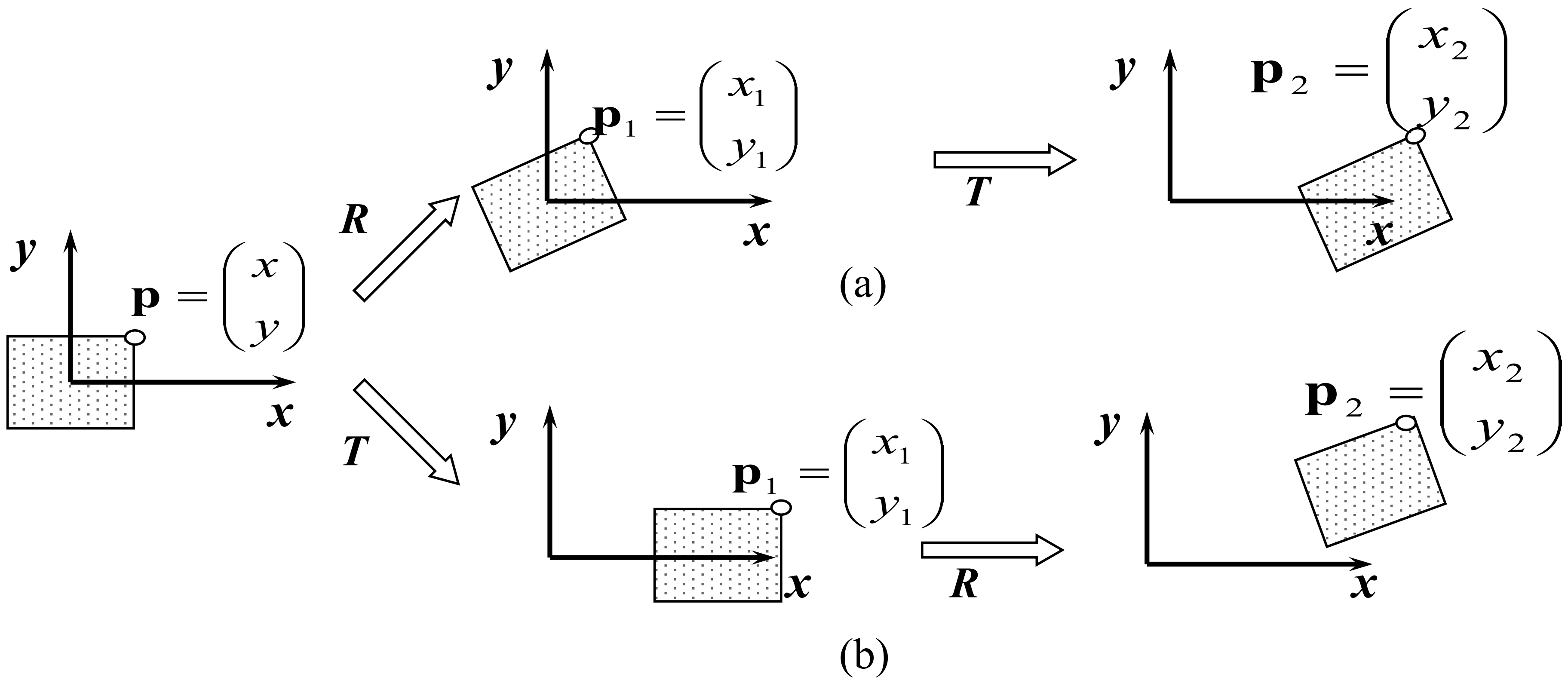
Hieraquia de transformações

Instanciação de objetos em cadeias cinemáticas

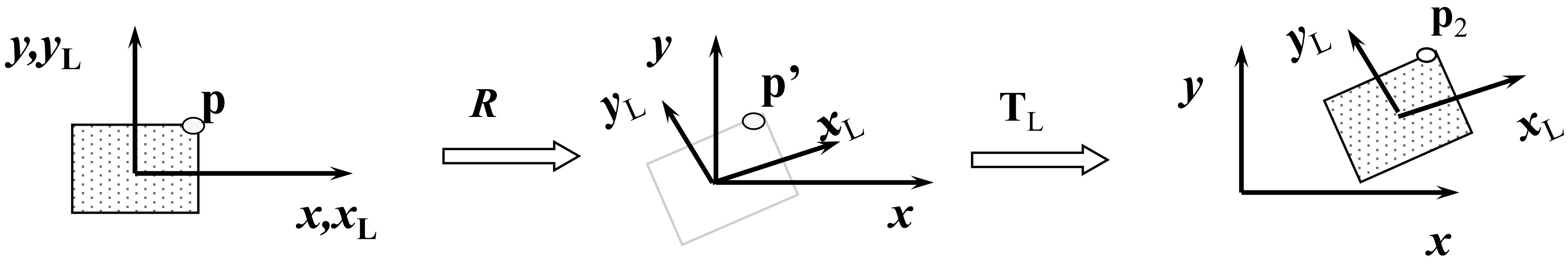




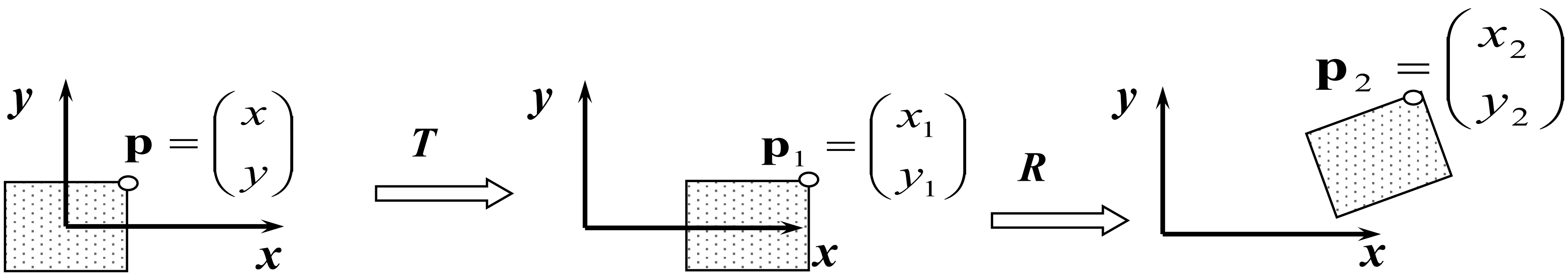
Ordem das transformações



Composição com sistema local móvel

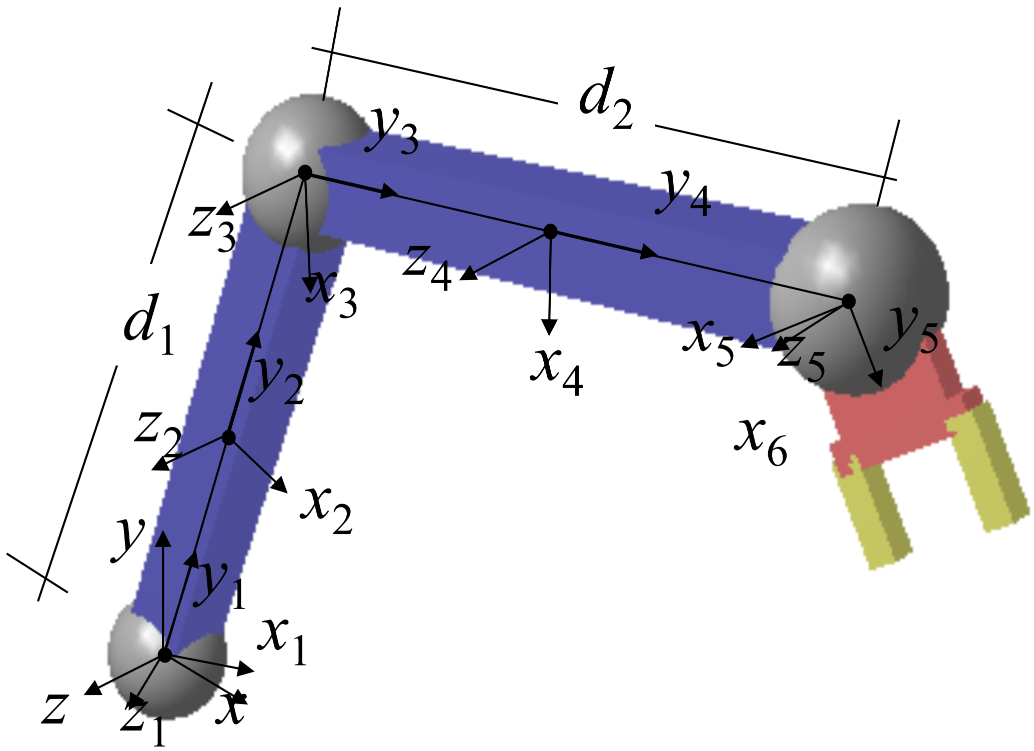


$p' = R\,p$ e $p_2 = T_L\,p'$ | $\Rightarrow p_2 = R\,T\,R^{-1}\,R\,p$ | ou | $p_2 = R\,T\,p$

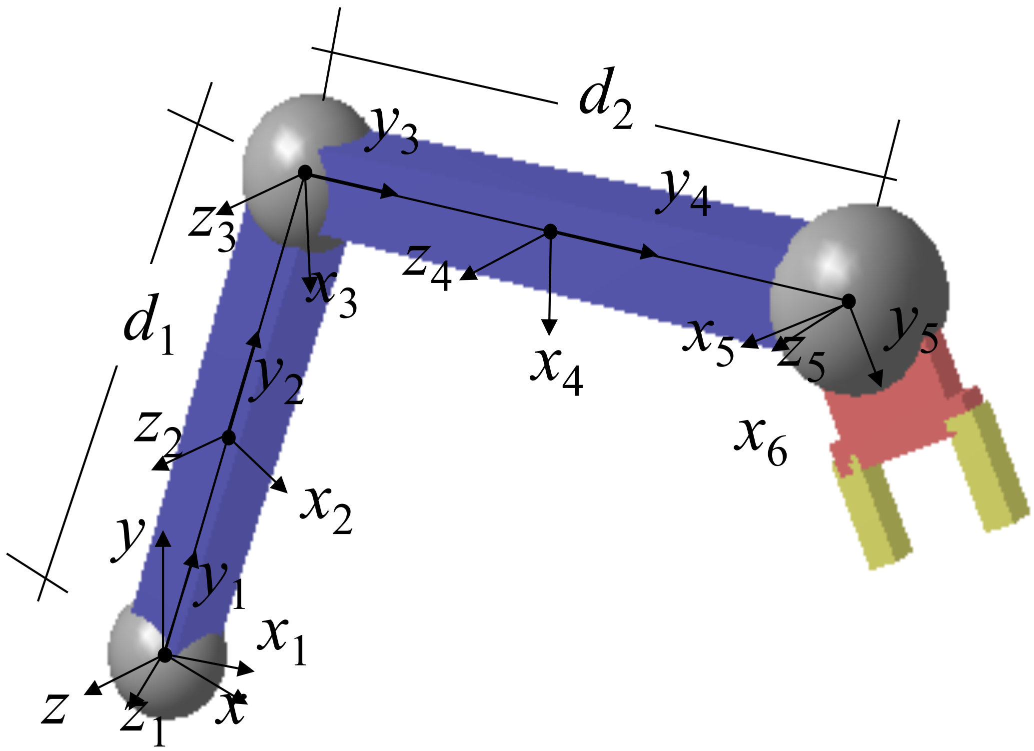


$p_1 = T\,p$ e $p_2 = R\,p_1$ $\Rightarrow$ $p_2 = R\,T\,p$

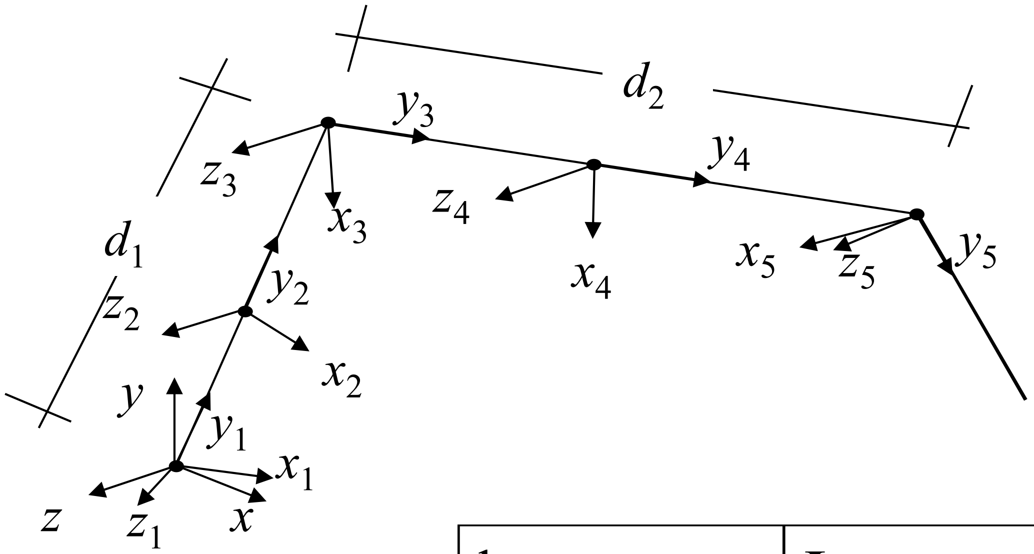
Instâncias de objetos com sistemas locais



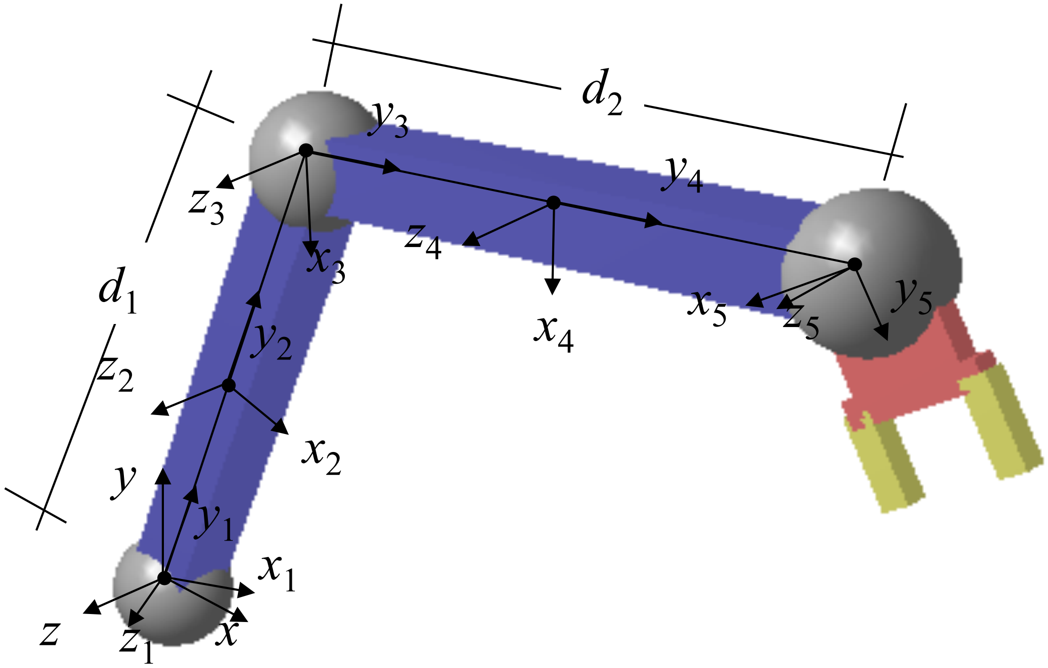
Instâncias de objetos com sistemas locais



Matrizes para desenho em cada sistema

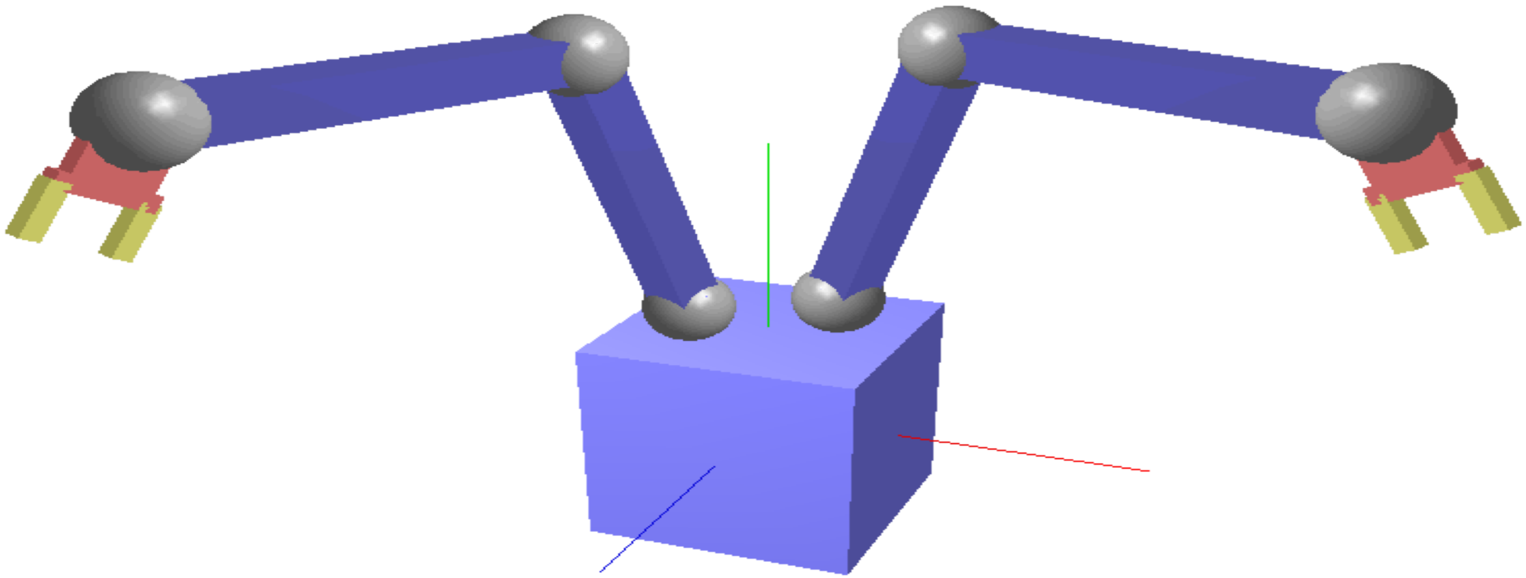


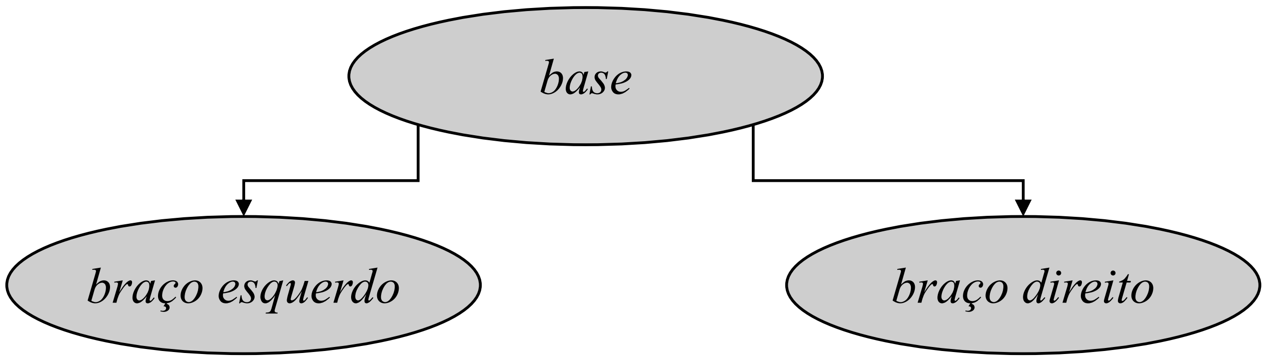
base	$\mathbf{I}$
ante-braço	$\mathbf{R_y\, R_{z1} T_{y1}}$
cotovelo	$\mathbf{R_y\, R_{z1} T_{y1} T_{y1}}$
braço	$\mathbf{R_y\, R_{z1} T_{y1} T_{y1}\; R_{z3} T_{y3}}$
pulso	$\mathbf{R_y\, R_{z1} T_{y1} T_{y1}\; R_{z3} T_{y3}\; T_{y3}}$
mão	$\mathbf{R_y\, R_{z1} T_{y1} T_{y1}\; R_{z3} T_{y3}\; T_{y3}\; R_{z5}}$



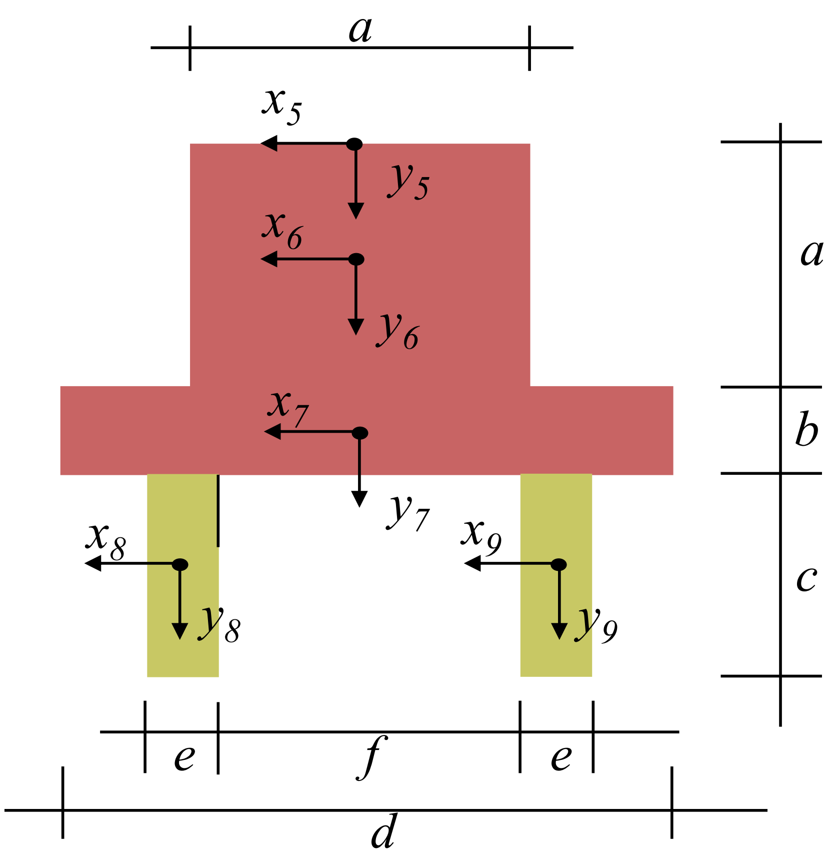

```
Desenha a base;
Roda em y;
Roda em z₁;
Translada em y₁ de d₁/2;
Desenha o ante-braço;
Translada em y₂ de d₁/2;
Desenha cotovelo;
Roda em z₃;
Translada em y₃ de d₂/2;
Desenha o braço;
Translada em y₃ de d₂/2;
Desenha o pulso;
Roda em z₅;
Desenha a mão;
```

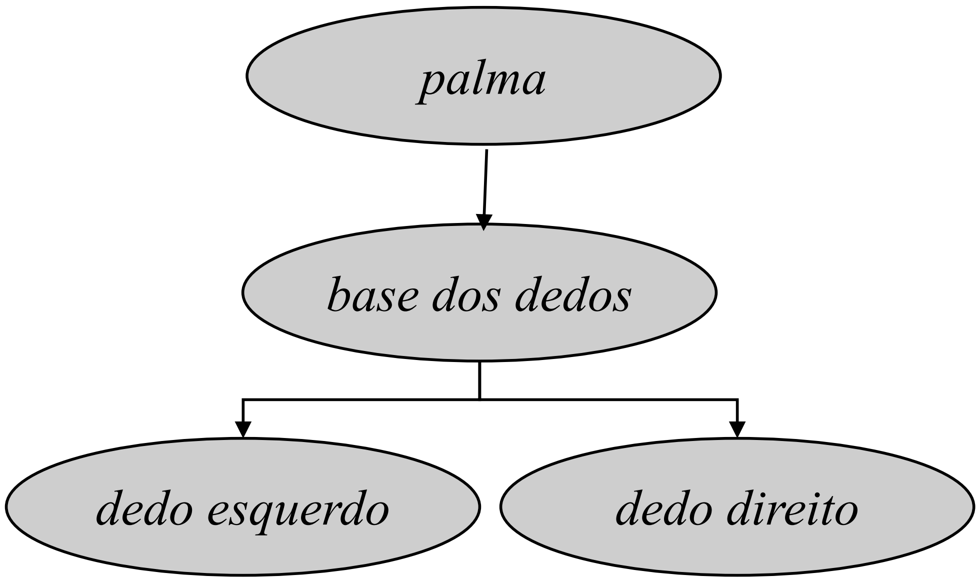
Hierarquia em árvore

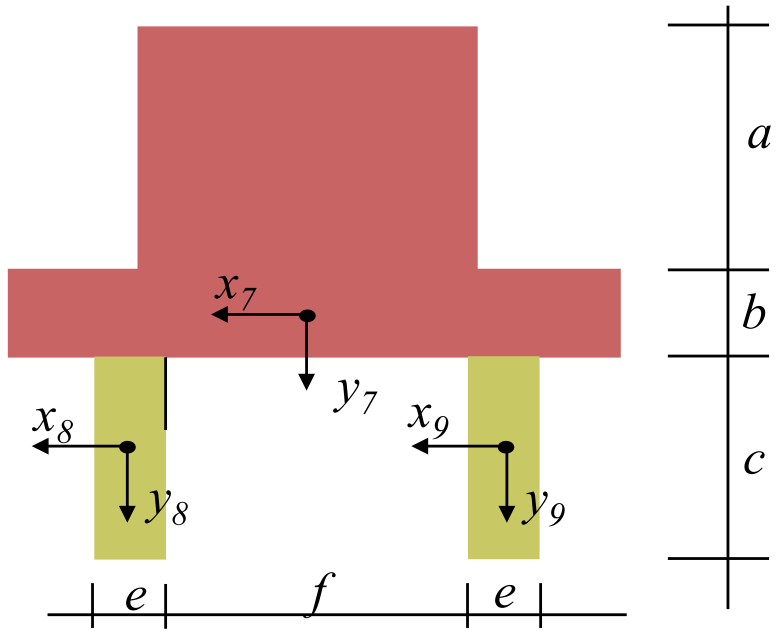




Hierarquia em árvore



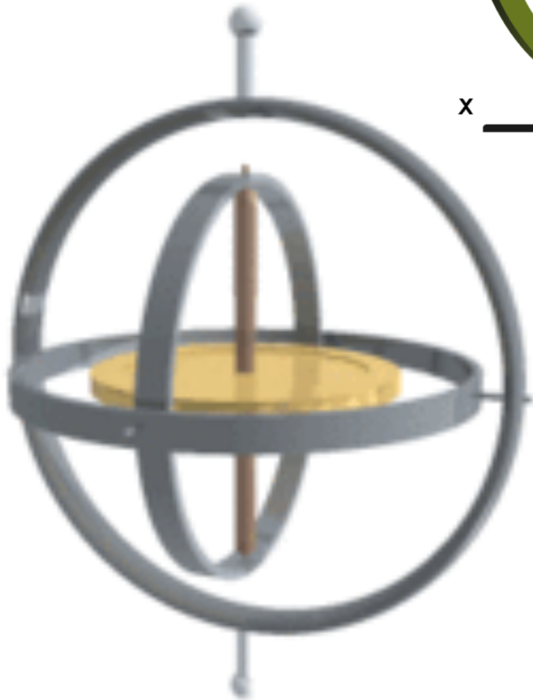


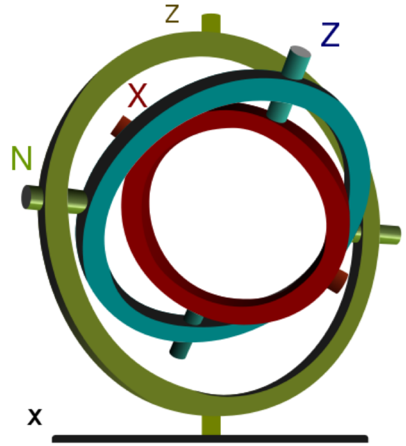



```
void desenhaDedos(float b,float c, float e, float f )
{
  /* dedo esquerdo */
  glPushMatrix();      /* Salva matriz corrente C0 */
    glTranslatef((f+e)/2,(b+c)/2,0.);   /* C=CTesq   */
    glScalef(e,c,e);                    /* C=CS      */
    glutSolidCube(1.0);
  glPopMatrix();        /* Recupera da pilha C=C0   */
  /* dedo direito */
  glPushMatrix();      /* Salva matriz corrente C0 */
    glTranslatef(-(f+e)/2,(b+c)/2,0.);   /* C=CTdir  */
    glScalef(e,c,e);                    /* C=CS  */
    glutSolidCube(1.0);
  glPopMatrix();        /* Recupera da pilha C=C0  */
}
```

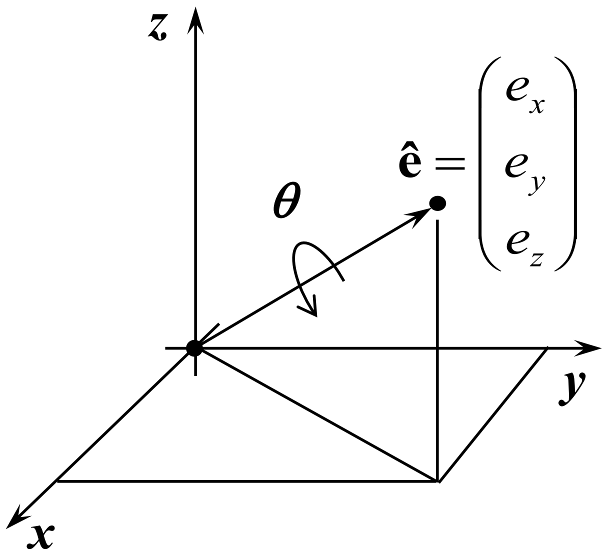
Complexidade da Rotação 3D

Gyroscope invented by Léon Foucault in 1852





Transformações em 3D
(rotação em torno de um eixo qualquer)



$$
\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} & 0 \\ m_{21} & m_{22} & m_{23} & 0 \\ m_{31} & m_{32} & m_{33} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}
$$

$$m_{11} = e_x^2 + \cos\theta\ (1 - e_x^2)$$

$$m_{12} = e_x e_y (1 - \cos\theta) - e_z\, sen\ \theta$$

$$m_{13} = e_z e_x (1 - \cos\theta) + e_y\, sen\ \theta$$

$$m_{21} = e_x e_y (1 - \cos\theta) + e_z\, sen\ \theta$$

$$m_{22} = e_y^2 + \cos\theta\ (1 - e_y^2)$$

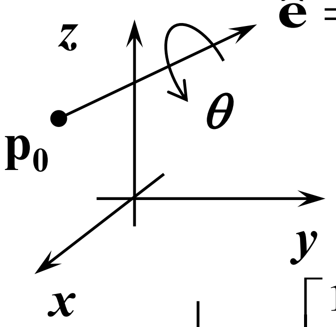
$$m_{23} = e_y e_z (1 - \cos\theta) - e_x\, sen\ \theta$$

$$m_{31} = e_x e_z\ (1 - \cos\theta) - e_y\, sen\ \theta$$

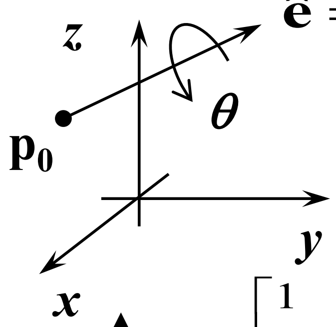
$$m_{32} = e_y e_z (1 - \cos\theta) + e_x\, sen\ \theta$$

$$m_{22} = e_z^2 + \cos\theta\ (1 - e_z^2)$$

Matriz de rotação em torno de um eixo ê que não passa pela origem



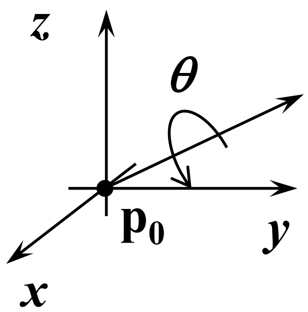
$$\hat{\mathbf{e}} = \begin{pmatrix} e_x \\ e_y \\ e_z \end{pmatrix}$$



$$\hat{\mathbf{e}} = \begin{pmatrix} e_x \\ e_y \\ e_z \end{pmatrix}$$

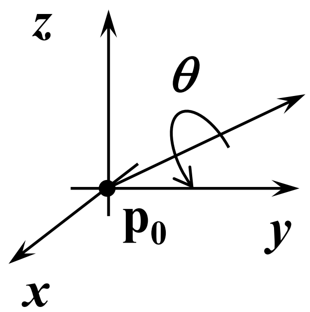
$$\mathbf{T} = \begin{bmatrix} 1 & 0 & 0 & -x_0 \\ 0 & 1 & 0 & -y_0 \\ 0 & 0 & 1 & -z_0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{T}^{-1} = \begin{bmatrix} 1 & 0 & 0 & x_0 \\ 0 & 1 & 0 & y_0 \\ 0 & 0 & 1 & z_0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

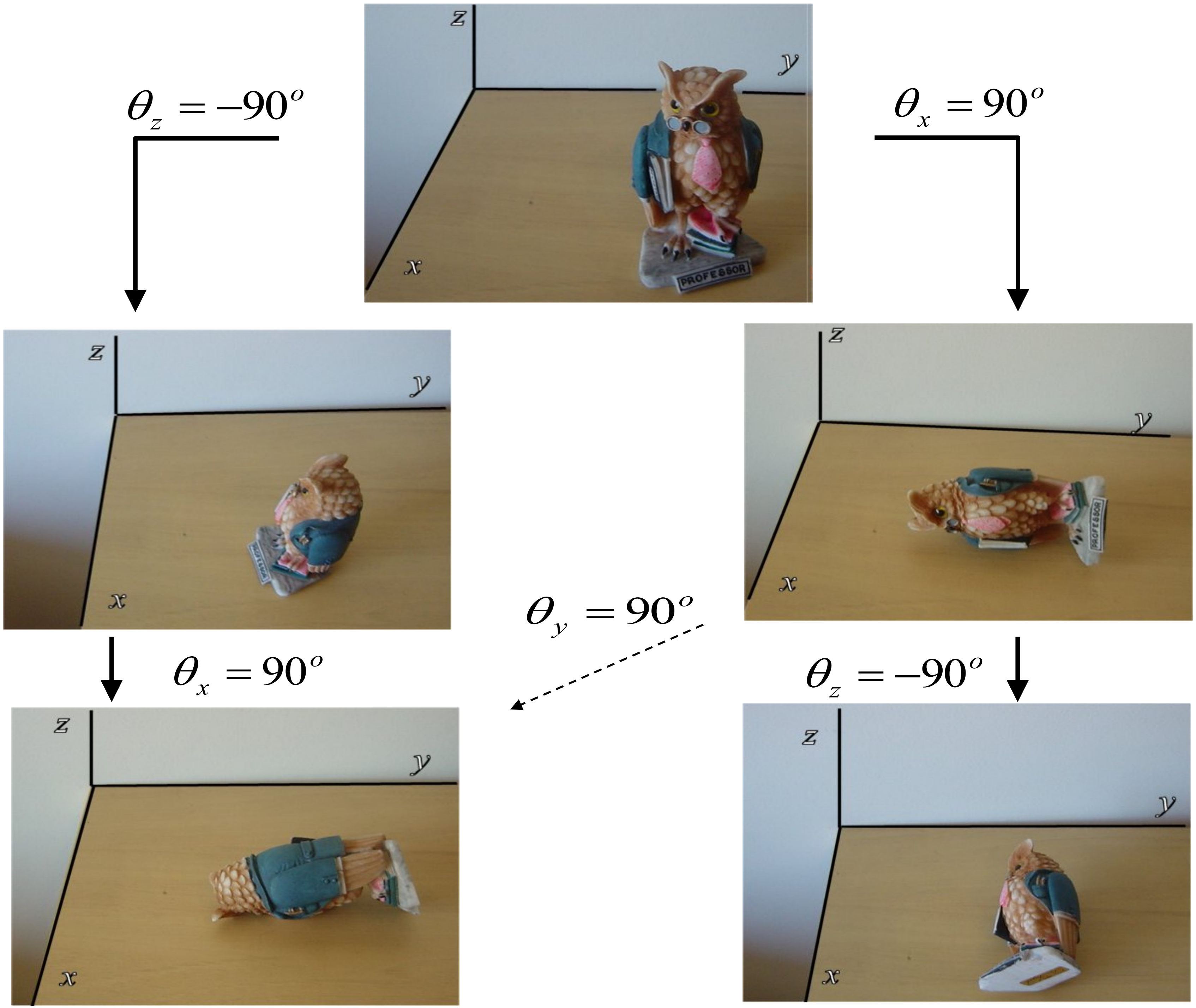


$$\hat{\mathbf{e}} = \begin{pmatrix} e_x \\ e_y \\ e_z \end{pmatrix}$$

M →

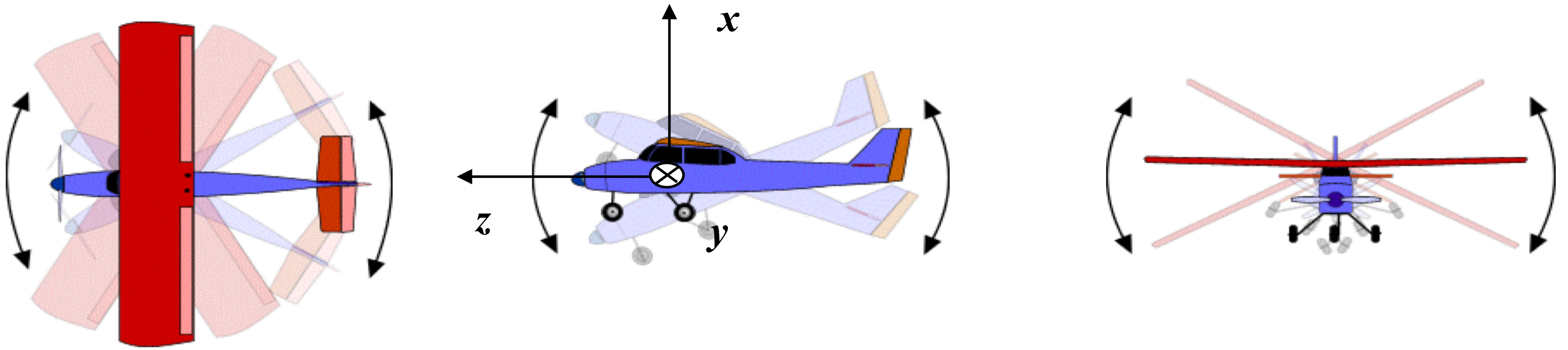


$$\hat{\mathbf{e}} = \begin{pmatrix} e_x \\ e_y \\ e_z \end{pmatrix}$$



$\theta_z = -90^o$
$\theta_x = 90^o$
$\theta_y = 90^o$
$\theta_x = 90^o$
$\theta_z = -90^o$
z
y
x
PROFESSOR

Yaw-Pitch-Rol



$$R_\varphi = \begin{bmatrix} \cos\varphi & \sin\varphi & 0 \\ -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad R_\theta = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \qquad R_\psi = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\psi & \sin\psi \\ 0 & -\sin\psi & \cos\psi \end{bmatrix}$$

$$R = \begin{bmatrix} \cos\theta\cos\varphi & \cos\theta\sin\varphi & -\sin\theta \\ \sin\psi\sin\theta\cos\varphi - \cos\psi\sin\varphi & \sin\psi\sin\theta\sin\varphi + \cos\psi\cos\varphi & \cos\theta\sin\psi \\ \cos\psi\sin\theta\cos\varphi + \sin\psi\sin\varphi & \cos\psi\sin\theta\sin\varphi - \sin\psi\cos\varphi & \cos\theta\cos\psi \end{bmatrix}$$

Ângulos de Euler

- **Transforma *x-y-z* em *x'-y'-z'* em 3 passos**

$$(x, y, z)$$ $$\mathbf{x}$$

Rotação de ϕ em torno eixo z

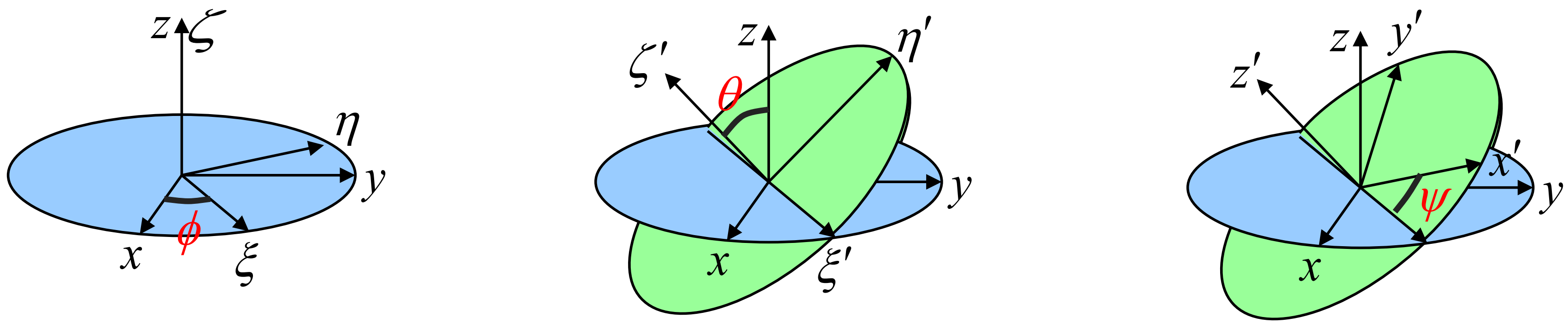
$$(\xi, \eta, \zeta)$$ $$\mathbf{Dx}$$

Rotação de θ em torno do eixo ξ

$$(\xi', \eta', \zeta')$$ $$\mathbf{CDx}$$

Rotação de ψ em torno do eixo ζ'

$$(x', y', z')$$ $$\mathbf{Ax} = \mathbf{BCDx}$$



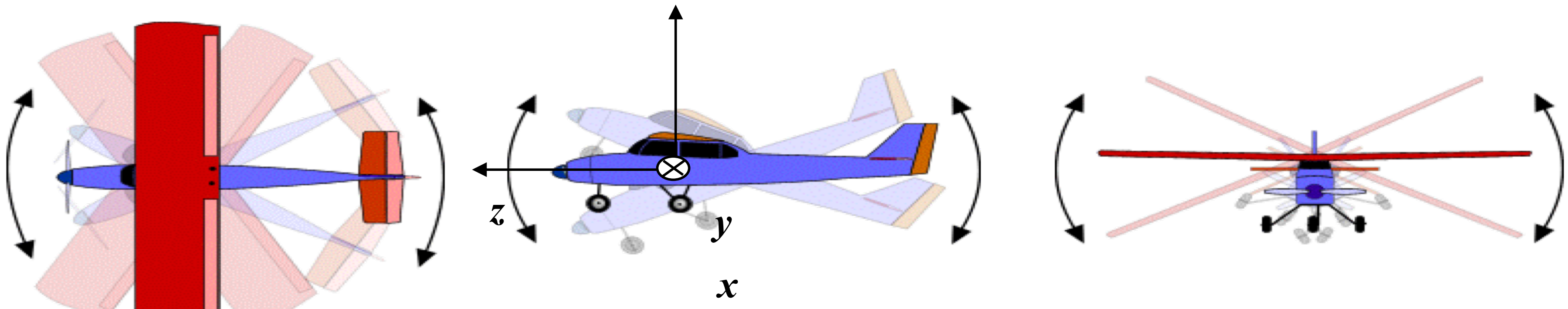
Ângulos de Euler

- **Transforma *x-y-z* em *x'-y'-z'* em 3 passos**

(x, y, z)		$\mathbf{x}$
	Rotação de ϕ em torno eixo z	
(ξ, η, ζ)		$\mathbf{Dx}$
	Rotação de θ em torno do eixo ξ	
(ξ', η', ζ')		$\mathbf{CDx}$
	Rotação de ψ em torno do eixo ζ'	
(x', y', z')		$\mathbf{Ax} = \mathbf{BCDx}$

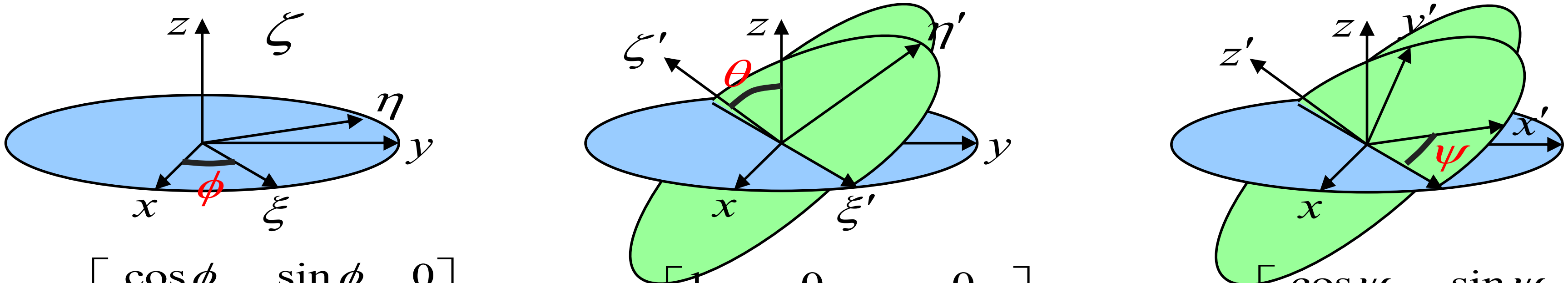
$$\mathbf{D} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \mathbf{C} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{A} = \begin{bmatrix} \cos\psi\cos\phi - \cos\theta\sin\phi\sin\psi & \cos\psi\sin\phi + \cos\theta\cos\phi\sin\psi & \sin\psi\sin\theta \\ -\sin\psi\cos\phi - \cos\theta\sin\phi\cos\psi & -\sin\psi\sin\phi + \cos\theta\cos\phi\cos\psi & \cos\psi\sin\theta \\ \sin\theta\sin\phi & -\sin\theta\cos\phi & \cos\theta \end{bmatrix}$$



$$R_\varphi = \begin{bmatrix} \cos\varphi & \sin\varphi & 0 \\ -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad R_\theta = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \qquad R_\psi = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\psi & \sin\psi \\ 0 & -\sin\psi & \cos\psi \end{bmatrix}$$

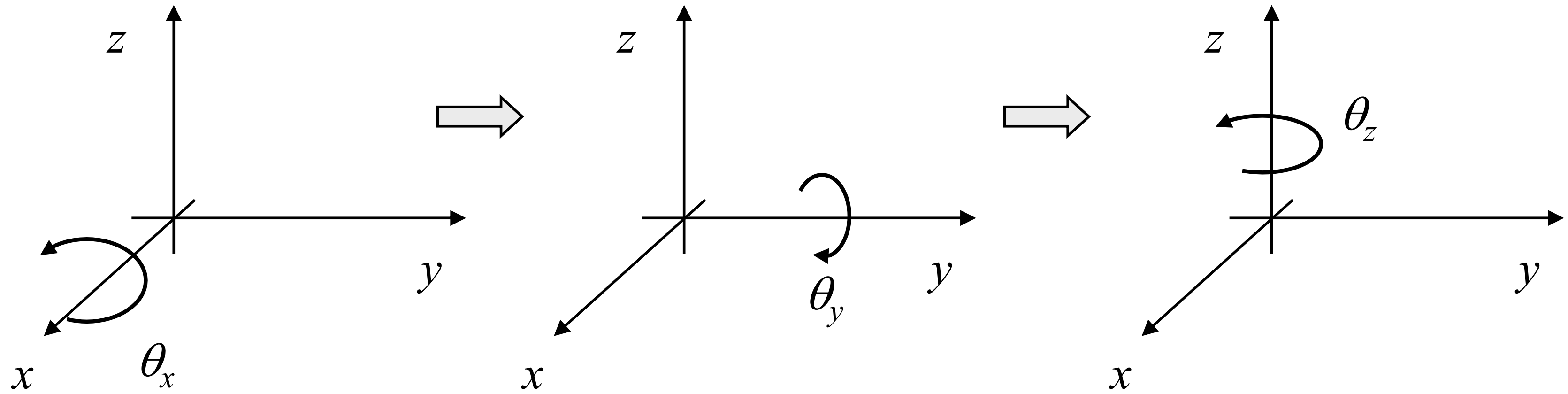
$$R = \begin{bmatrix} \cos\theta\cos\varphi & \cos\theta\sin\varphi & -\sin\theta \\ \sin\psi\sin\theta\cos\varphi - \cos\psi\sin\varphi & \sin\psi\sin\theta\sin\varphi + \cos\psi\cos\varphi & \cos\theta\sin\psi \\ \cos\psi\sin\theta\cos\varphi + \sin\psi\sin\varphi & \cos\psi\sin\theta\sin\varphi - \sin\psi\cos\varphi & \cos\theta\cos\psi \end{bmatrix}$$



$$\mathbf{D} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \mathbf{C} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

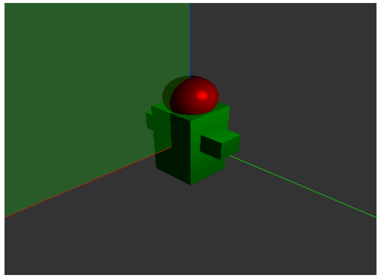
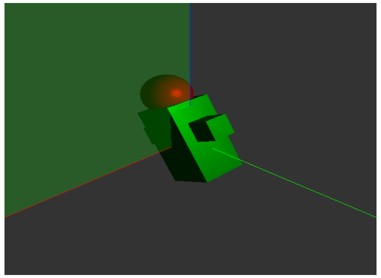
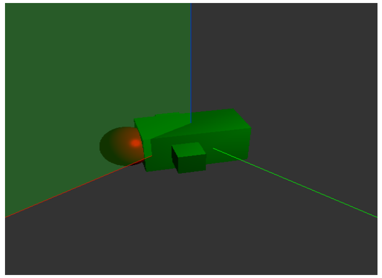
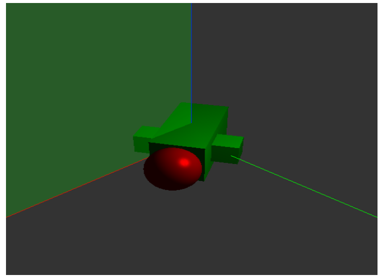
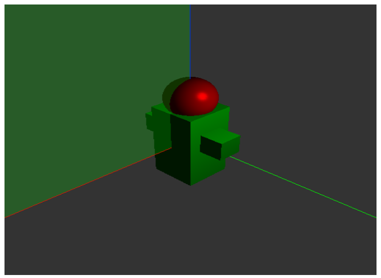
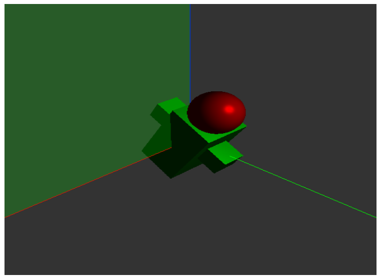
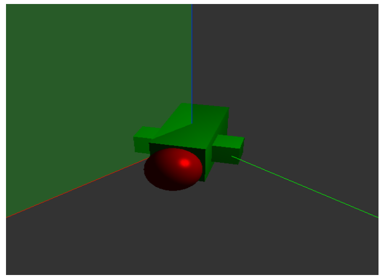
$$\mathbf{A} = \begin{bmatrix} \cos\psi\cos\phi - \cos\theta\sin\phi\sin\psi & \cos\psi\sin\phi + \cos\theta\cos\phi\sin\psi & \sin\psi\sin\theta \\ -\sin\psi\cos\phi - \cos\theta\sin\phi\cos\psi & -\sin\psi\sin\phi + \cos\theta\cos\phi\cos\psi & \cos\psi\sin\theta \\ \sin\theta\sin\phi & -\sin\theta\cos\phi & \cos\theta \end{bmatrix}$$

Parametrização de rotações: Ângulos de Euler

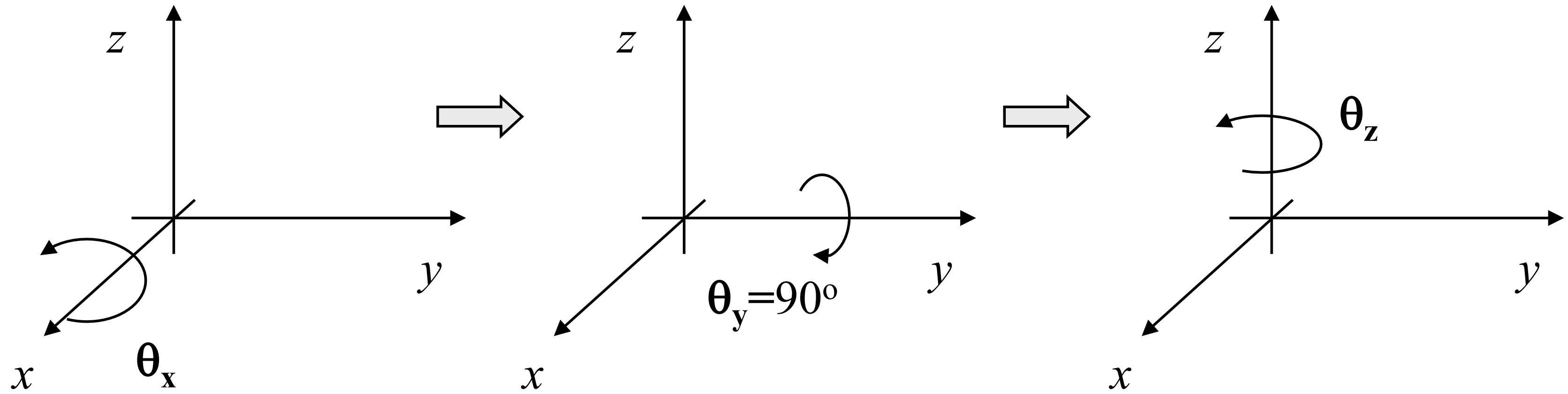


$$\mathbf{R}(\theta_x, \theta_y, \theta_z) = \begin{bmatrix} c_y c_z & c_y s_z & -s_y & 0 \\ s_x s_y c_z - c_x s_z & s_x s_y s_z + c_x c_z & s_x c_y & 0 \\ c_x s_y c_z + s_x s_z & c_x s_y s_z - s_x c_z & c_x c_y & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Ângulos de Euler
Gimbal lock

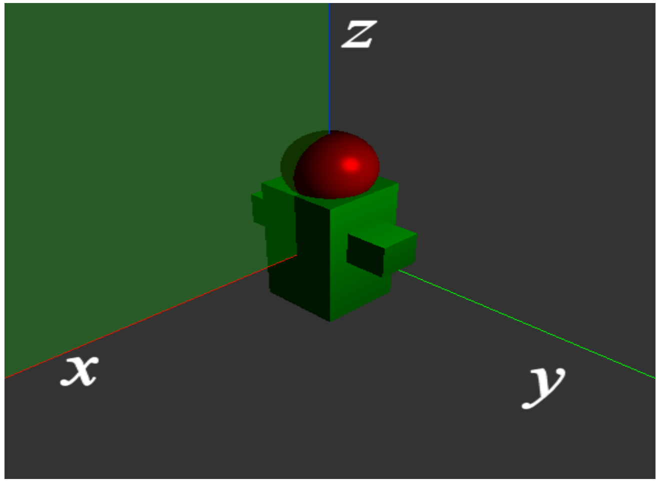
Ângulos de Euler
Gimbal lock

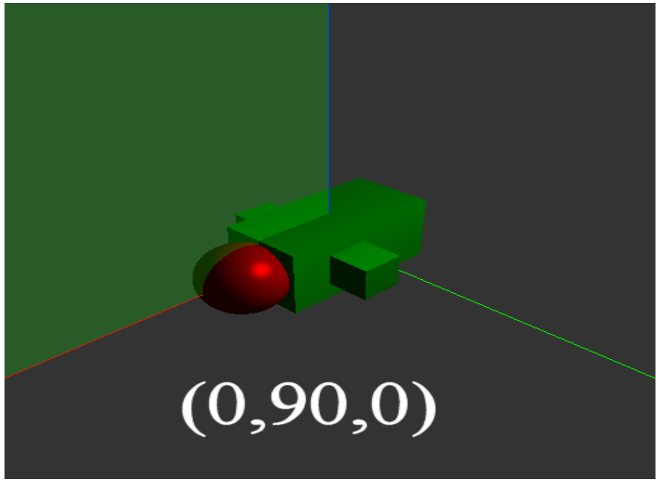


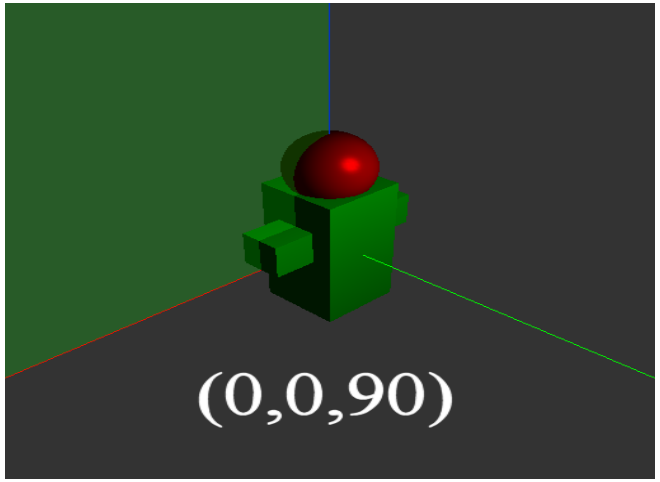
$$\mathbf{R}(\theta_x,\theta_y,\theta_z)=\begin{bmatrix} c_y c_z & c_y s_z & -s_y & 0 \\ s_x s_y c_z - c_x s_z & s_x s_y s_z + c_x c_z & s_x c_y & 0 \\ c_x s_y c_z + s_x s_z & c_x s_y s_z - s_x c_z & c_x c_y & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

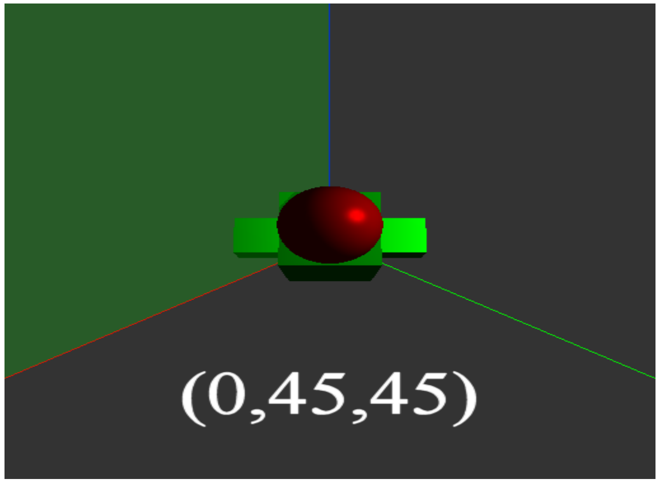
$$\mathbf{R}(\theta_x,90^o,\theta_z)=\begin{bmatrix} 0 & 0 & -1 & 0 \\ s_x c_z - c_x s_z & s_x s_z + c_x c_z & 0 & 0 \\ c_x c_z + s_x s_z & c_x s_z - s_x c_z & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 & 0 \\ \sin(\theta_x-\theta_z) & \cos(\theta_x-\theta_z) & 0 & 0 \\ \cos(\theta_x-\theta_z) & \sin(\theta_x-\theta_z) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Interpolação não gera posições "entre"

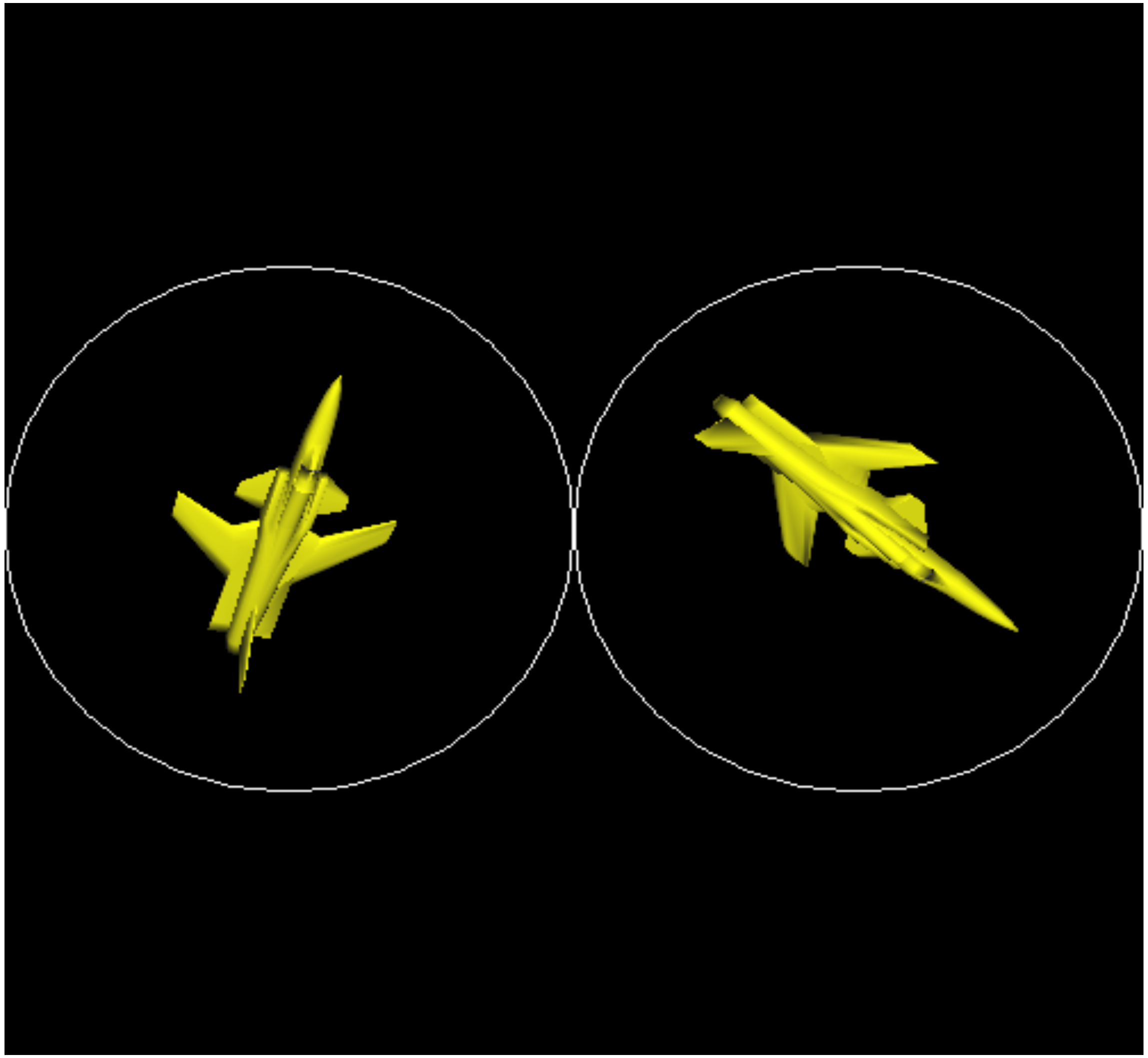




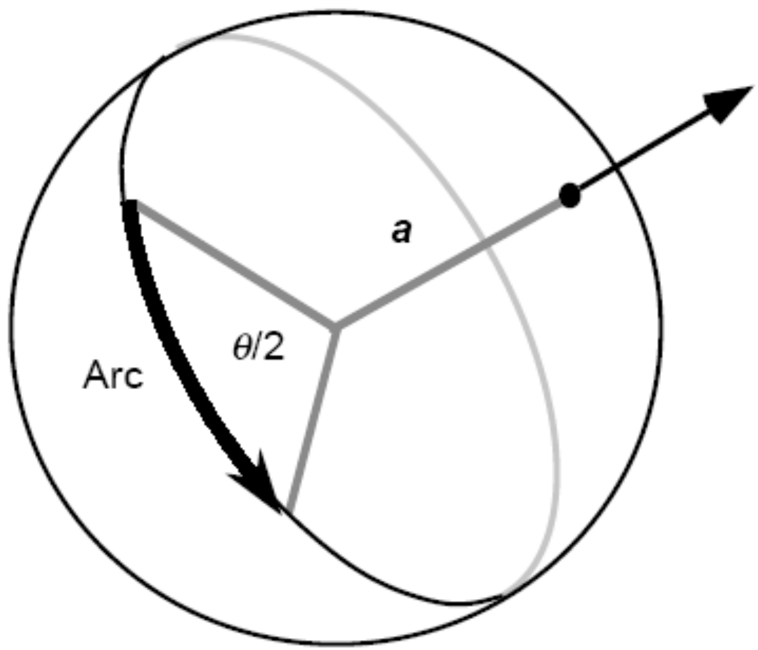




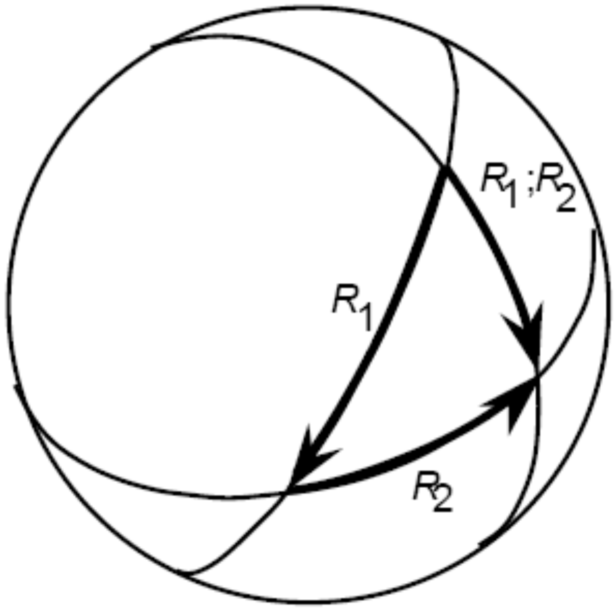
Interface para rotações tipo ArcBall



Rotação do ArcBall

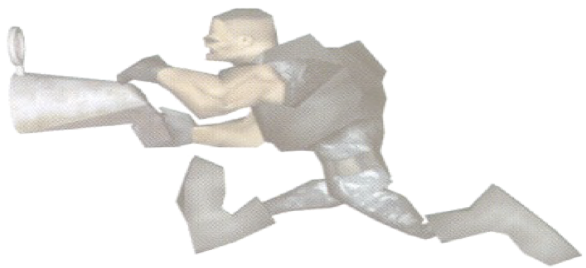


Conservativo



Quatérnios

$$\underline{\mathbf{q}} = s + x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$$



$$\underline{\mathbf{q}} = (s, \mathbf{v})$$

Figure 13.2
An example of manual animation of a character. (Courtesy of Consequência Animação)

Soma e multiplicação por escalar

$$\underline{\mathbf{q}}_1 + \underline{\mathbf{q}}_2 = (s_1, \mathbf{v}_1) + (s_2, \mathbf{v}_2) = (s_1 + s_2, \mathbf{v}_1 + \mathbf{v}_2)$$

$$\underline{a\mathbf{q}} = a(s, \mathbf{v}) = (as, a\mathbf{v})$$

Produto de dois quatérnios

$$\underline{\mathbf{q}}_1\underline{\mathbf{q}}_2 = (s_1 + x_1\mathbf{i} + y_1\mathbf{j} + z_1\mathbf{k})(s_2 + x_2\mathbf{i} + y_2\mathbf{j} + z_2\mathbf{k})$$

$$\begin{aligned}\underline{\mathbf{q}}_1\underline{\mathbf{q}}_2 &= s_1 s_2 + s_1 x_2\mathbf{i} + s_1 y_2\mathbf{j} + s_1 z_2\mathbf{k} \\ &+ x_1 s_2\mathbf{i} + x_1 x_2\mathbf{ii} + x_1 y_2\mathbf{ij} + x_1 z_2\mathbf{ik} \\ &+ y_1 s_2\mathbf{j} + y_1 x_2\mathbf{ji} + y_1 y_2\mathbf{jj} + y_1 z_2\mathbf{jk} \\ &+ y_1 s_2\mathbf{k} + z_1 x_2\mathbf{ki} + z_1 y_2\mathbf{kj} + z_1 z_2\mathbf{kk}\end{aligned}$$

$$\mathbf{ii} = \mathbf{jj} = \mathbf{kk} = -1$$

$$\mathbf{ij} = -\mathbf{ji} = \mathbf{k}, \quad \mathbf{jk} = -\mathbf{kj} = \mathbf{i}, \quad \mathbf{ki} = -\mathbf{ik} = \mathbf{j}$$

$$\begin{aligned}\underline{\mathbf{q}}_1\underline{\mathbf{q}}_2 &= s_1 s_2 - (x_1 x_2 + y_1 y_2 + z_1 z_2) \\ &+ s_1(x_2\mathbf{i} + y_2\mathbf{j} + z_2\mathbf{k}) + s_2(x_1\mathbf{i} + y_1\mathbf{j} + z_1\mathbf{k}) \\ &+ (y_1 z_2 - z_1 y_2)\mathbf{i} + (z_1 x_2 - x_1 z_2)\mathbf{j} + (x_1 y_2 - y_1 x_2)\mathbf{k}\end{aligned}$$

Produto de dois quatérnios(cont.)

$$\underline{\mathbf{q}}_1 \underline{\mathbf{q}}_2 = s_1 s_2 - (x_1 x_2 + y_1 y_2 + z_1 z_2)$$
$$+ s_1 (x_2 \mathbf{i} + y_2 \mathbf{j} + z_2 \mathbf{k}) + s_2 (x_1 \mathbf{i} + y_1 \mathbf{j} + z_1 \mathbf{k})$$
$$+ (y_1 z_2 - z_1 y_2)\mathbf{i} + (z_1 x_2 - x_1 z_2)\mathbf{j} + (x_1 y_2 - y_1 x_2)\mathbf{k}$$

$$\underline{\mathbf{q}}_1 \underline{\mathbf{q}}_2 = (s_1 s_2 - \mathbf{v}_1 \cdot \mathbf{v}_2 , s_1 \mathbf{v}_2 + s_2 \mathbf{v}_1 + \mathbf{v}_1 \times \mathbf{v}_2)$$

Conjugado, normas e produto interno

conjugado de um quatérnio

$$\underline{\mathbf{q}}^* = (s, \mathbf{v})^* = (s, -\mathbf{v})$$

norma de um quatérnio

$$n(\underline{\mathbf{q}}) = \underline{\mathbf{q}}\,\underline{\mathbf{q}}^* = (s, \mathbf{v})(s, -\mathbf{v}) = s^2 + \mathbf{v} \cdot \mathbf{v} = s^2 + x^2 + y^2 + z^2$$

produto interno de dois quatérnios

$$\underline{\mathbf{q}}_1 \cdot \underline{\mathbf{q}}_2 = s_1 s_2 + x_1 x_2 + y_1 y_2 + z_1 z_2$$

norma euclidiana

$$n(\underline{\mathbf{q}}) = \|\underline{\mathbf{q}}\|^2$$

Quatérnio inverso e unitário

inverso de um quatérnio

$$\underline{\mathbf{q}}^{-1} = \frac{1}{n(\underline{\mathbf{q}})}\underline{\mathbf{q}}^{*}$$

$$\underline{\mathbf{q}}\underline{\mathbf{q}}^{-1} = \frac{1}{n(\underline{\mathbf{q}})}\underline{\mathbf{q}}\underline{\mathbf{q}}^{*} = \frac{n(\underline{\mathbf{q}})}{n(\underline{\mathbf{q}})} = 1$$

unitário de um quatérnio

$$\hat{\underline{\mathbf{q}}} = \frac{1}{\left\|\underline{\mathbf{q}}\right\|}\underline{\mathbf{q}}$$

$$\hat{\underline{\mathbf{q}}} = \left(\cos\phi, \sin\phi\,\hat{\mathbf{v}}\right)$$

Quatérnios e rotações

Dada uma rotação definida por um eixo **ê** e um ângulo θ construímos o quatérnio unitário:

$$\underline{\hat{\mathbf{q}}} = \left(\cos\left(\frac{\theta}{2}\right), \sin\left(\frac{\theta}{2}\right)\hat{\mathbf{e}} \right)$$

Dado um ponto qualquer **p** do R^3 construímos o quatérnio:

$$\underline{\mathbf{p}} = (0, \mathbf{p})$$

Calculamos o produto:

$$\underline{\mathbf{p}}' = \underline{\hat{\mathbf{q}}}\,\underline{\mathbf{p}}\,\underline{\hat{\mathbf{q}}}^{-1}$$

→

$$\underline{\mathbf{p}'} = (0, \mathbf{p}')$$

Demostração

$$(0,\mathbf{p}') = \left(\cos\left(\frac{\theta}{2}\right), \sin\left(\frac{\theta}{2}\right)\hat{\mathbf{e}} \right)(0,\mathbf{p})\left(\cos\left(\frac{\theta}{2}\right), -\sin\left(\frac{\theta}{2}\right)\hat{\mathbf{e}} \right)$$

...

$$\boxed{\mathbf{p}' = (\cos\theta)\,\mathbf{p} + (1-\cos\theta)(\hat{\mathbf{e}}\cdot\mathbf{p})\hat{\mathbf{e}} + (sen\,\theta)(\hat{\mathbf{e}}\times\mathbf{p})}$$

Composição de rotações

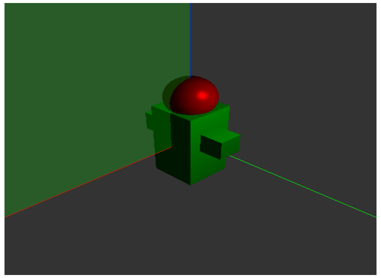
$$\widehat{\underline{\mathbf{q}}}_1 \xrightarrow{\textit{seguida de}} \widehat{\underline{\mathbf{q}}}_2$$

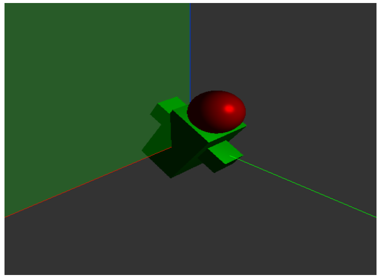
$$\underline{\hat{\mathbf{q}}_2}(\underline{\hat{\mathbf{q}}_1}\,\underline{\mathbf{p}}\,\underline{\hat{\mathbf{q}}_1^{-1}})\underline{\hat{\mathbf{q}}_2^{-1}}$$

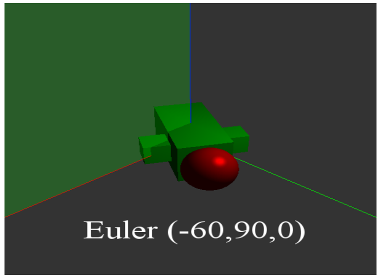
$$(\underline{\hat{\mathbf{q}}_2}\,\underline{\hat{\mathbf{q}}_1})\underline{\mathbf{p}}(\underline{\hat{\mathbf{q}}_2}\,\underline{\hat{\mathbf{q}}_1})^{-1}$$

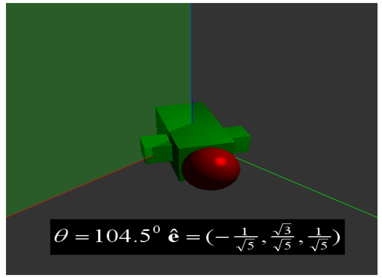
$$\hat{\mathbf{q}}_2\hat{\mathbf{q}}_1$$

Composição de rotações









$$\underline{\hat{\mathbf{q}}}_1 = \left(\cos(-30^o), \sin(-30^o)(1,0,0)\right) = \left(\tfrac{\sqrt{3}}{2}, -\tfrac{1}{2}(1,0,0)\right)$$

$$\underline{\hat{\mathbf{q}}}_2 = \left(\cos(45^o), \sin(45^o)(0,1,0)\right) = \left(\tfrac{1}{\sqrt{2}}, \tfrac{1}{\sqrt{2}}(0,1,0)\right)$$

$$\underline{\hat{\mathbf{q}}}_2 \underline{\hat{\mathbf{q}}}_1 = \left(\tfrac{1}{\sqrt{2}}, \tfrac{1}{\sqrt{2}}(0,1,0)\right)\left(\tfrac{\sqrt{3}}{2}, -\tfrac{1}{2}(1,0,0)\right) = \left(\tfrac{\sqrt{6}}{4}, \tfrac{\sqrt{10}}{4}\left(-\tfrac{1}{\sqrt{5}}, \tfrac{\sqrt{3}}{\sqrt{5}}, \tfrac{1}{\sqrt{5}}\right)\right)$$

$$\theta = 104.5^0 \qquad \hat{\mathbf{e}} = \left(-\tfrac{1}{\sqrt{5}}, \tfrac{\sqrt{3}}{\sqrt{5}}, \tfrac{1}{\sqrt{5}}\right)$$

Exemplo

$$\theta_z = -90^o$$

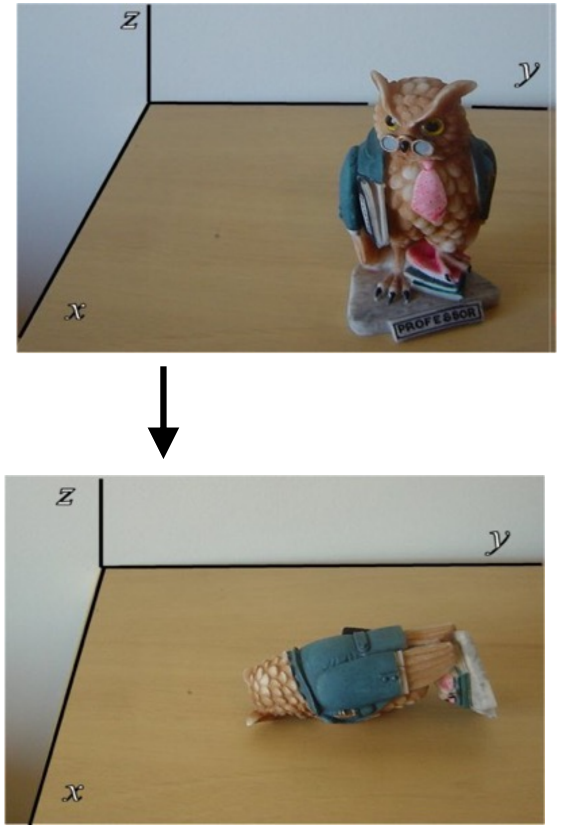
$$\hat{\underline{\mathbf{q}}}_1 = \left(\cos(-45^0), \sin(-45^0)(0,0,1)\right)$$

$$\hat{\underline{\mathbf{q}}}_1 = \left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}(0,0,1)\right)$$

$$\theta_x = 90^o$$

$$\hat{\underline{\mathbf{q}}}_1 = \left(\cos(45^0), \sin(45^0)(1,0,0)\right)$$

$$\hat{\underline{\mathbf{q}}}_2 = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}(1,0,0)\right)$$



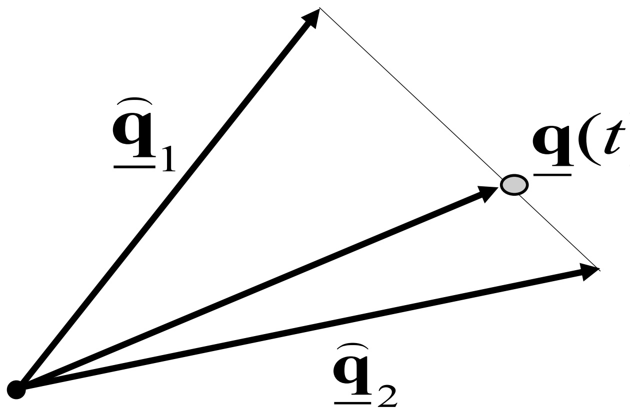
$$\theta = 120^{o}$$

$$\hat{\mathbf{e}} = \begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} \end{pmatrix}$$

$$\underline{\hat{\mathbf{q}}}_2 \underline{\hat{\mathbf{q}}}_1 = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}(1,0,0) \right) \left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}(0,0,1) \right)$$

$$\underline{\hat{\mathbf{q}}}_2 \underline{\hat{\mathbf{q}}}_1 = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right) \right)$$

Interpolação de quatérnios



$$\underline{\mathbf{q}}(t) = t\widehat{\underline{\mathbf{q}}}_1 + (1-t)\widehat{\underline{\mathbf{q}}}_2$$

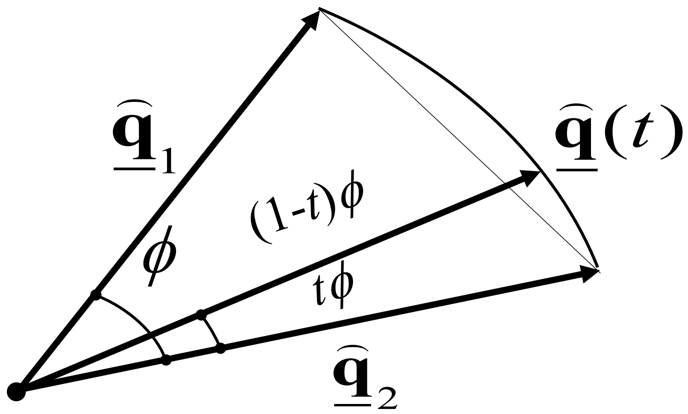
$$t \in [0,1]$$

não é unitário

⇓

não representa rotação

Interpolação de quatérnios



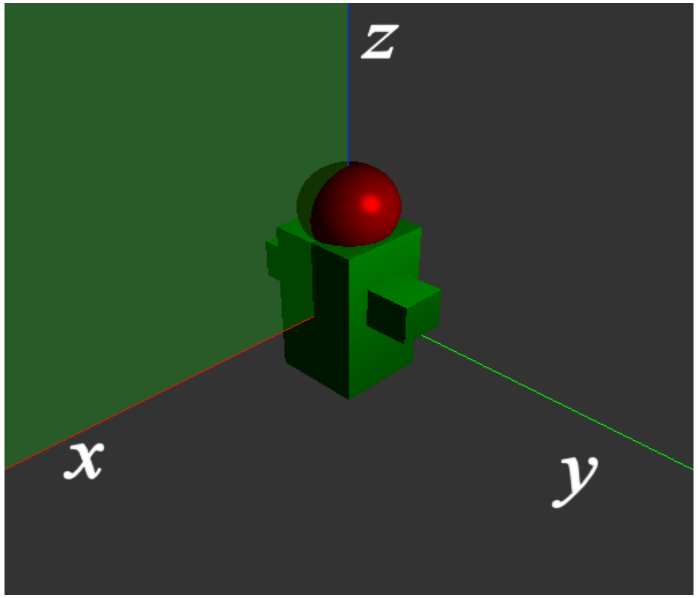
$$\hat{\underline{\mathbf{q}}}(t) = a\hat{\underline{\mathbf{q}}}_1 + b\hat{\underline{\mathbf{q}}}_2$$

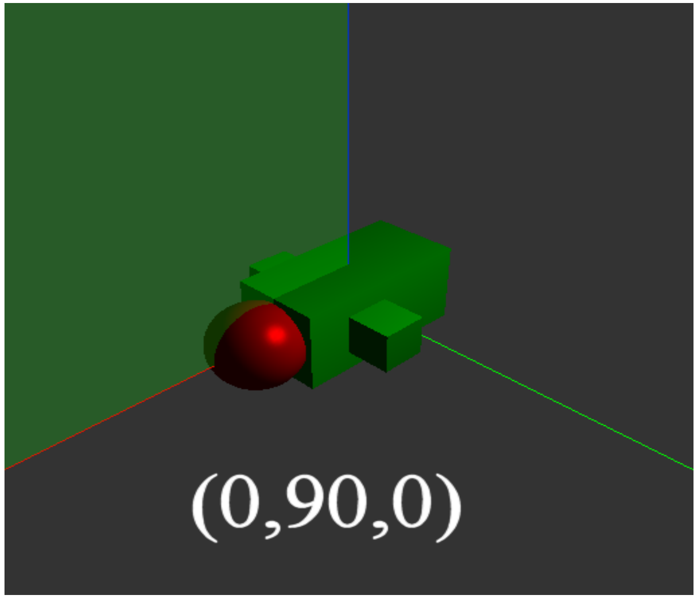
$$\left\|\hat{\underline{\mathbf{q}}}(t)\right\| = 1$$

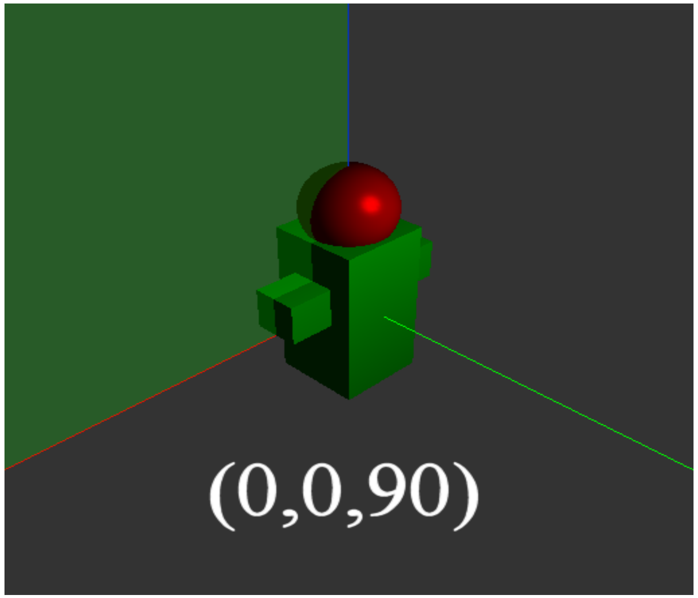
$$\hat{\underline{\mathbf{q}}}(t)\cdot\hat{\underline{\mathbf{q}}}_2 = \cos(t\phi)$$

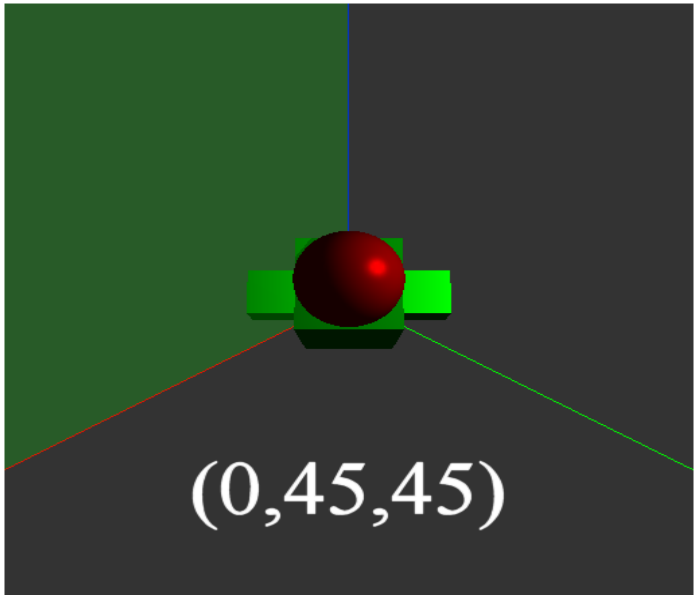
$$\hat{\underline{\mathbf{q}}}_1\cdot\hat{\underline{\mathbf{q}}}_2 = \cos(\phi)$$

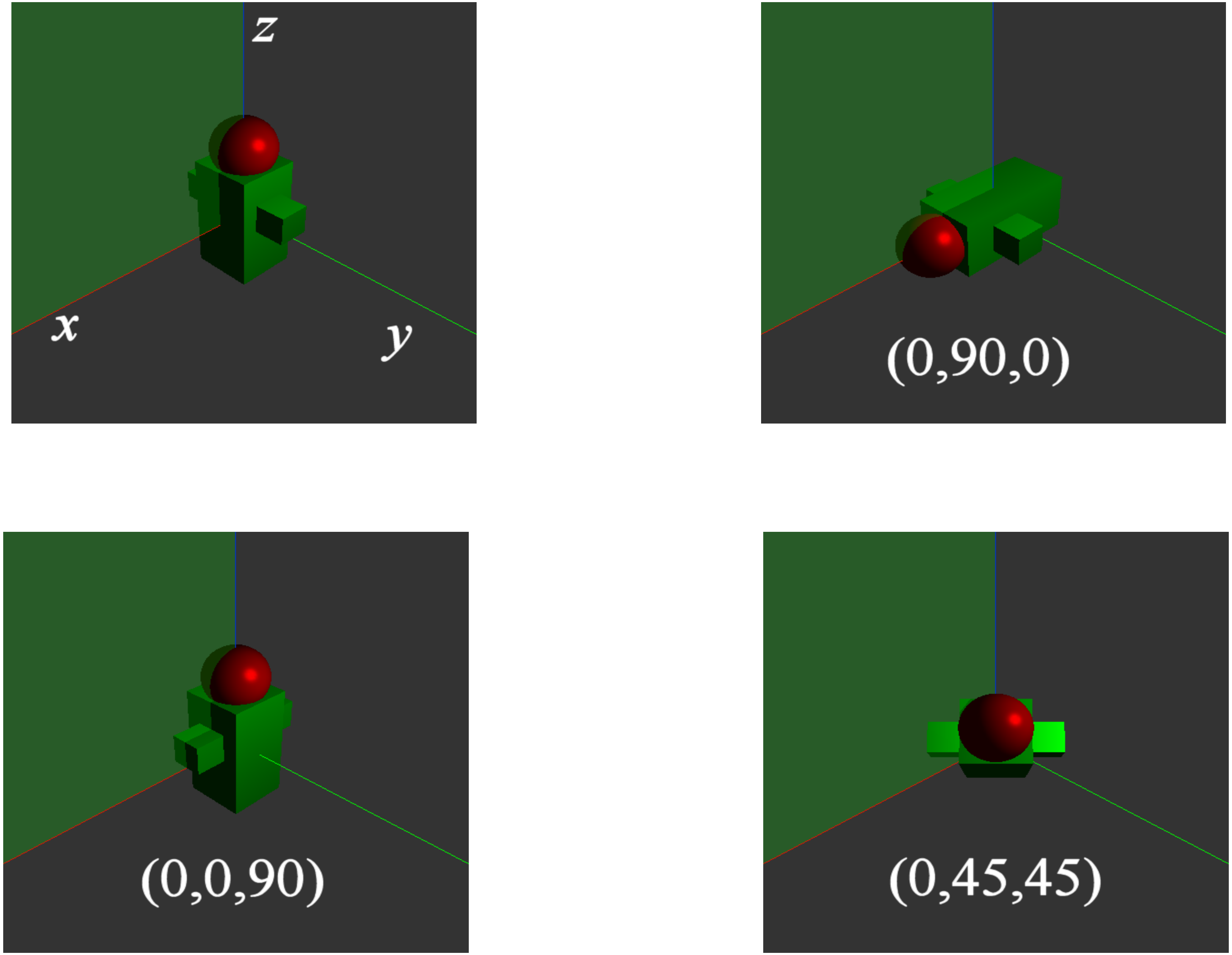
$$\hat{\underline{\mathbf{q}}}(t) = Slerp\left(\hat{\underline{\mathbf{q}}}_1, \hat{\underline{\mathbf{q}}}_2, t\right) = \hat{\underline{\mathbf{q}}}_1 \frac{\sin((1-t)\phi)}{\sin(\phi)} + \hat{\underline{\mathbf{q}}}_2 \frac{\sin(t\phi)}{\sin(\phi)}$$


z
x
y


(0,90,0)


(0,0,90)


(0,45,45)


z
x
y
(0,90,0)
(0,0,90)
(0,45,45)

Quatérnios e matrizes

$$\underline{\hat{\mathbf{q}}} = (w, x, y, z)$$

$$\underline{\hat{\mathbf{q}}}\mathbf{p}\underline{\hat{\mathbf{q}}}^{-1}$$

$$\mathbf{M}_{\underline{\hat{\mathbf{q}}}} = \begin{bmatrix} 1-2(y^2+z^2) & 2xy-2zw & 2xz+2yw & 0 \\ 2xy+2zw & 1-2(x^2+z^2) & 2yz-2xw & 0 \\ 2xz-2yw & 2yz+2xw & 1-2(x^2+y^2) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Matrizes e quatérnios

$$\begin{bmatrix} 1-2(y^2+z^2) & 2xy-2zw & 2xz+2yw & 0 \\ 2xy+2zw & 1-2(x^2+z^2) & 2yz-2xw & 0 \\ 2xz-2yw & 2yz+2xw & 1-2(x^2+y^2) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} m_{11} & m_{12} & m_{13} & 0 \\ m_{21} & m_{22} & m_{23} & 0 \\ m_{31} & m_{32} & m_{33} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$4-2(2x^2+2y^2+2z^2) = 1+m_{11}+m_{22}+m_{33}$$

$$4-4(1-w^2) = 1+m_{11}+m_{22}+m_{33}$$

$$w = \pm\tfrac{1}{2}\sqrt{1+m_{11}+m_{22}+m_{33}}$$

$$x = \frac{m_{32}-m_{23}}{4w} \qquad y = \frac{m_{13}-m_{31}}{4w} \qquad z = \frac{m_{21}-m_{12}}{4w}$$

FIM