

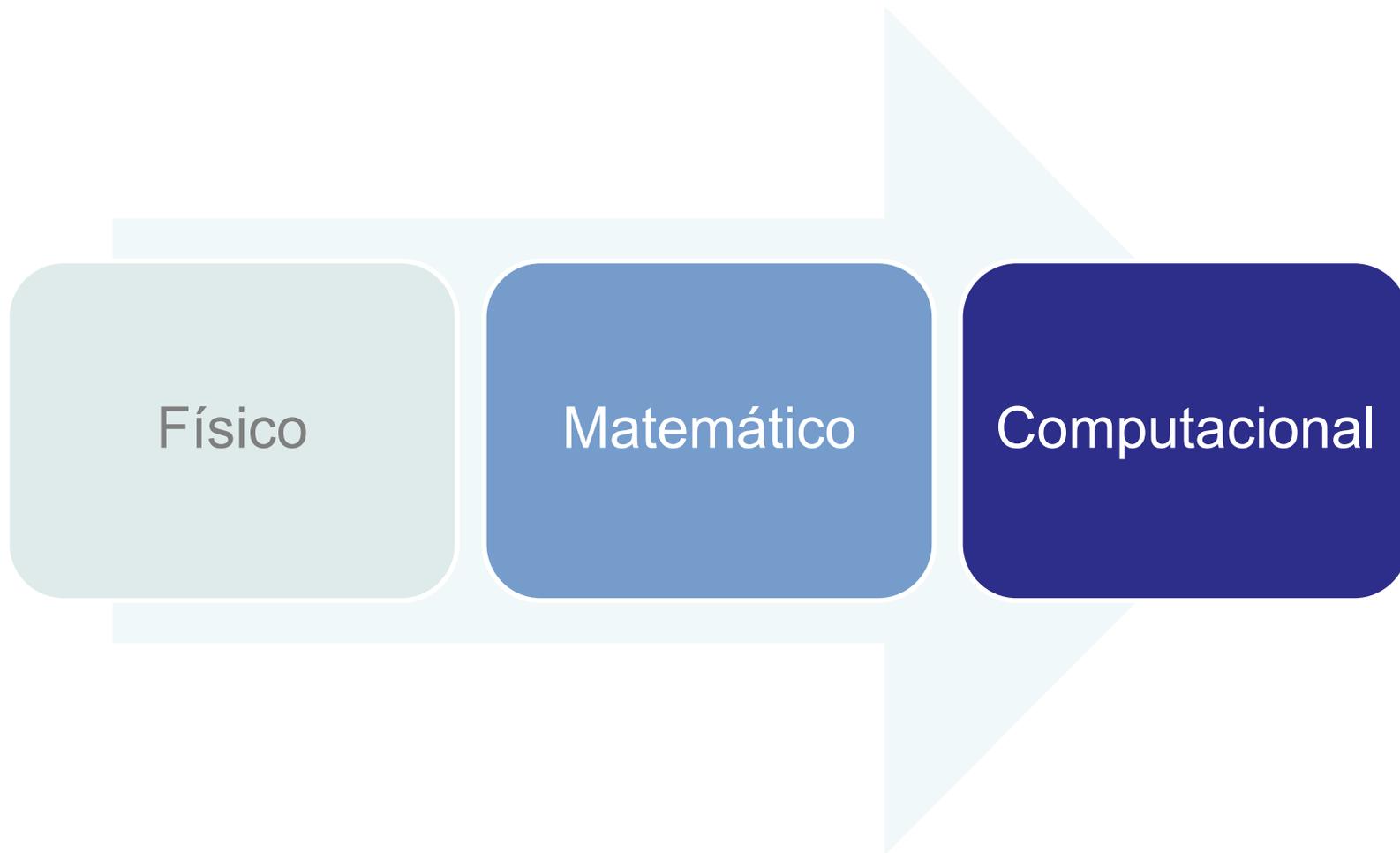
Imagens Digitais

Processamento de Imagens

Visão Computacional

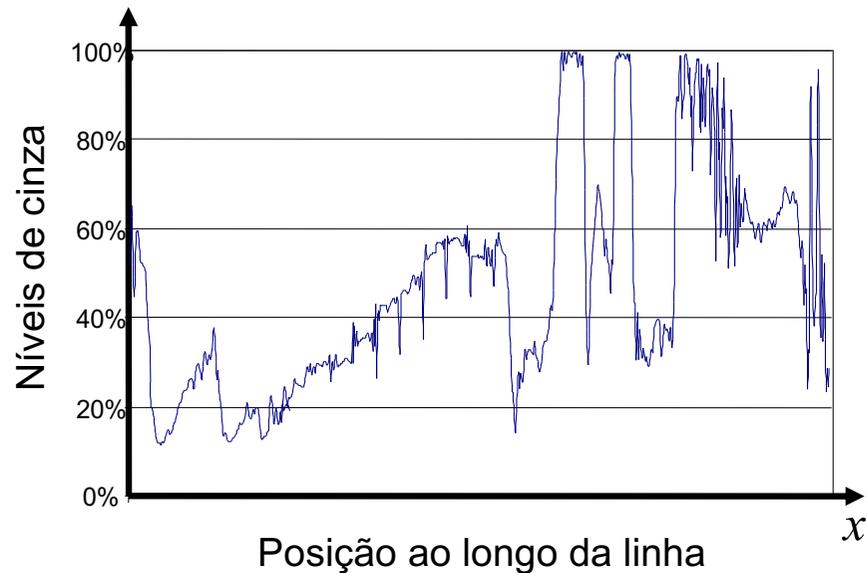
Compressão

Modelos de uma Imagem Digital



Matematicamente imagem é uma função

Imagem em tons de cinza



$$L : [0, w] \times [0, h] \subset \mathbb{R}^2 \rightarrow \mathbf{C}$$

$$\begin{pmatrix} u \\ v \end{pmatrix} \rightarrow L$$

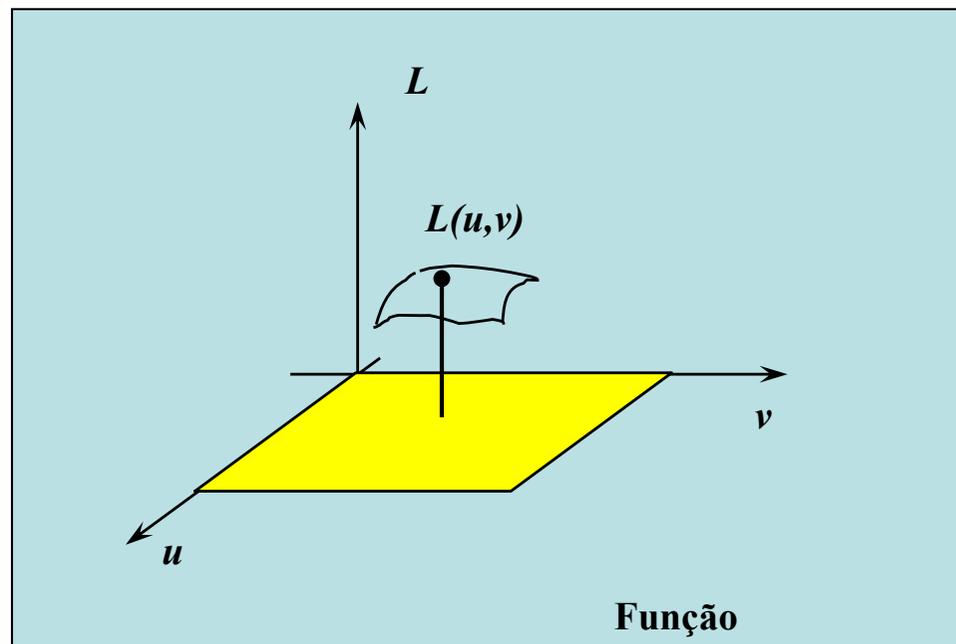


Imagem colorida como uma função do $\mathbb{R}^2 \rightarrow \mathbb{R}^3$

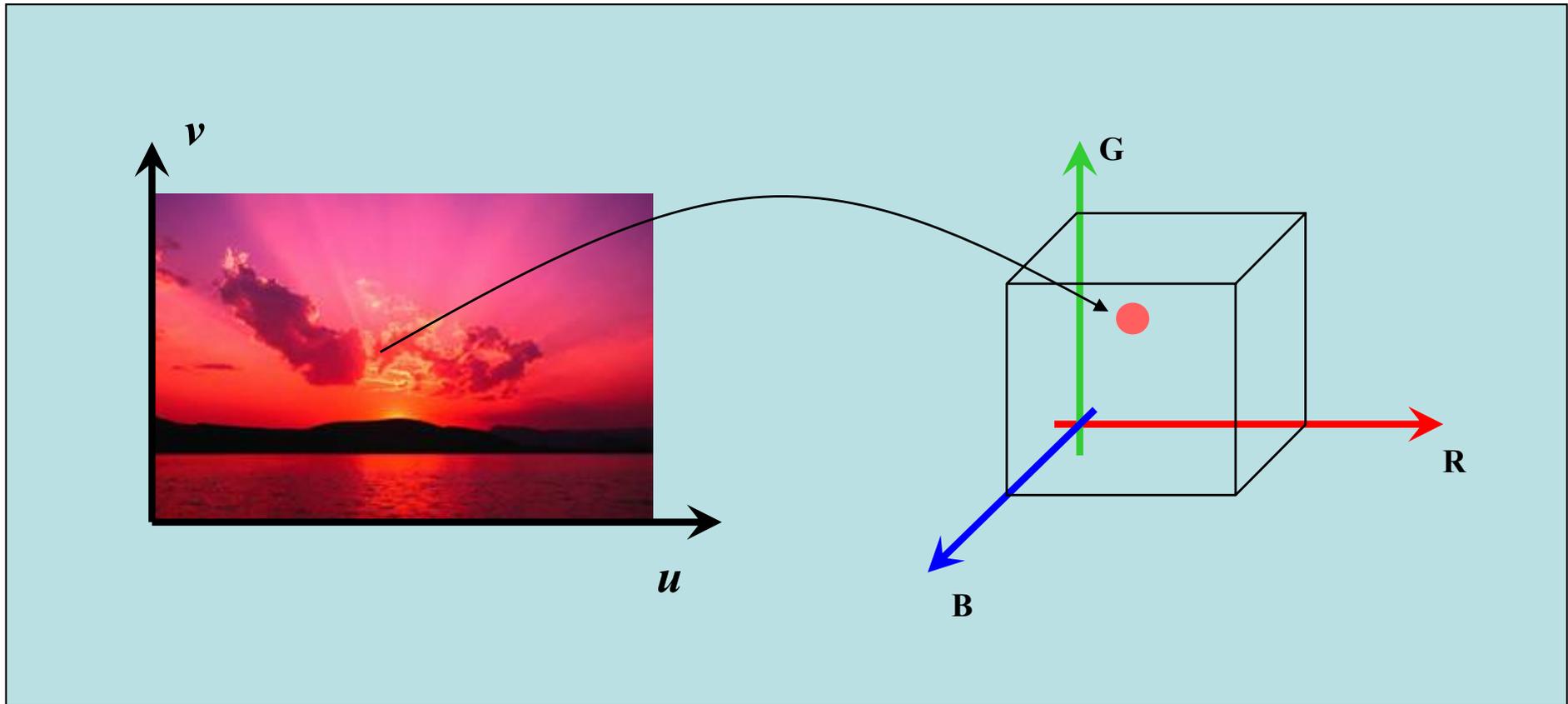
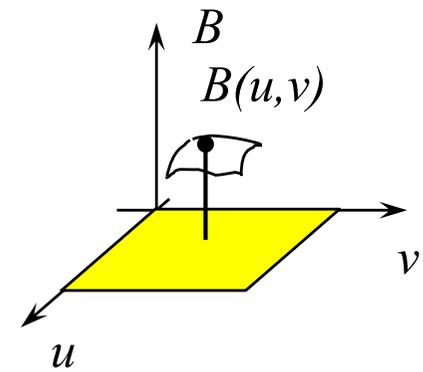
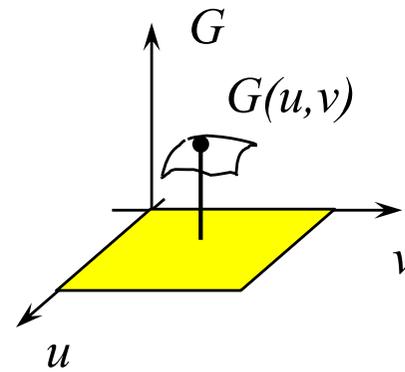
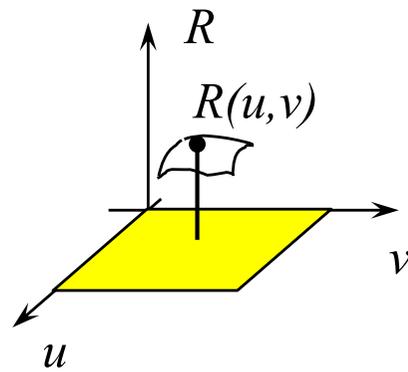


Imagem coloridas como 3 “canais” de cor



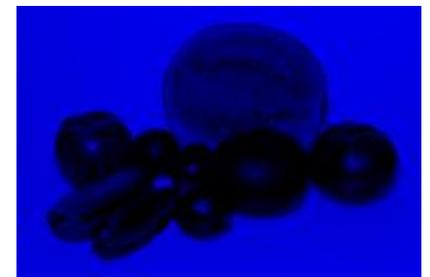
=



+



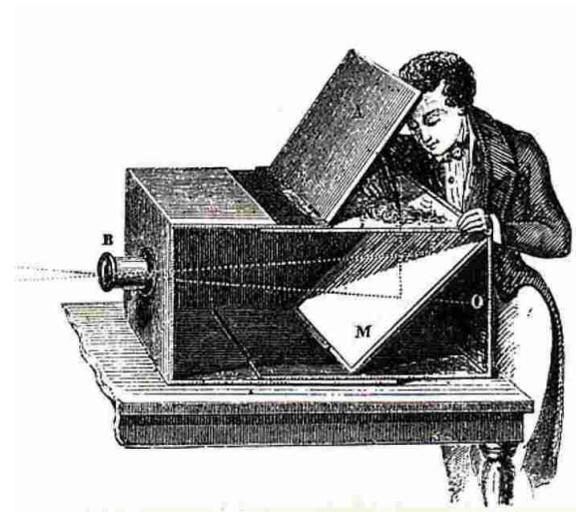
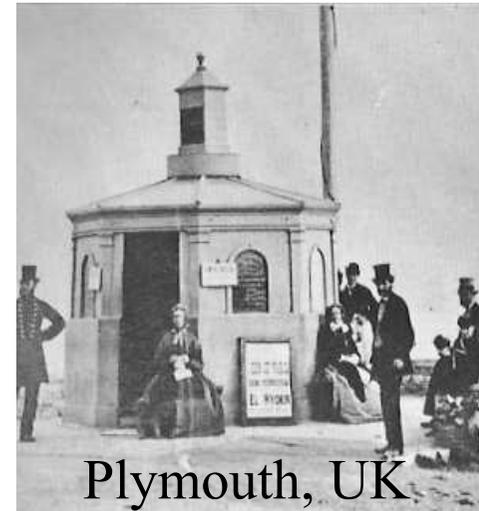
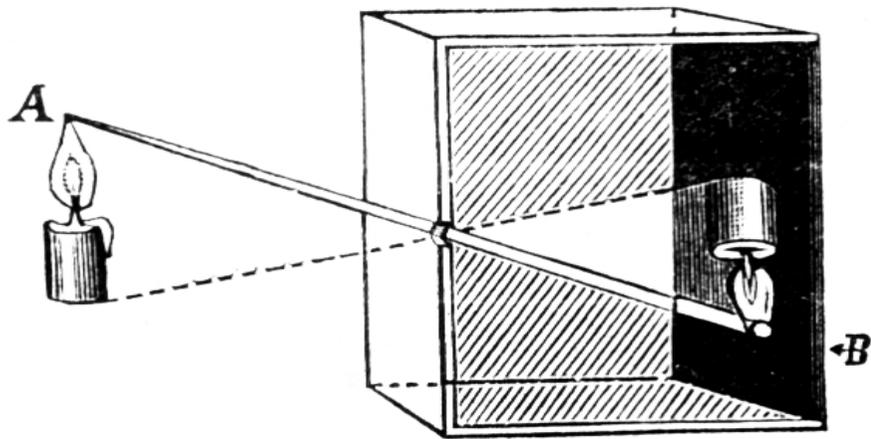
+



Processos que ocorrem na captura de uma imagem

Amostragem, quantização e
codificação

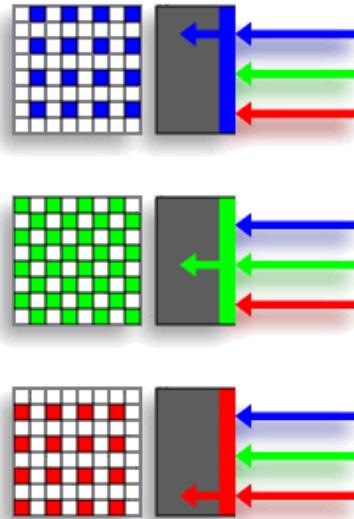
Câmera obscura e Câmera “pin-hole”



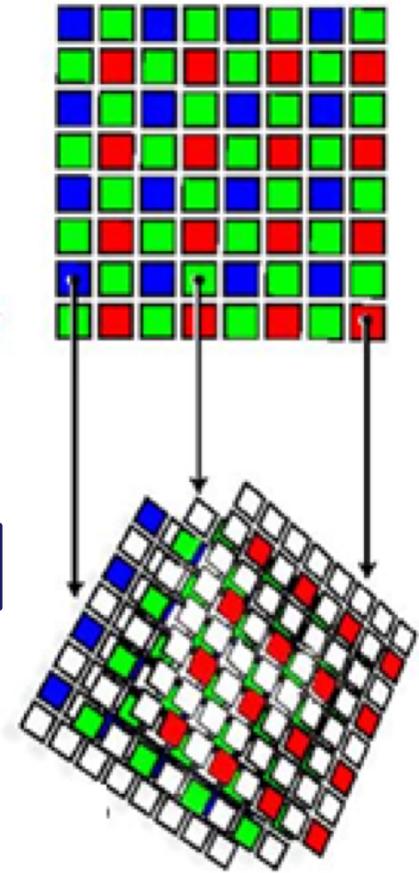
1) No plano de imagem a câmara tem um banco de sensores.



2) Filtros controlam a cor que cada sensor amostra.



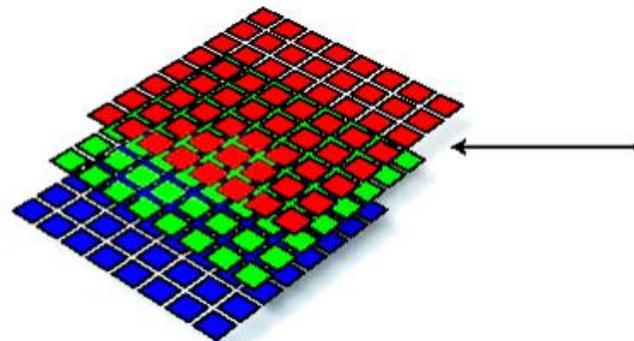
3) Um padrão comum é o de Bayer, que privilegia o verde.



Aquisição de imagem numa camera fotografica



6) A imagem comprimida e armazenada num formato mais eficiente (jpg ou png).

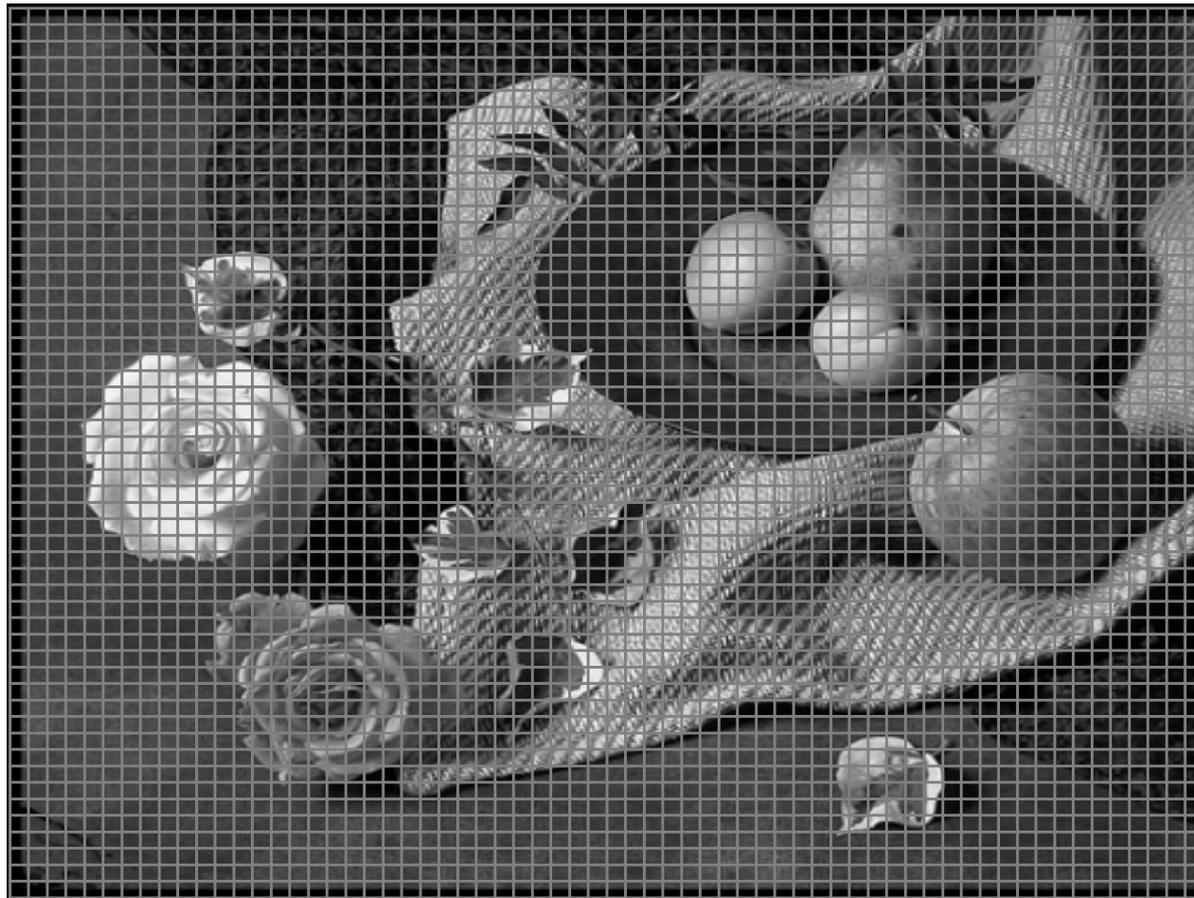


5) Os mapas incompletos são reconstituídos e reamostrados (formato RAW).

4) A imagem mista é decomposta em tres canais incompletos.

Digitalização de Imagens

Discretização espacial (amostragem)



Processos básicos



Imagem de tons contínuos

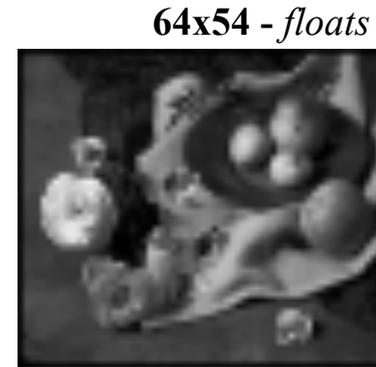
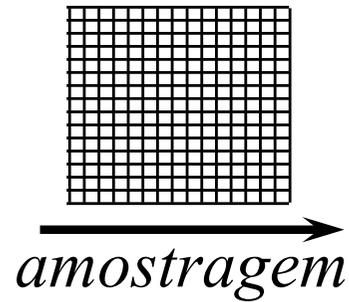


Imagem amostrada

↓
quantização

64x54 - 256 tons



Imagem amostrada e quantizada

55	55	55	55	55	55	55
55	20	22	23	45	55	55
55	55	10	09	11	55	55
55	55	43	42	70	55	55
55	55	28	76	22	55	55
55	55	55	55	55	55	55

8*55, 1*20, 1*22, 1*23,

←
codificação

Imagem amostrada, quantizada e codificada

Quantização



Seis fotos da mesma cena com variação do tempo de abertura da câmera



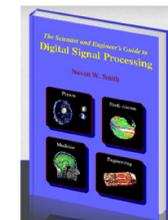
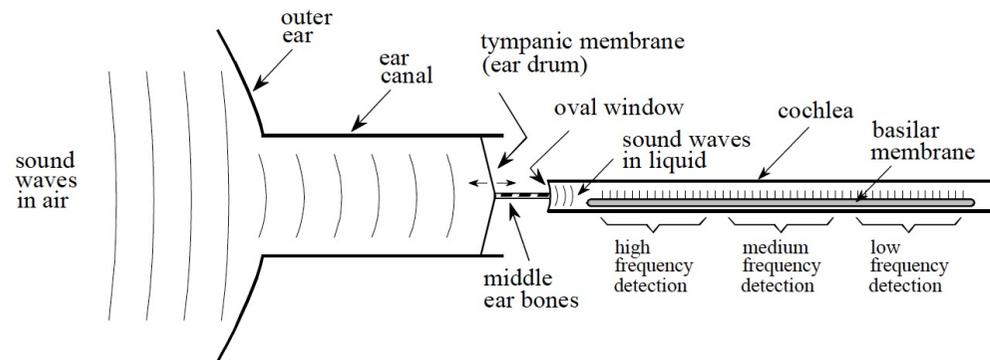
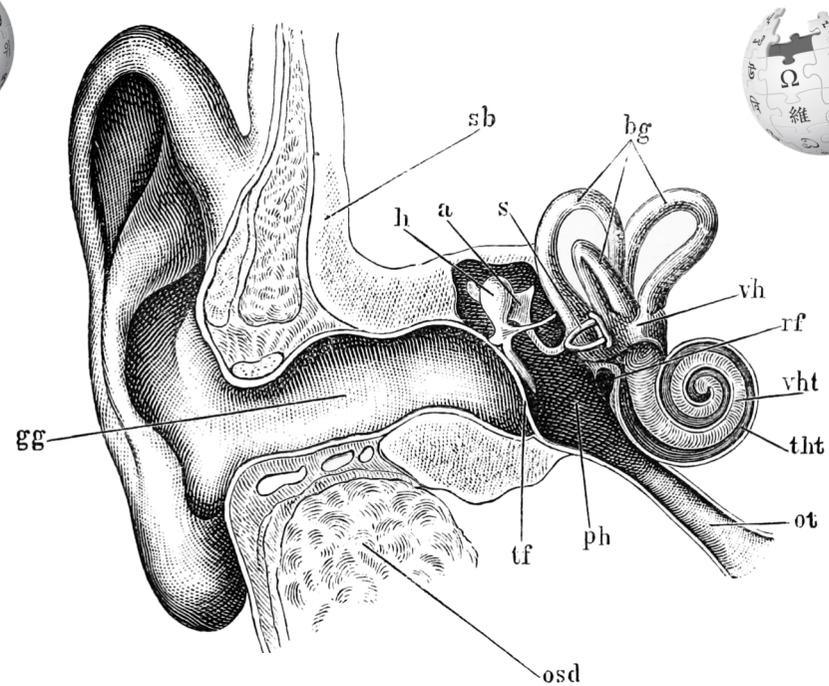
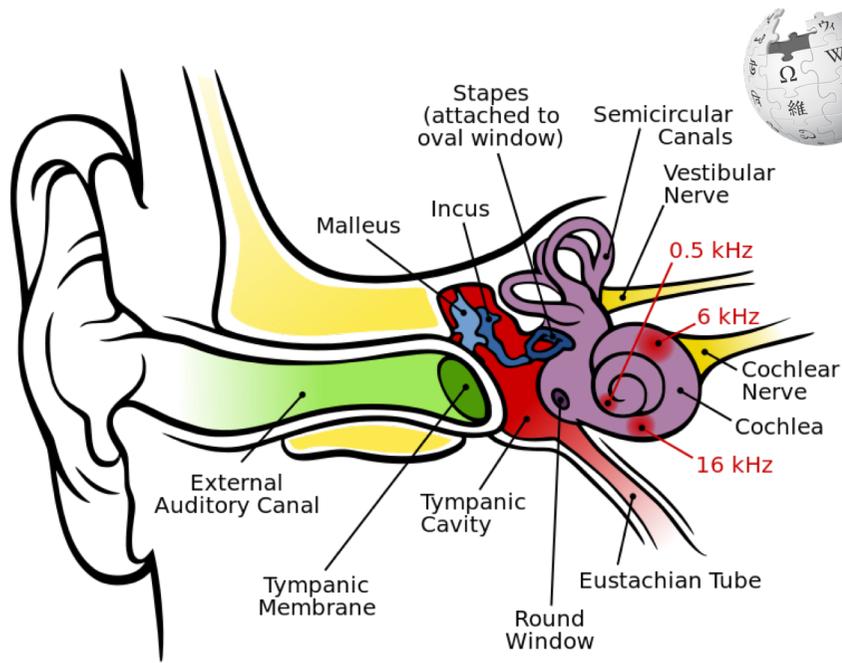
Razões de lados e resolução típicas

16:10	1920×1200 ou 1280×800
16:9	1920×1080 (HDTV) ou 1600×900 ou
4:3	1024×768 ou 800×600 ou 640×480

Amostragem e reconstrução

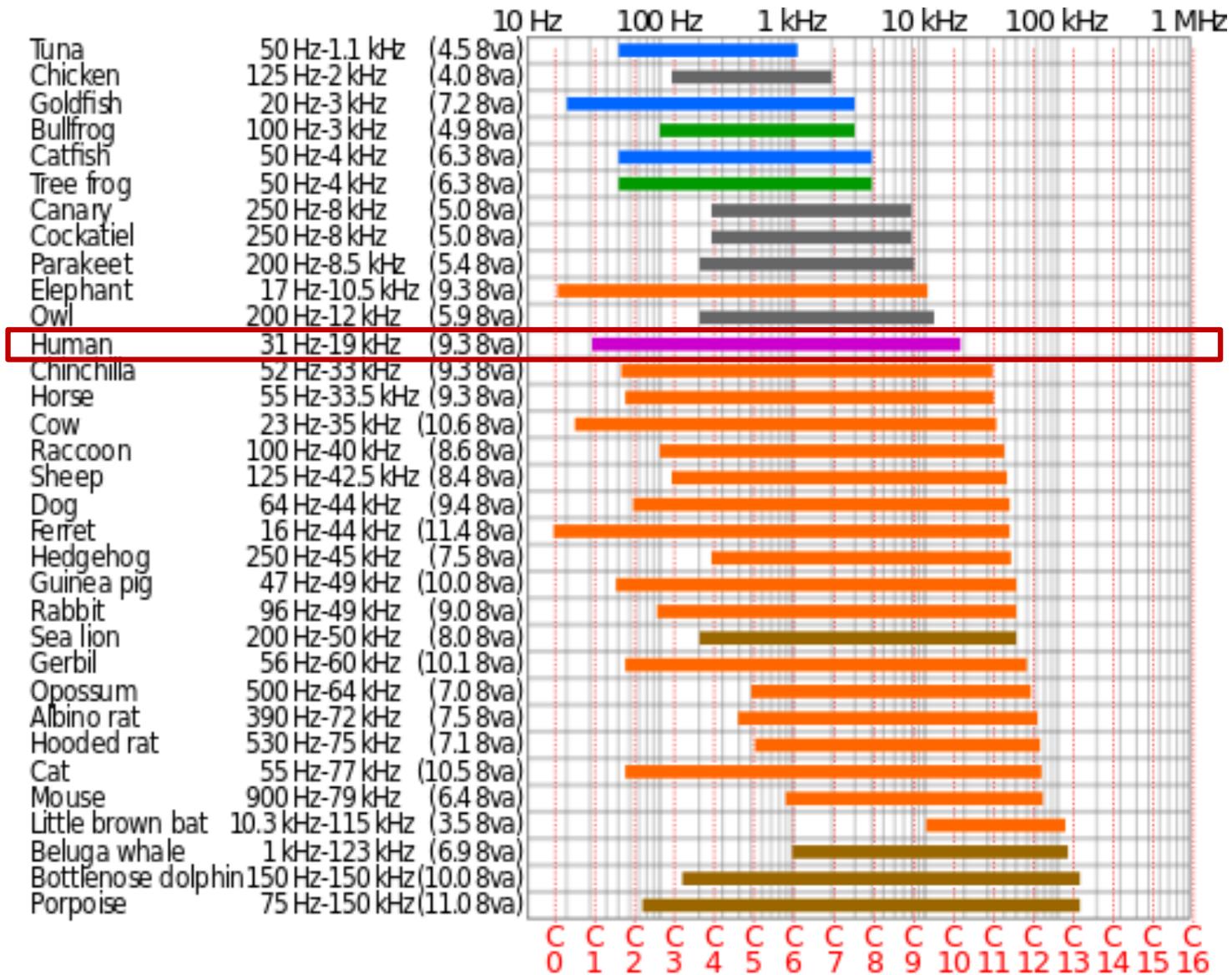
1D - SOM (VOZ, FALA E MÚSICA)



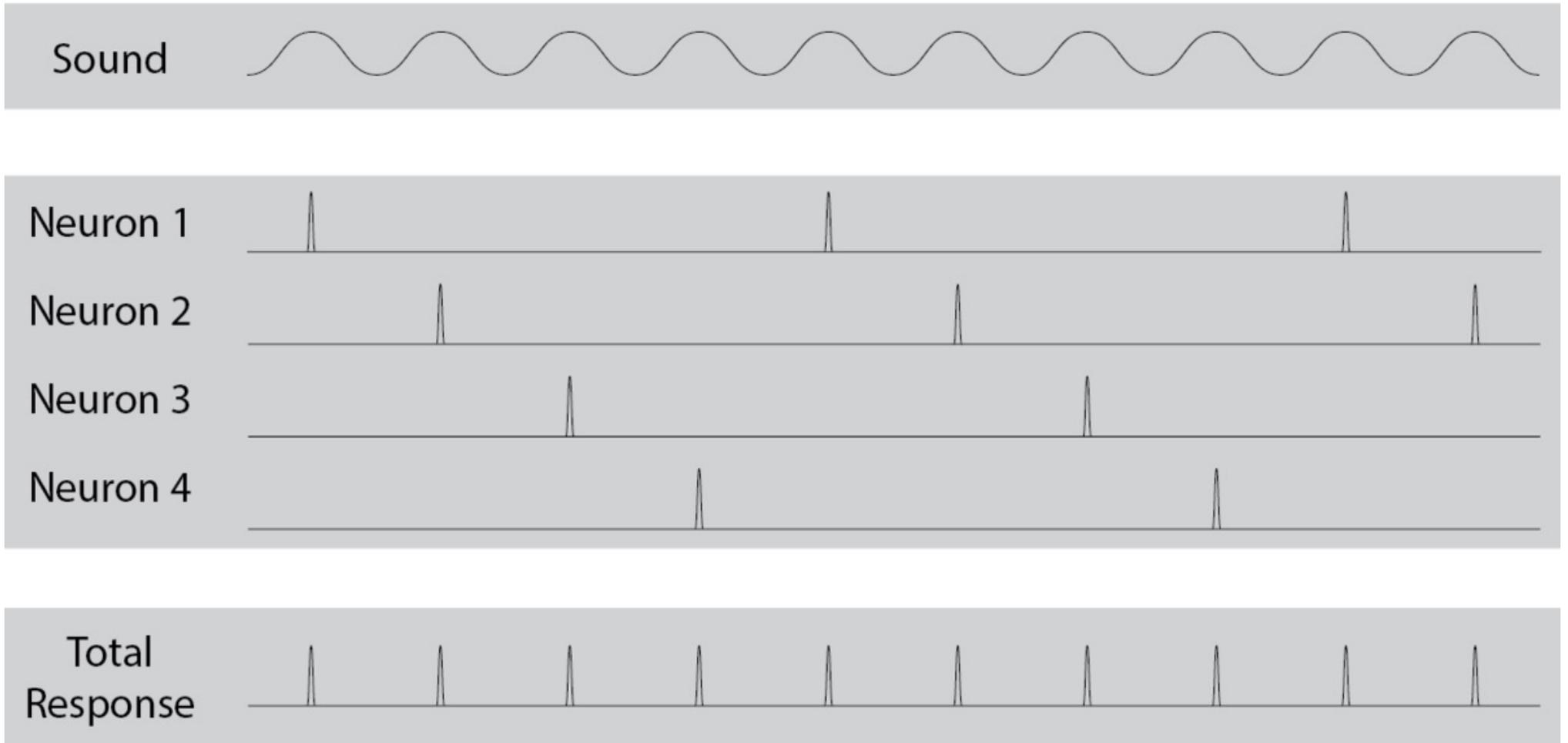


Steven W. Smith

Diagrama funcional da orelha humana. A orelha externa coleta ondas sonoras do ambiente e canaliza elas para a membrana timpânica (tambor da orelha), uma fina camada de tecido que vibra em sincronia com o forma da onda de ar. Os ossos do ouvido médio (martelo, bigorna e estribo) transmitem essas vibrações para a janela oval, uma membrana flexível na cóclea cheia de líquido. Contido dentro da cóclea temos a membrana basilar, que é estrutura que suporta para cerca de 12.000 células nervosas que formam o nervo coclear. Devido à rigidez variável da membrana de base, cada célula nervosa apenas responde a uma faixa estreita de frequências de áudio, tornando a orelha um analisador do espectro de a frequência.



Volley theory



“Lei” de Weber



Ernst Heinrich Weber
(24 June 1795 – 26 January 1878)

$$k = \frac{\Delta x / \Delta y}{x}$$

$$k = \frac{dx / dy}{x}$$

$$dx / dy = k x$$

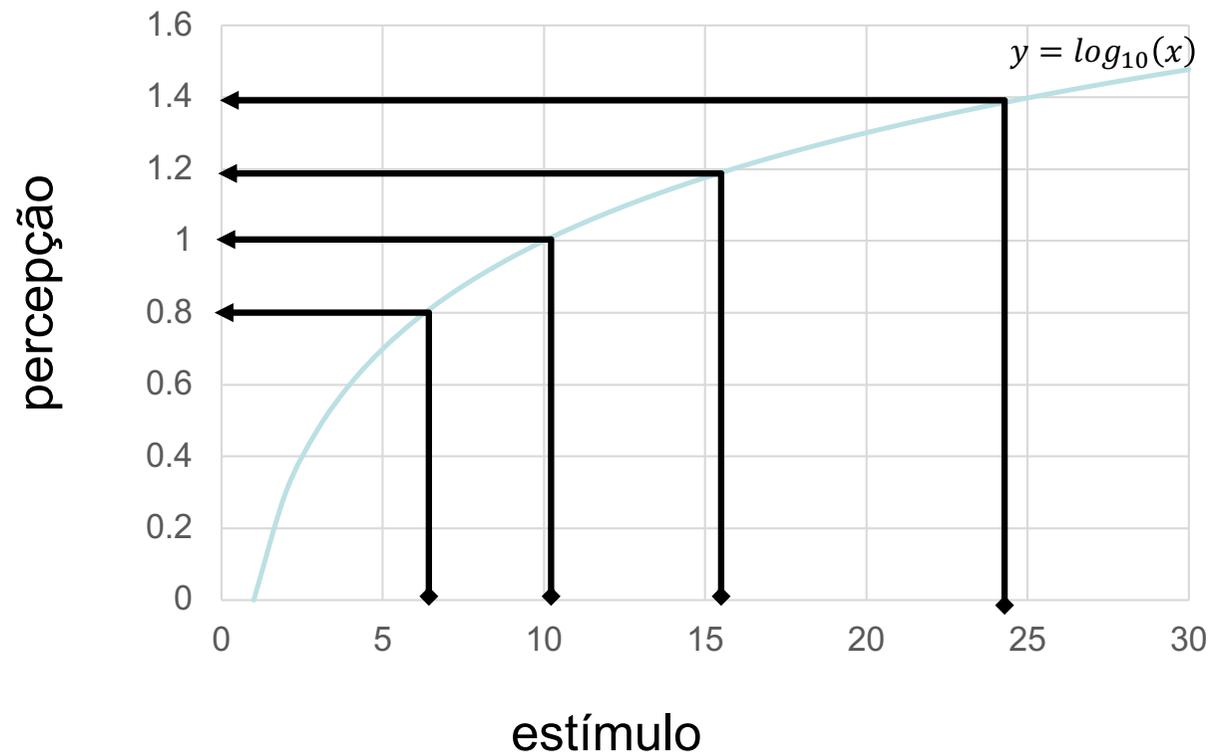
$$x = e^{ky}$$

$$y = \frac{1}{k} \ln(x)$$

$$\log_{10}(x) = \log_{10}(e^{ky})$$

$$\log_{10}(x) = k \log_{10}(e)y$$

$$y \sim \log_{10}(x)$$



Intensidade do som

$$1 \text{ dB} = 10 \log_{10} \left(\frac{\text{valor da intensidade}}{\text{intensidade de referencia}} \right)$$

	Watts/cm ²	Decibels SPL	Example sound
	10 ⁻²	140 dB	Pain
	10 ⁻³	130 dB	
	10 ⁻⁴	120 dB	Discomfort
	10 ⁻⁵	110 dB	Jack hammers and rock concerts
	10 ⁻⁶	100 dB	
	10 ⁻⁷	90 dB	OSHA limit for industrial noise
	10 ⁻⁸	80 dB	
	10 ⁻⁹	70 dB	
	10 ⁻¹⁰	60 dB	Normal conversation
	10 ⁻¹¹	50 dB	
	10 ⁻¹²	40 dB	Weakest audible at 100 hertz
	10 ⁻¹³	30 dB	
	10 ⁻¹⁴	20 dB	Weakest audible at 10kHz
	10 ⁻¹⁵	10 dB	
	10 ⁻¹⁶	0 dB	Weakest audible at 3 kHz
	10 ⁻¹⁷	-10 dB	
	10 ⁻¹⁸	-20 dB	



Louder

Softer



Intensidade do som

$$1 \text{ dB} = 10 \log_{10} \left(\frac{\text{valor da intensidade}}{\text{intensidade de referencia}} \right)$$

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Louder

Softer

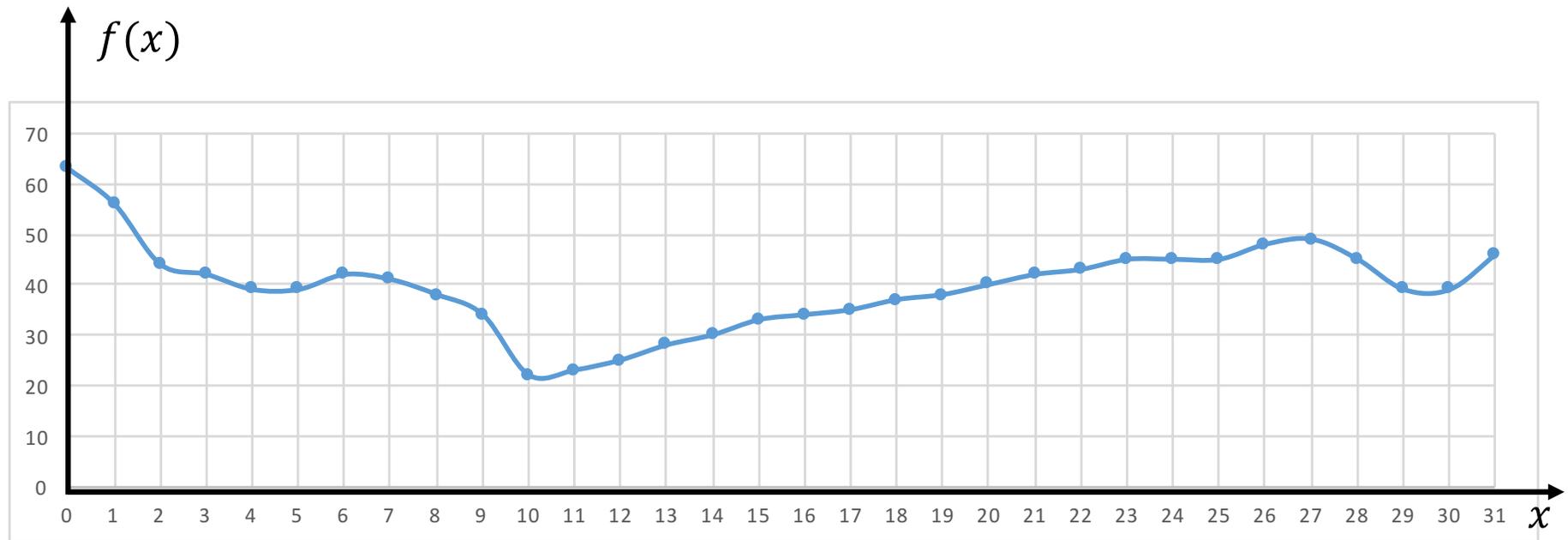
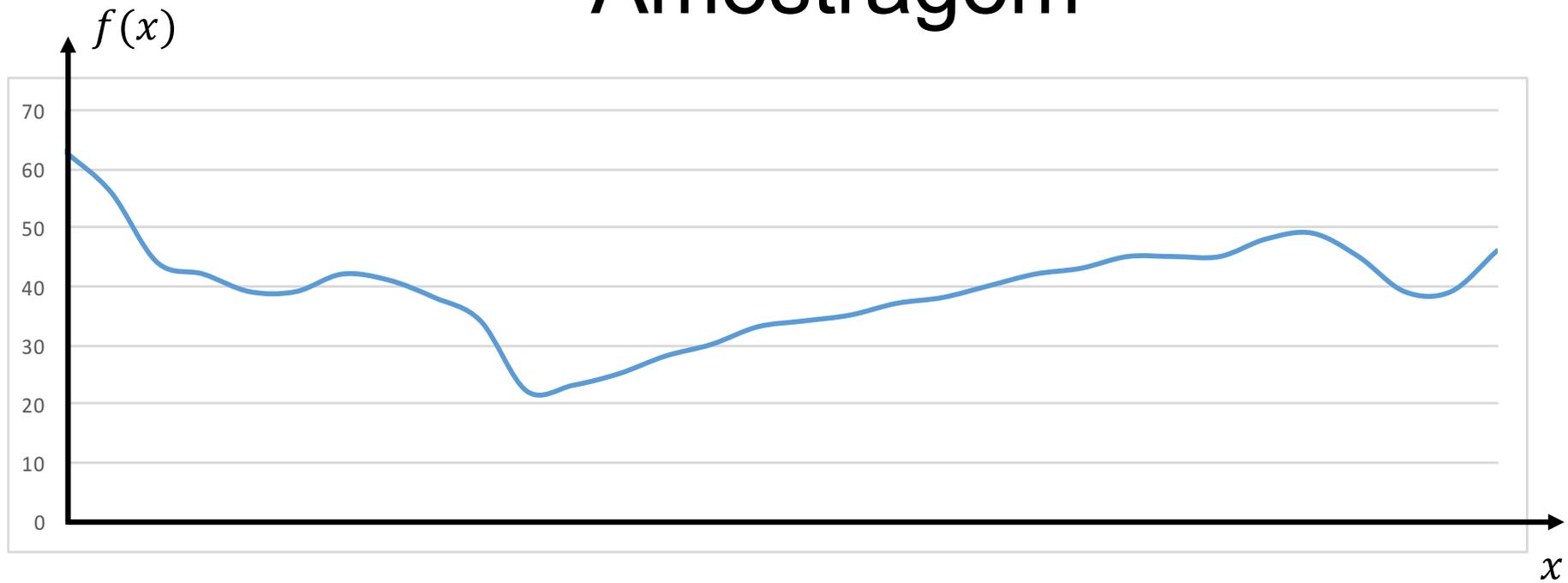


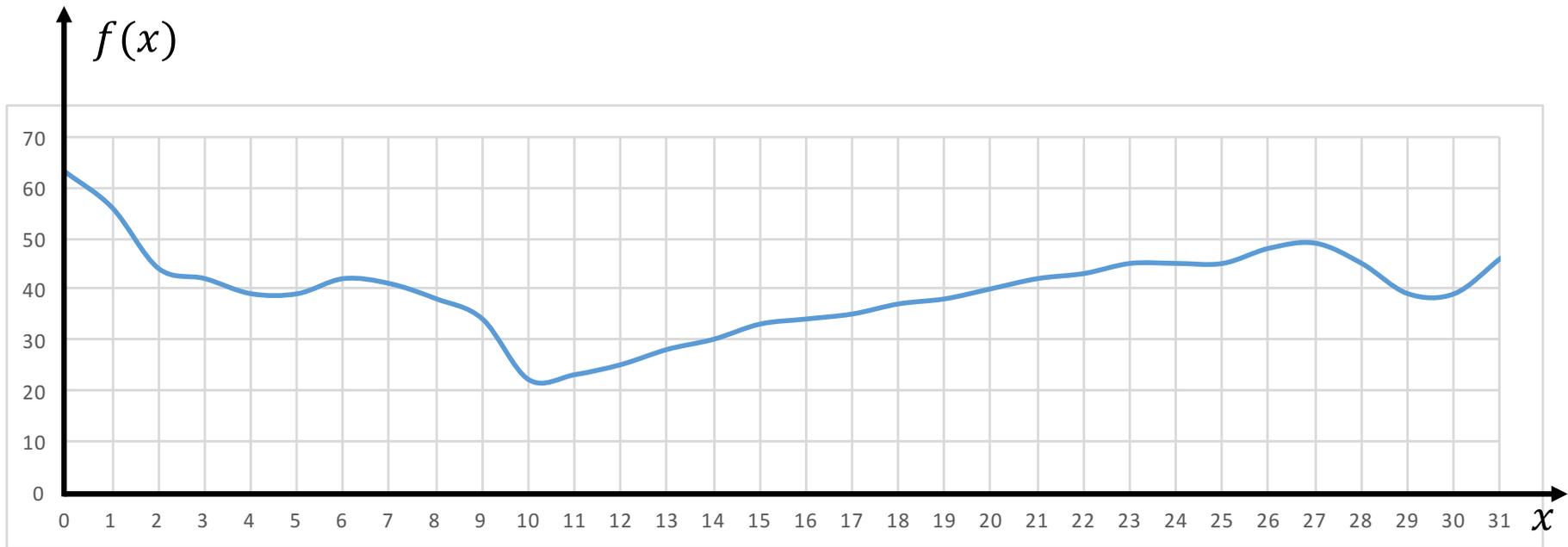
<https://www.youtube.com/watch?v=5tGEDgkZIC8>

<https://www.youtube.com/watch?v=XPbLYD9KFAo>

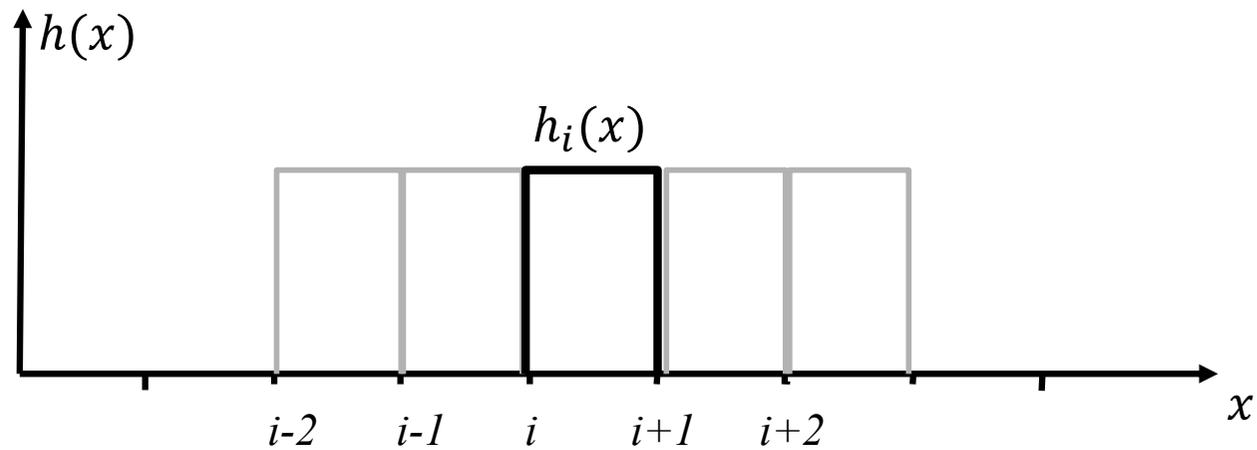
<https://www.youtube.com/watch?v=0ImS5IQ5MSU>

Amostragem





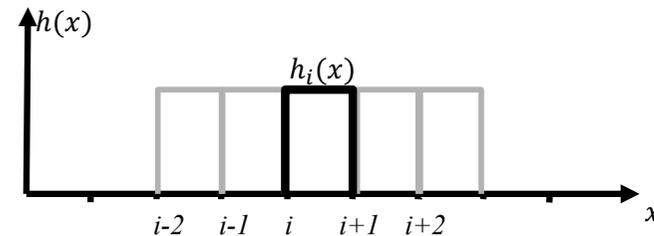
$$f(x) = \sum_i f_i h_i(x)$$



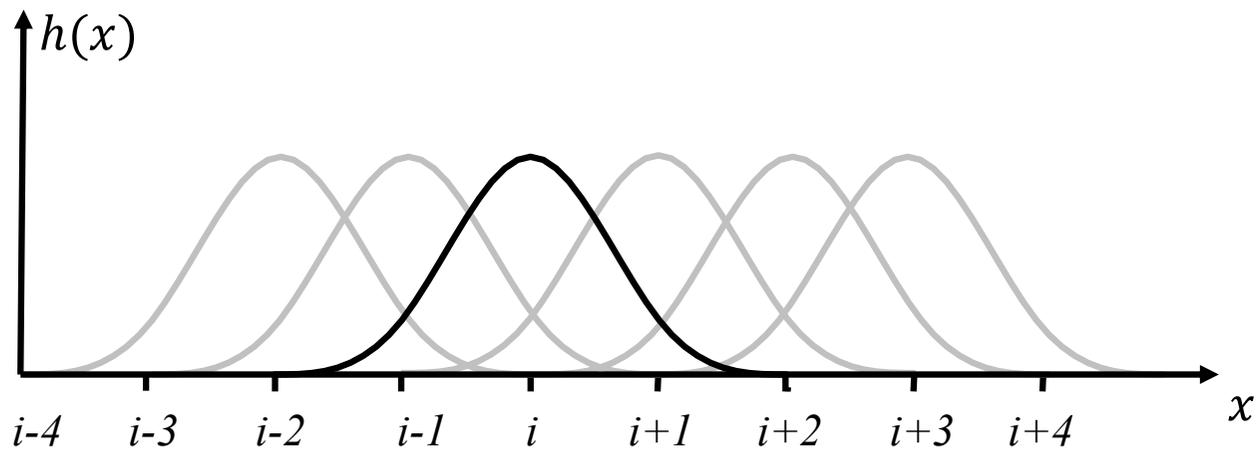
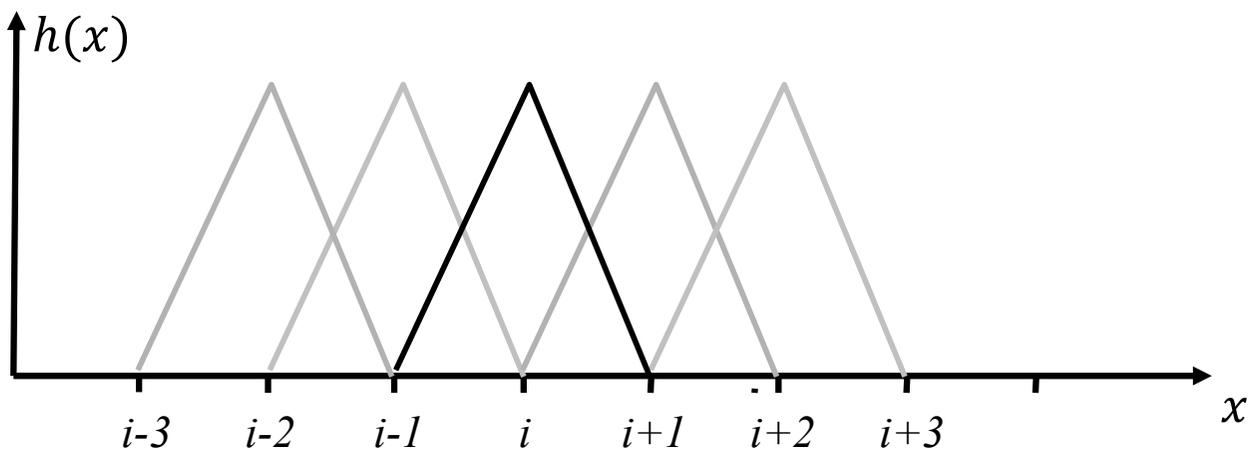
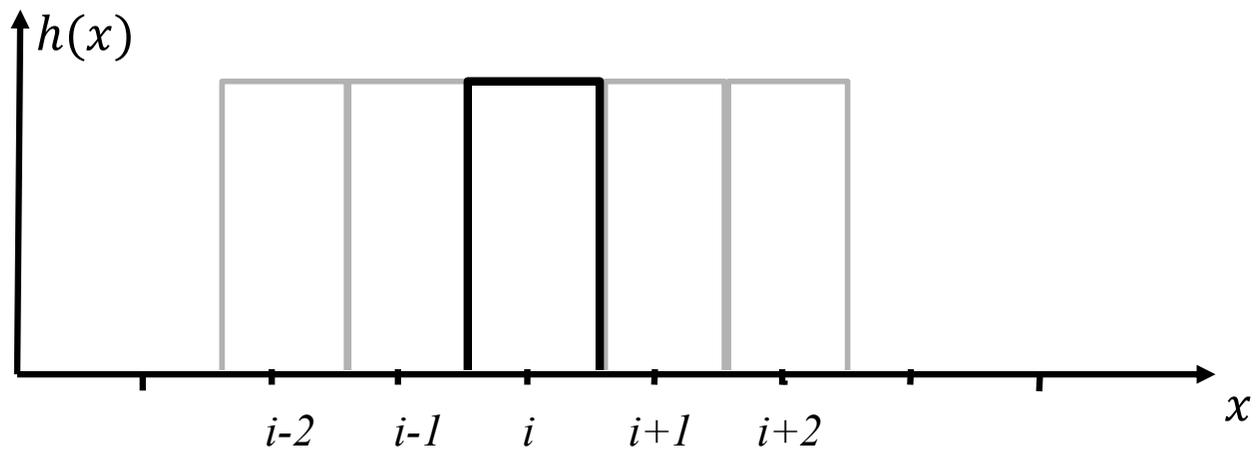
$$f(x) = \sum_i f_i h_i(x)$$

$$\int h_k(x) f(x) dx = \int h_k(x) (\sum_i f_i h_i(x)) dx = \sum_i f_i \int h_k(x) h_i(x) dx$$

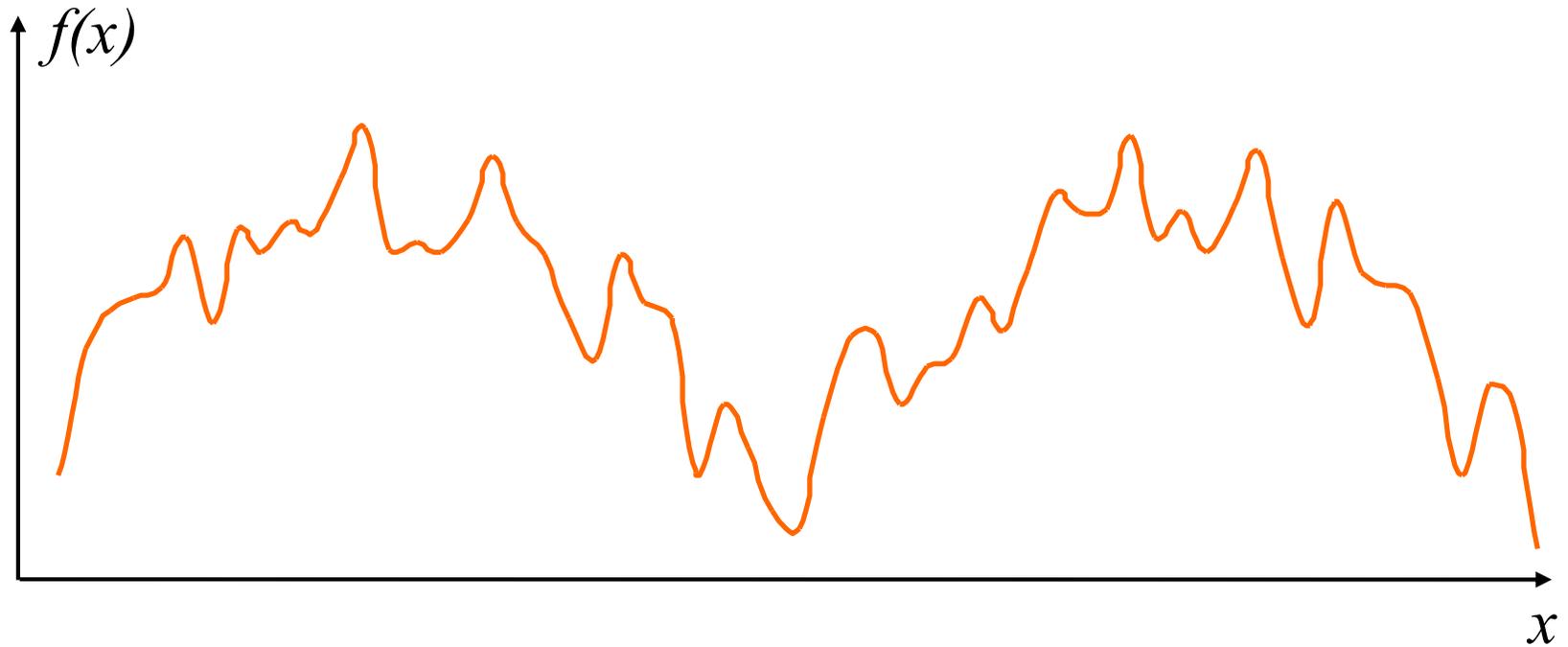
$$\int h_k(x) h_i(x) dx = \begin{cases} 0, & \text{se } i \neq j \\ 1, & \text{se } i = j \end{cases}$$



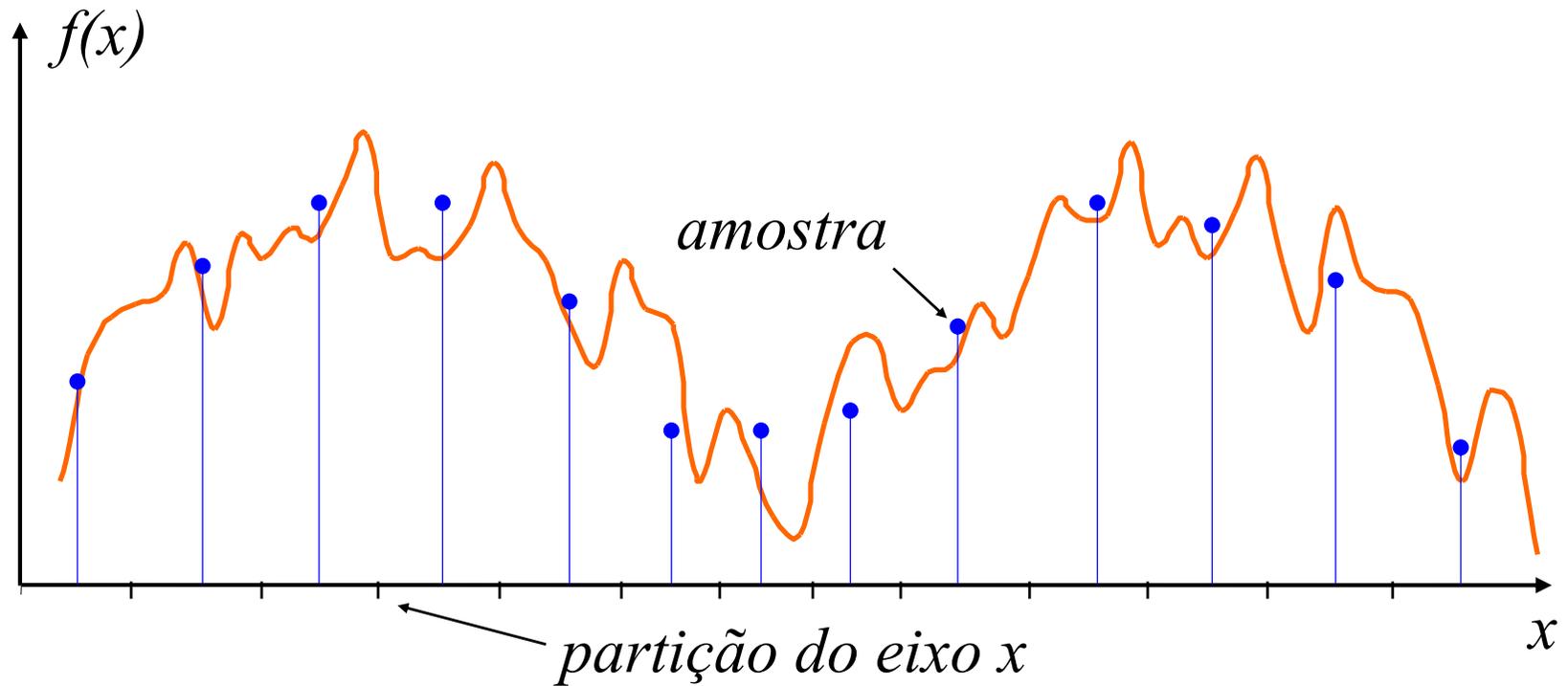
$$f_k = \int f(x) h_k(x) dx = \int_k^{k+1} f(x) dx$$



Amostragem, quantização e codificação de $f(x)$



Amostragem, quantização e codificação de $f(x)$

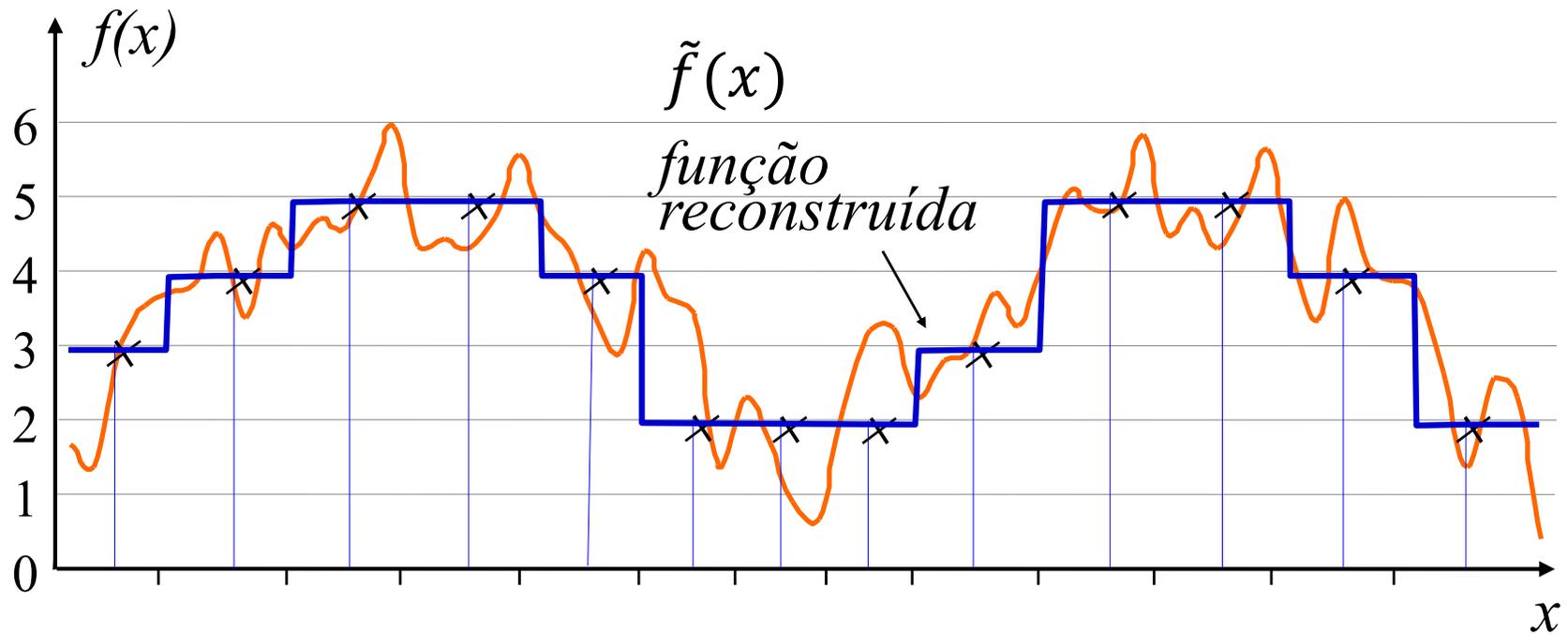


Amostragem, quantização e codificação de $f(x)$

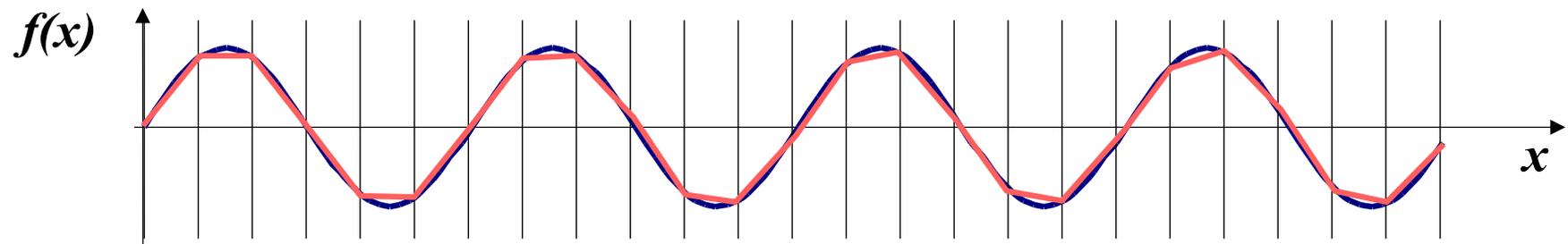
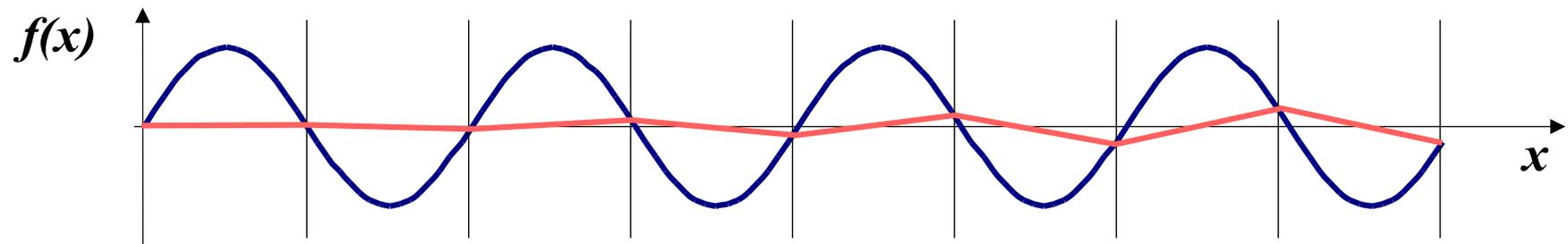


$$f_i = (3, 4, 5, 5, 4, 2, 2, 3, 5, 5, 4, 2)$$

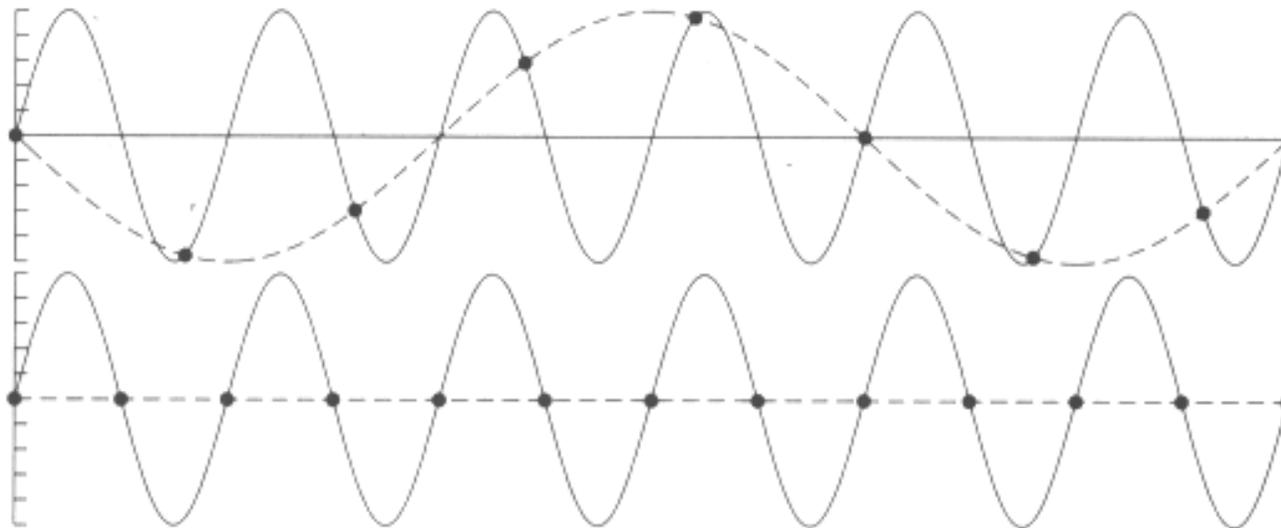
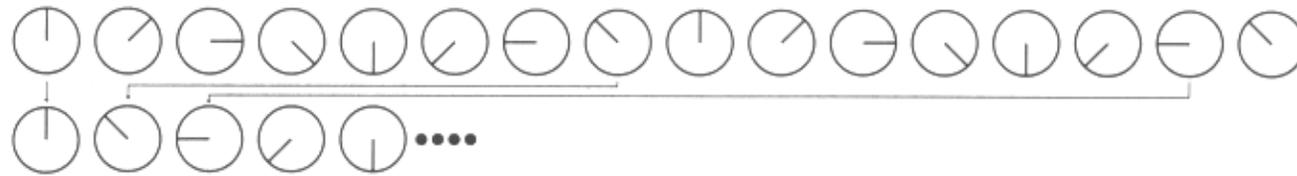
Amostragem, quantização e codificação de $f(x)$



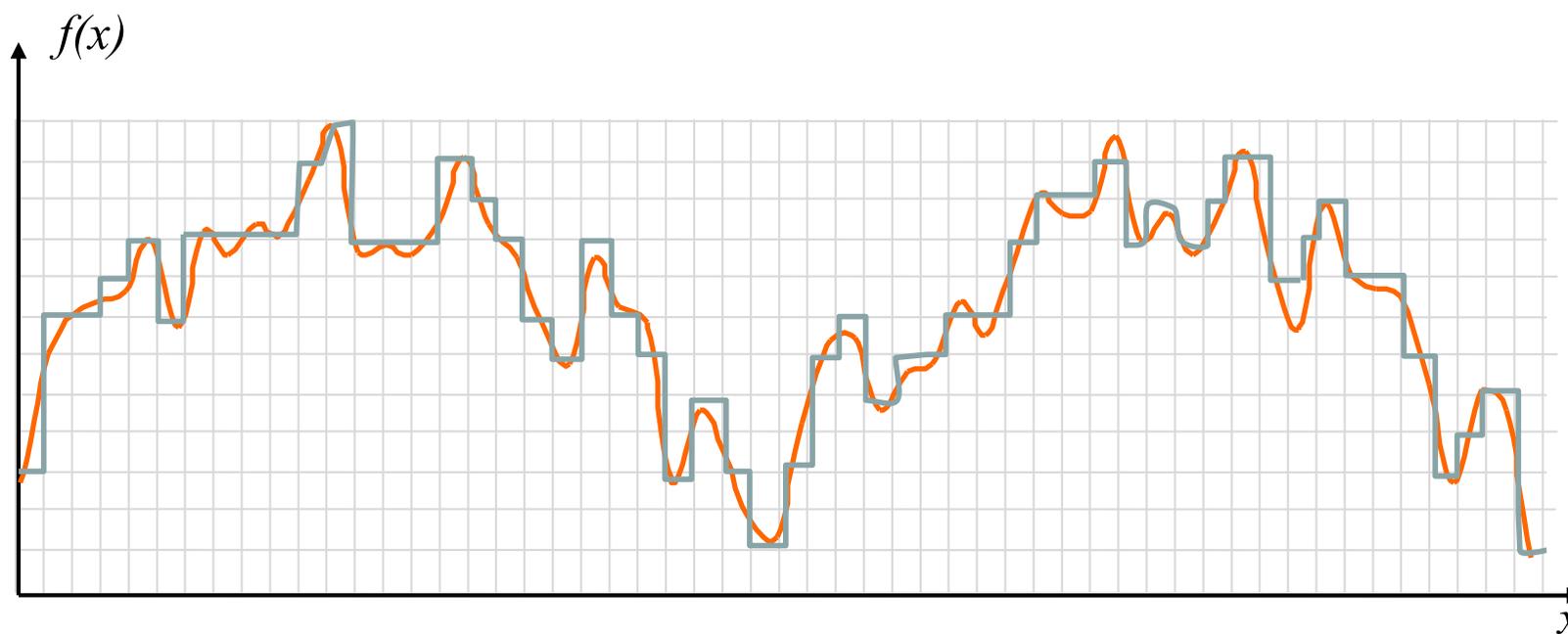
Frequência de Amostragem



Sinal sub-amostrado



Amostragem mínima $f(x)$



Convolução

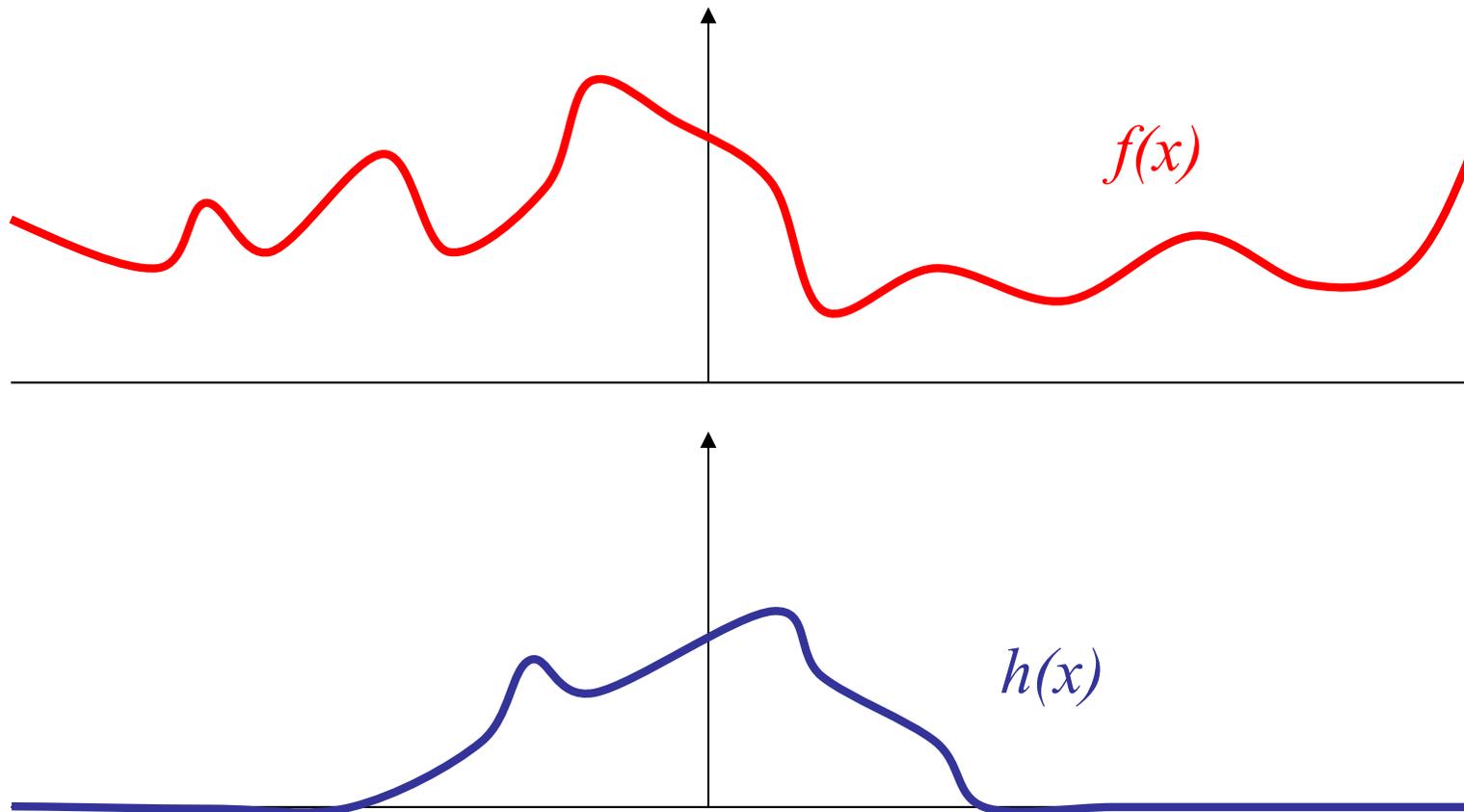
$$h(x) = f \otimes g = \int_{-\infty}^{\infty} f(u)g(x-u)du$$

$$h(x) = \int_{t=-\infty}^{t=\infty} g(t-x)f(x)dt$$

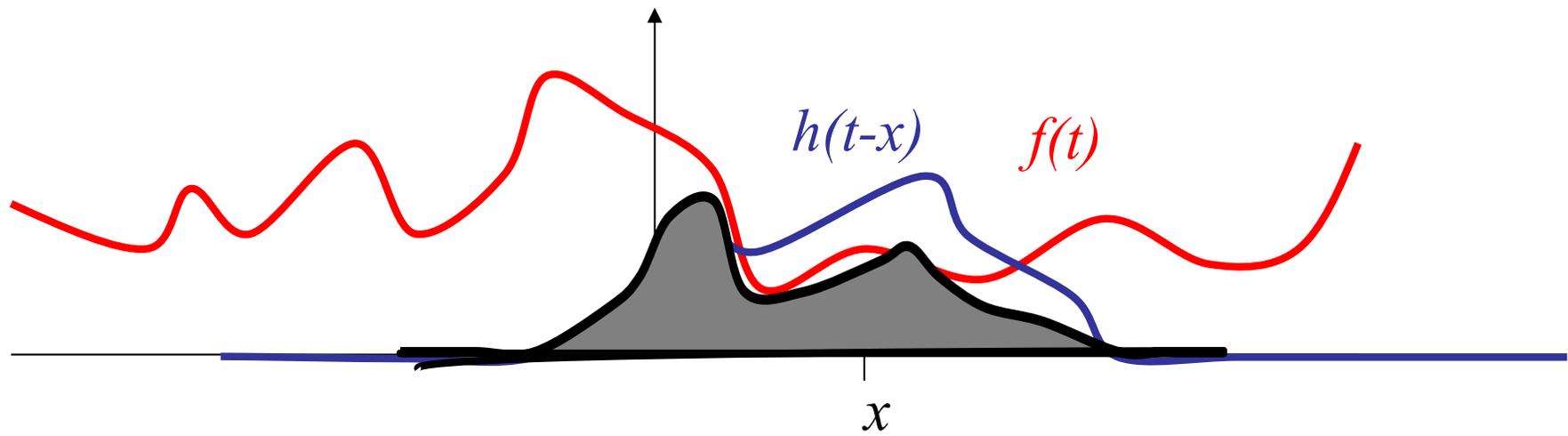
$$h_i = \sum_{k=0}^{n-1} g_{(k-i)}f_i$$

Convolution

- Pictorially



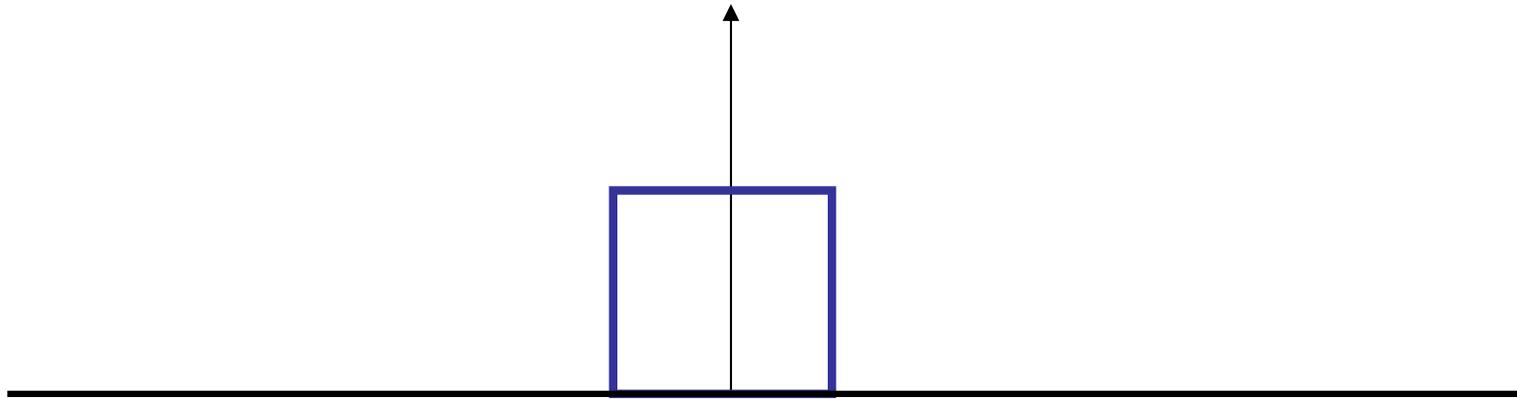
Convolution



Convolution

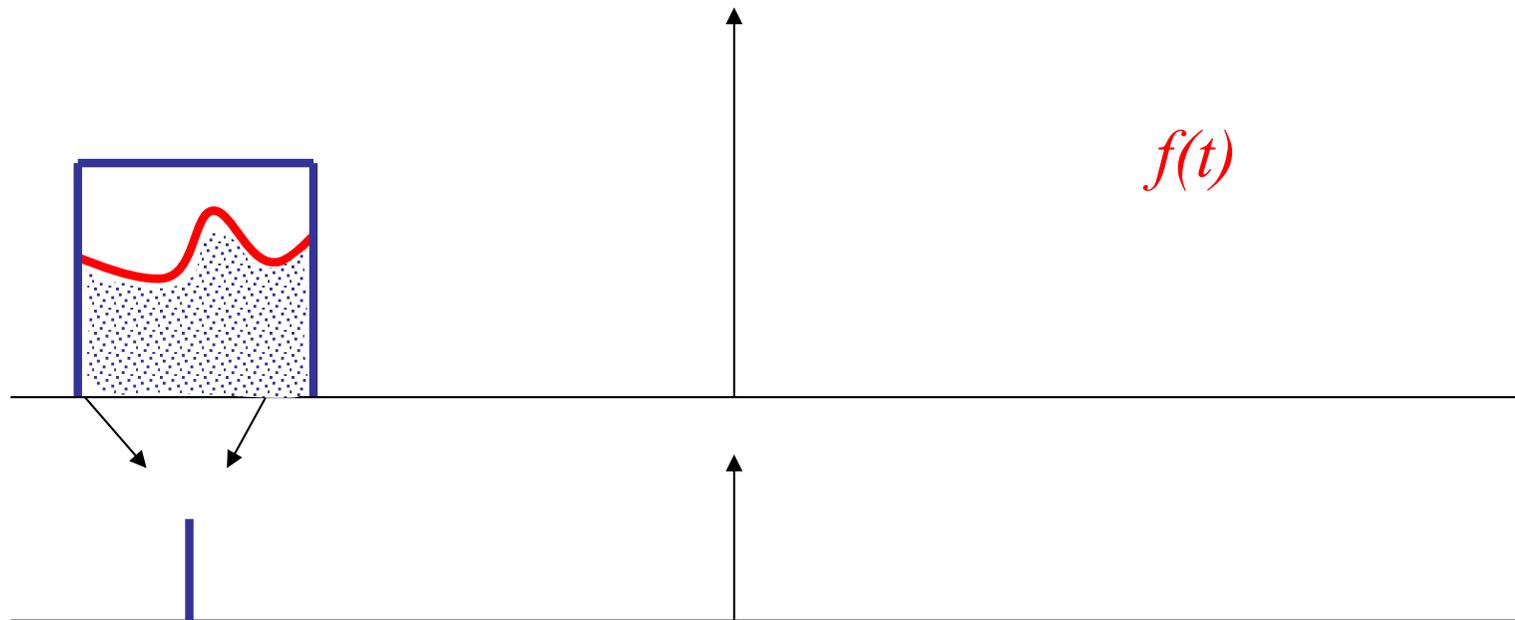
- Consider the function (box filter):

$$h(x) = \begin{cases} 0 & x < -\frac{1}{2} \\ 1 & -\frac{1}{2} \leq x \leq \frac{1}{2} \\ 0 & x > \frac{1}{2} \end{cases}$$



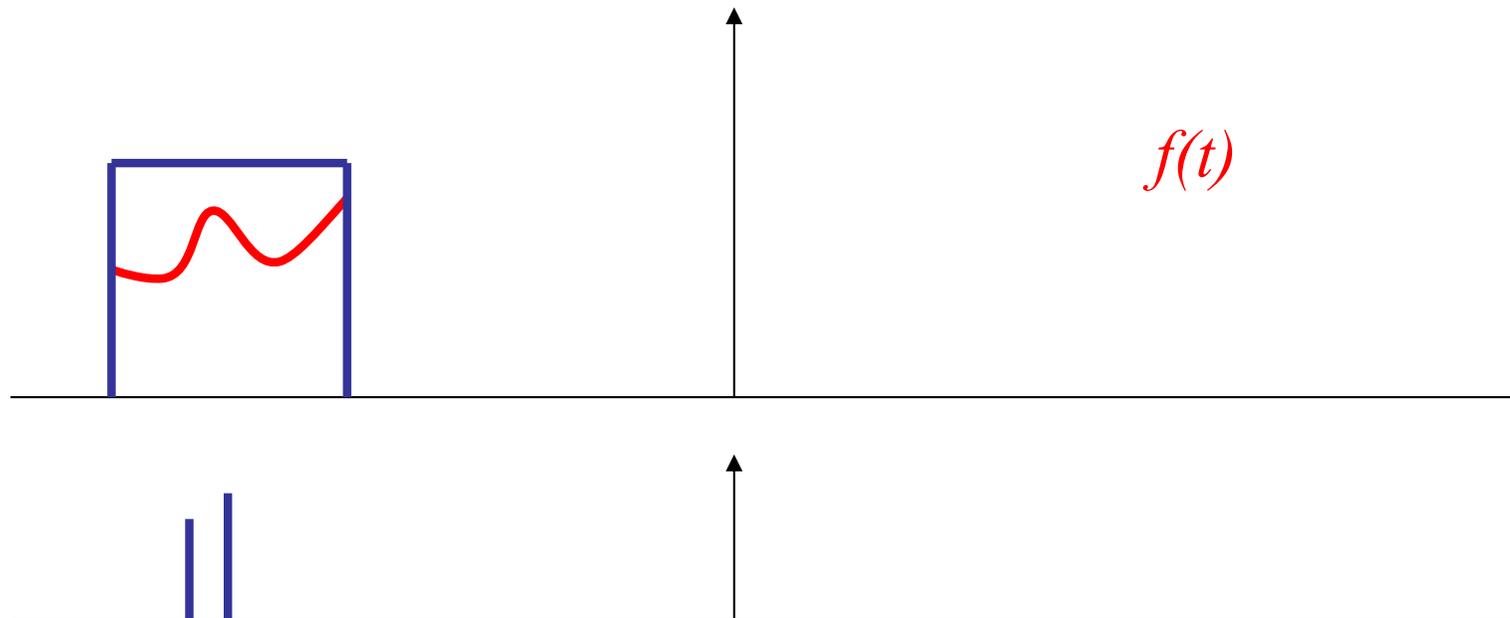
Convolution

- This function ***windows*** our function $f(x)$.



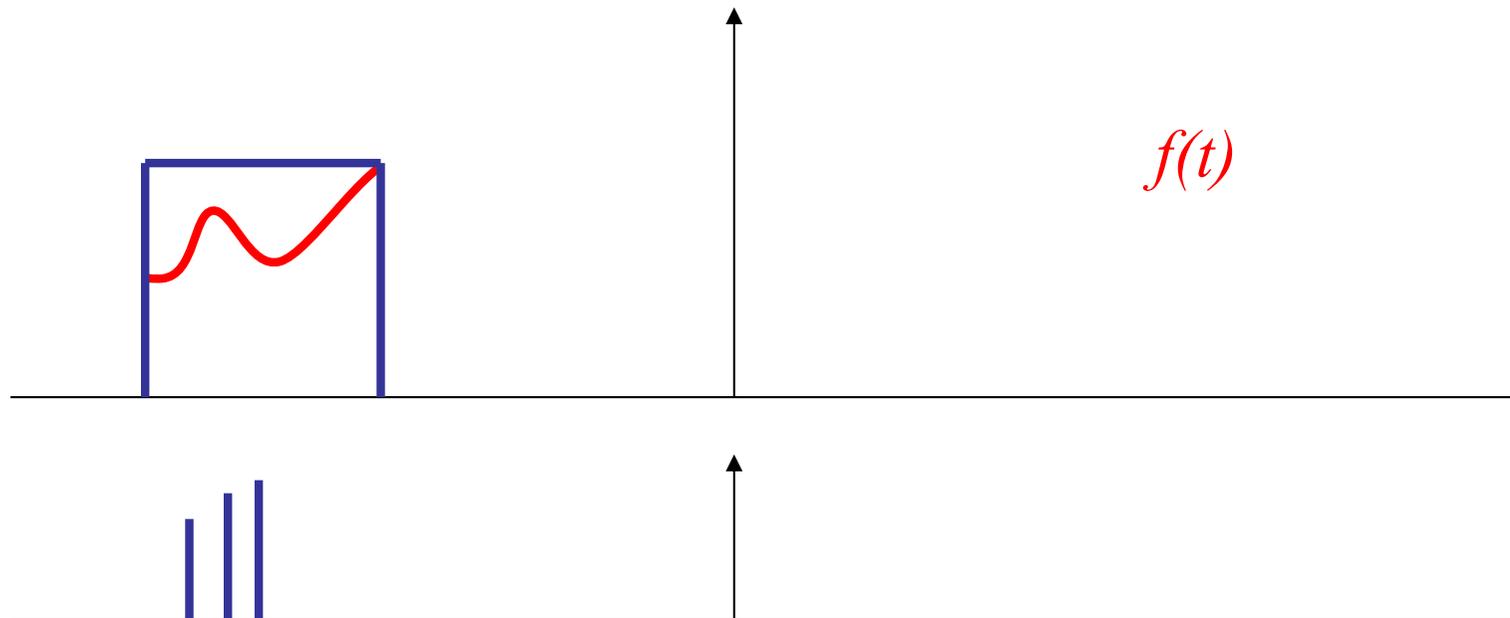
Convolution

- This function *windows* our function $f(x)$.



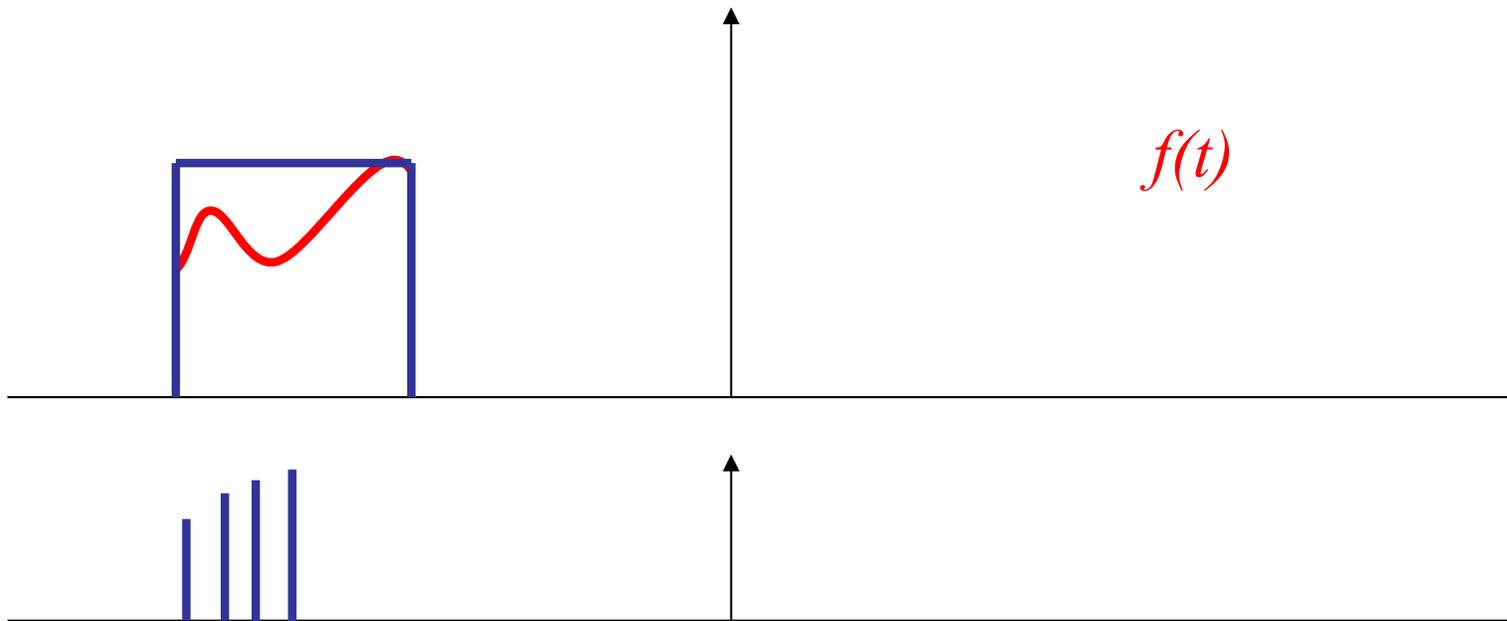
Convolution

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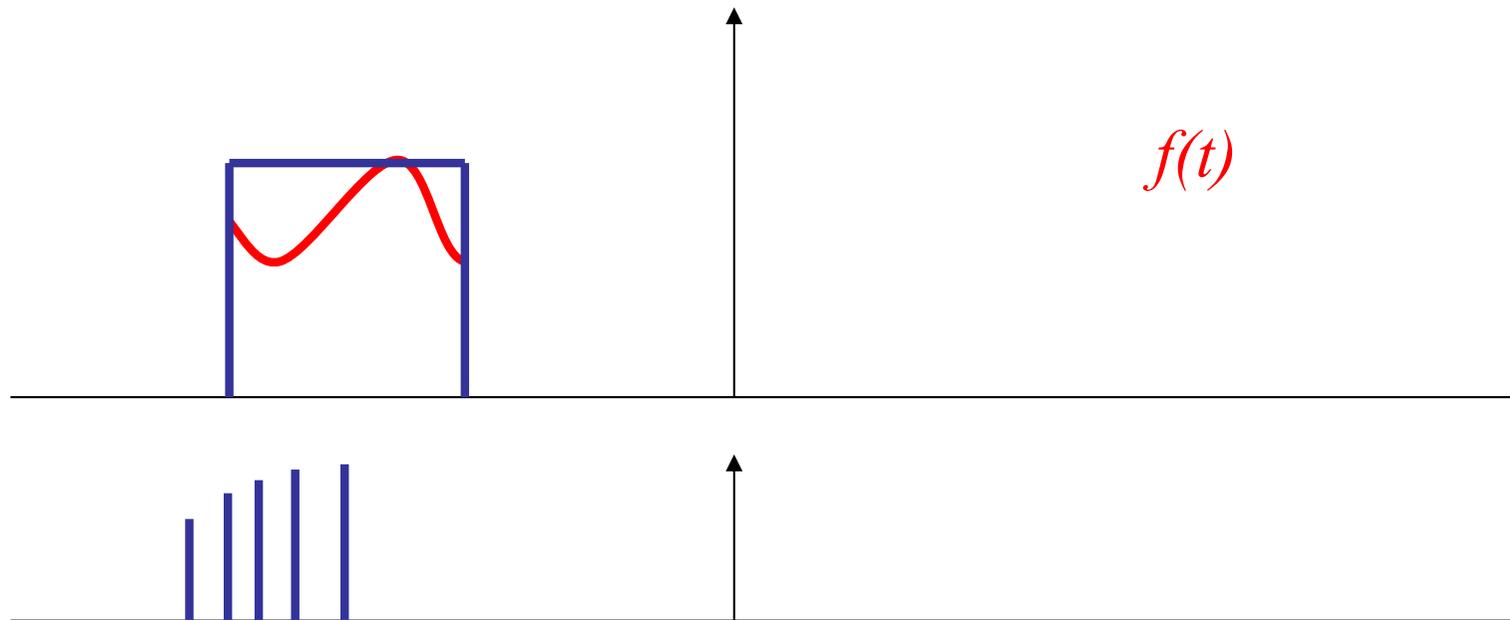
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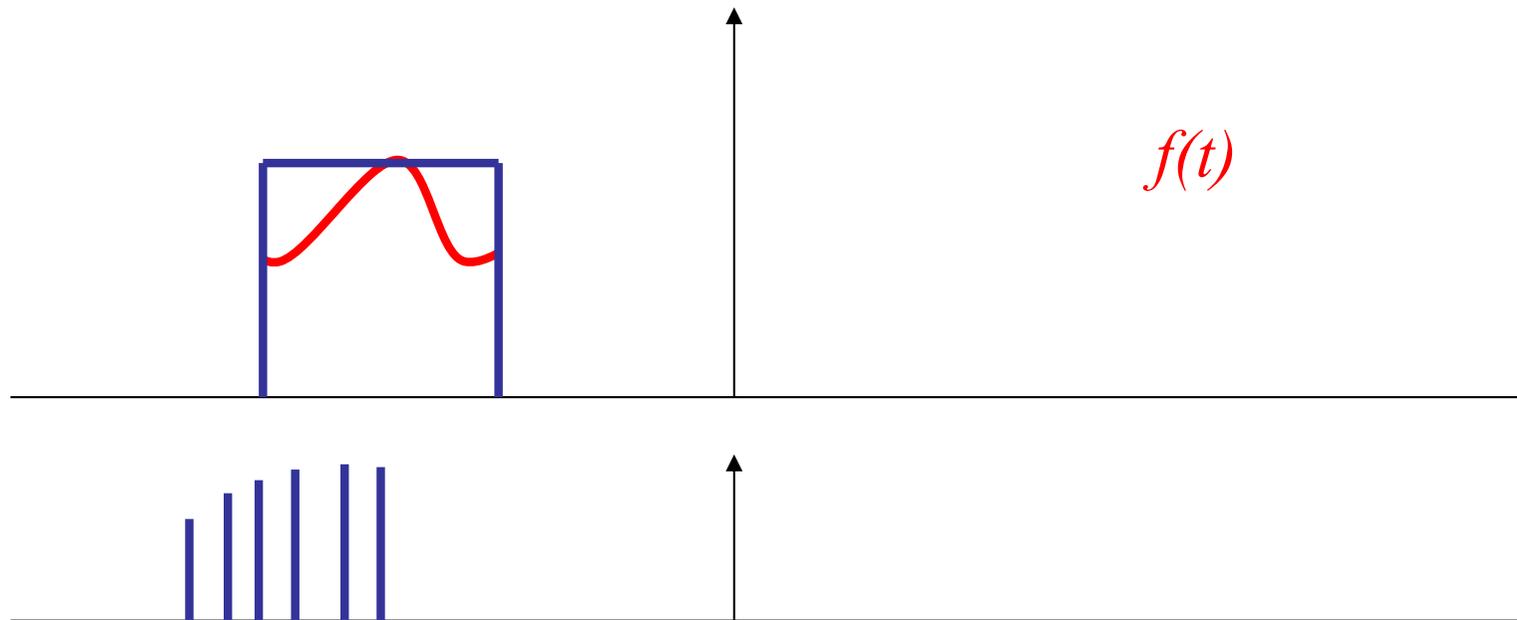
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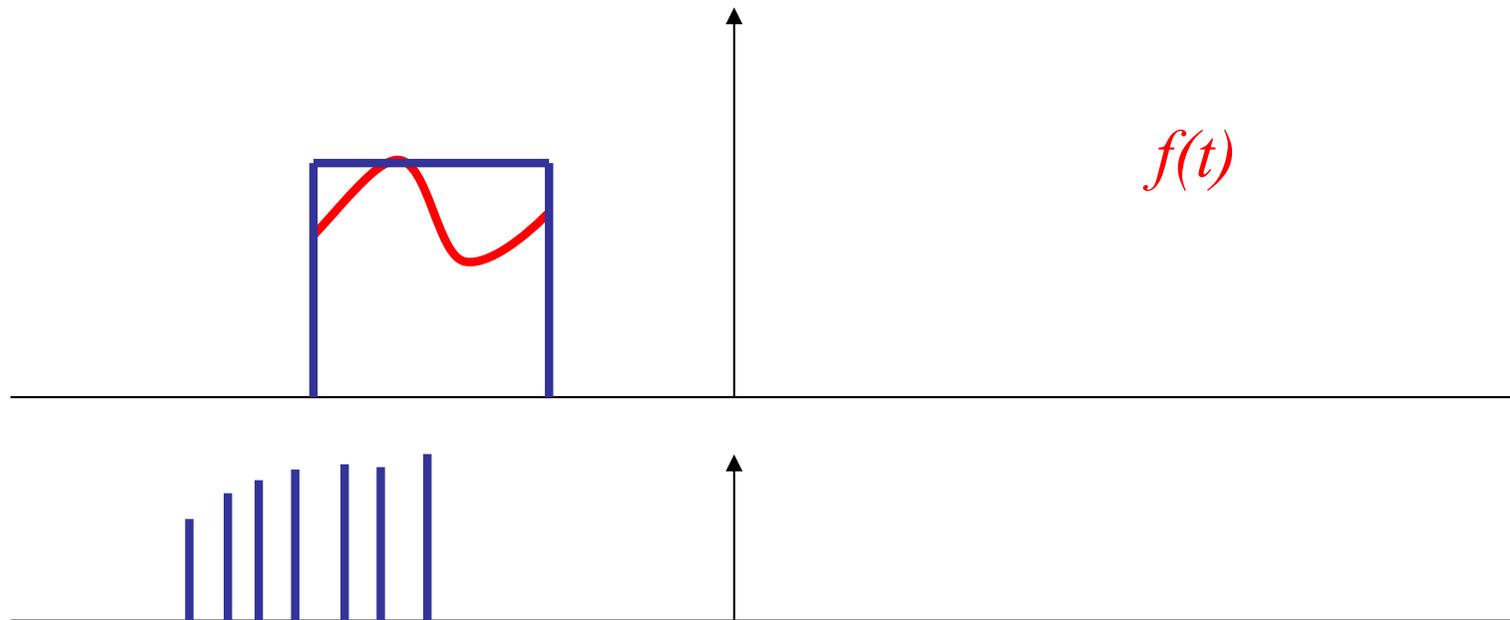
Convolution

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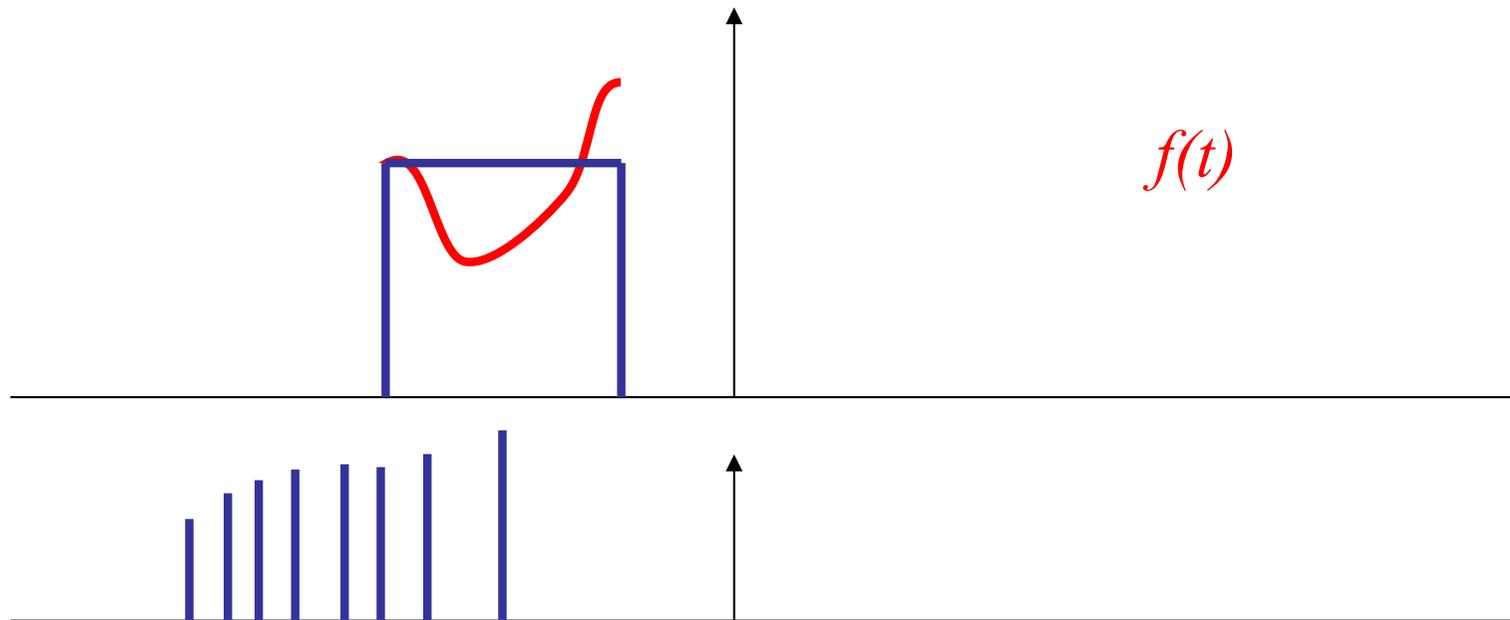
Convolution

- This function *windows* our function $f(x)$.



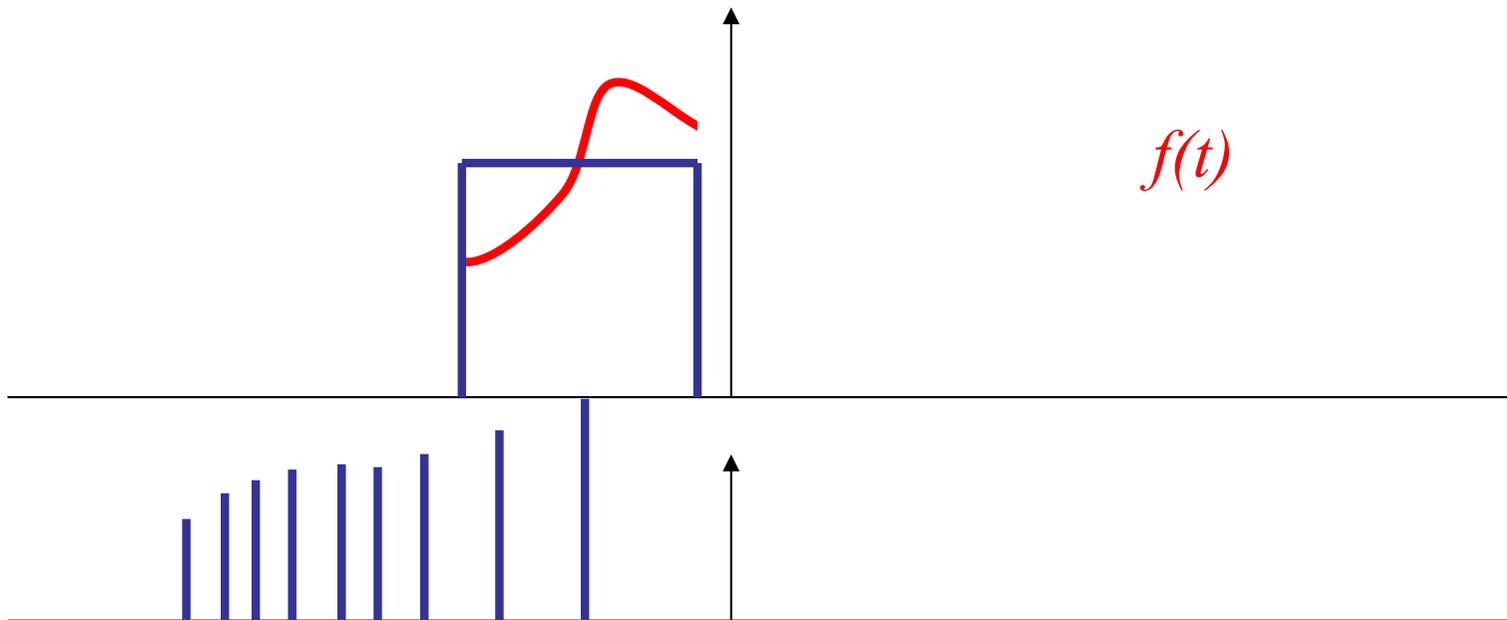
Convolution

- This function *windows* our function $f(x)$.



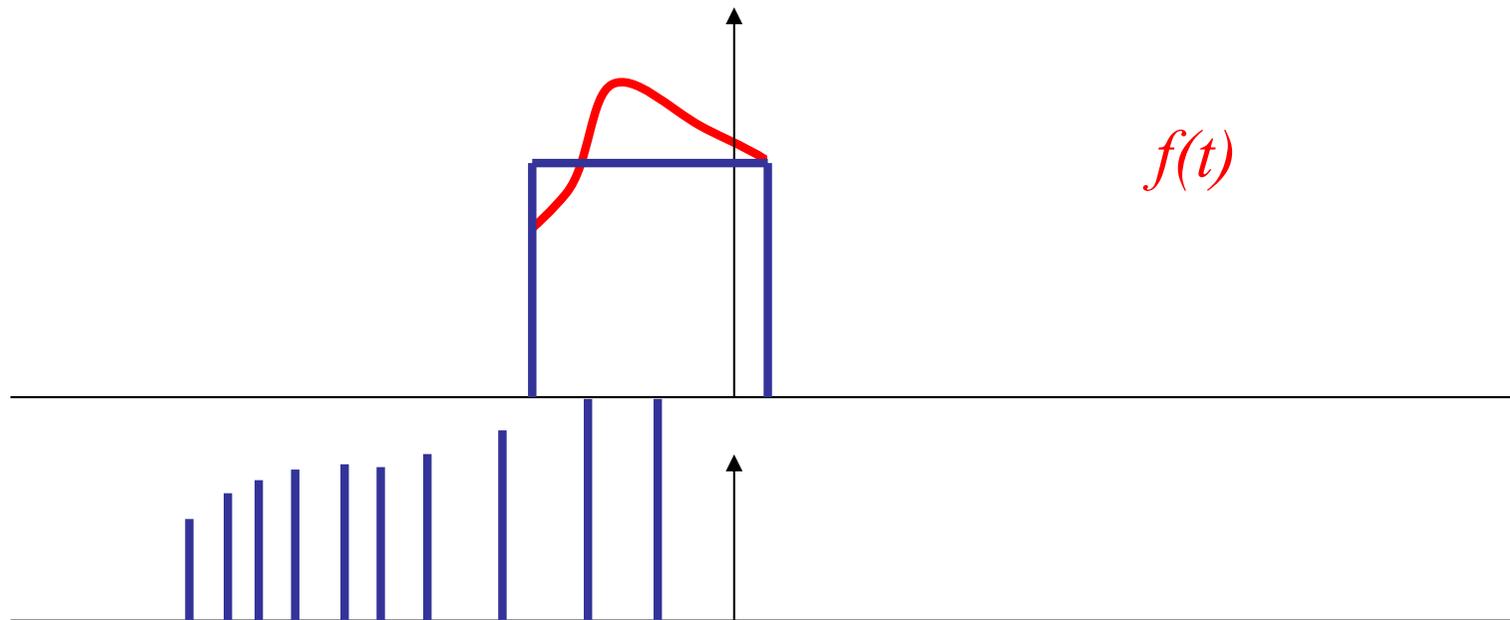
Convolution

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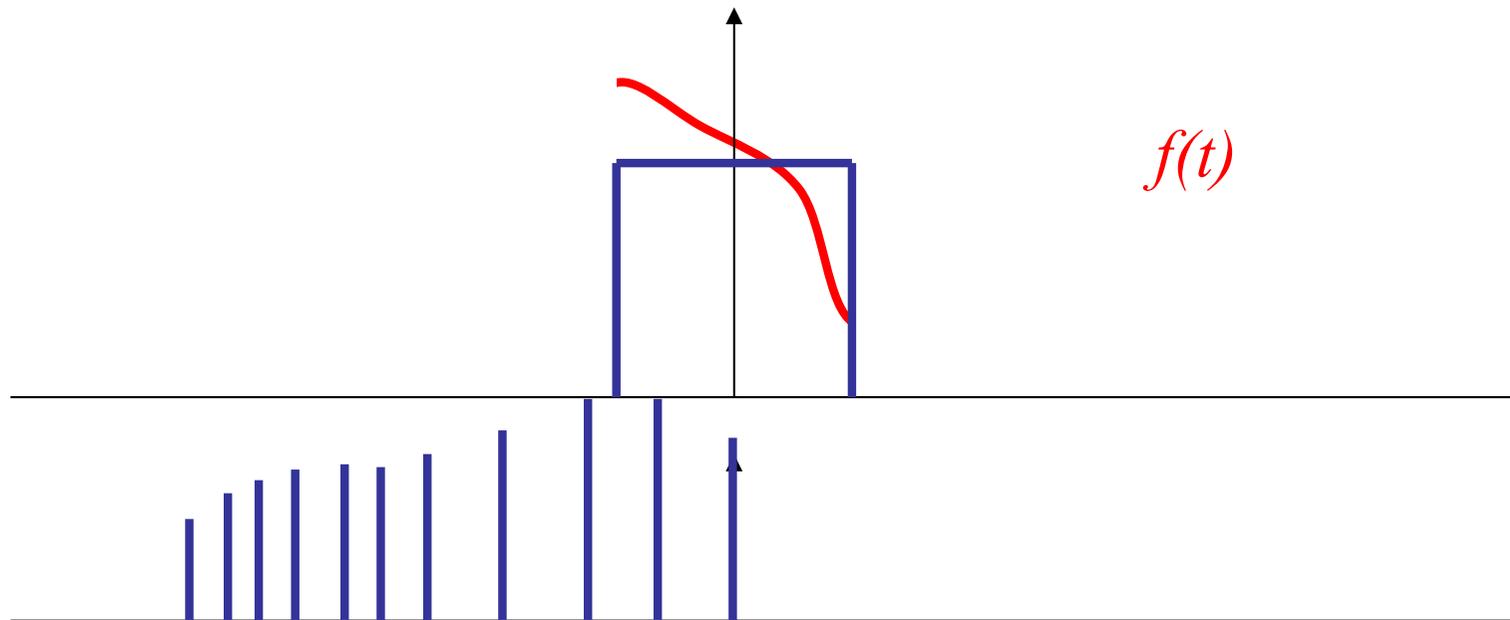
Convolution

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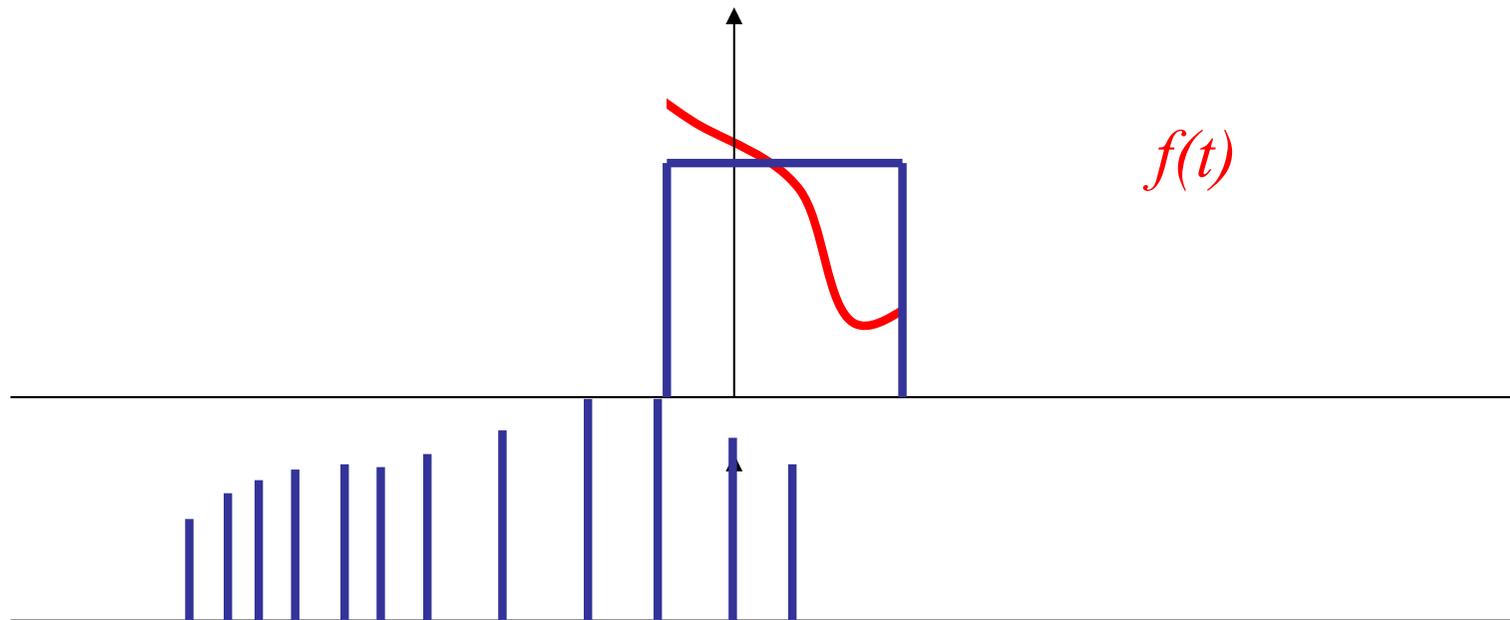
Convolution

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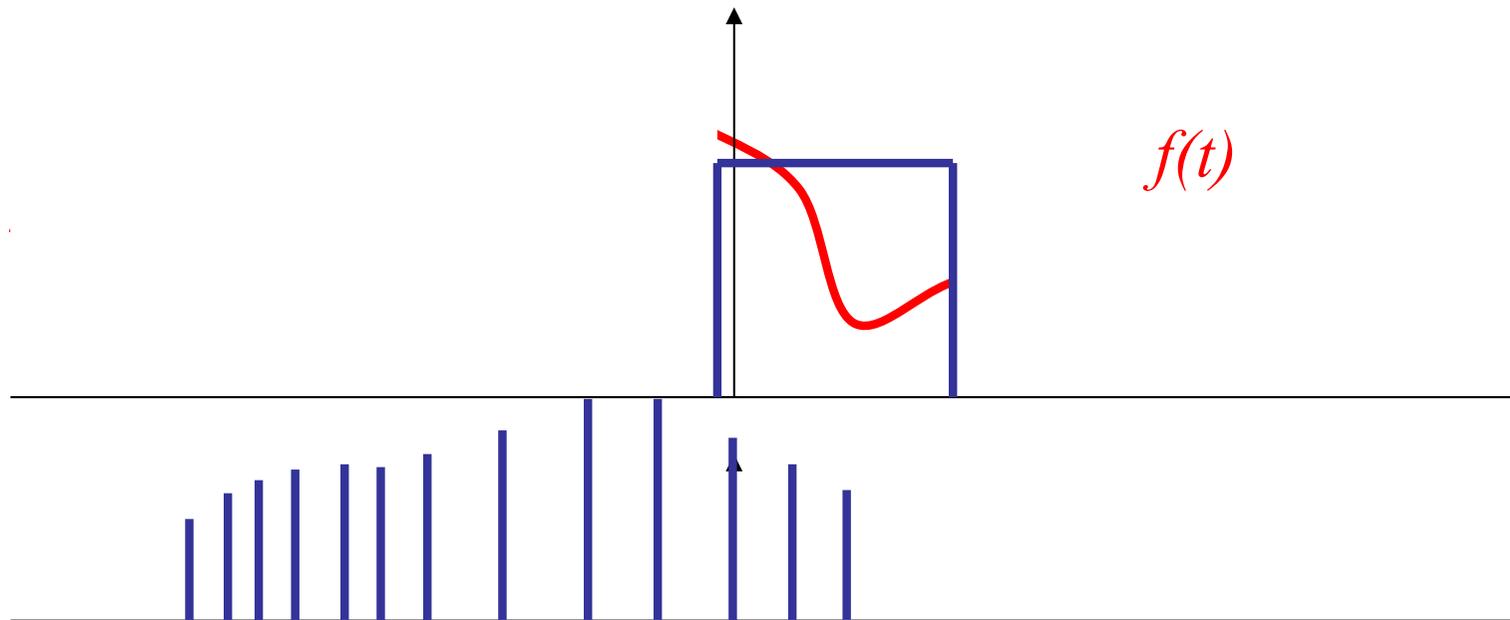
Convolution

- This function *windows* our function $f(x)$.



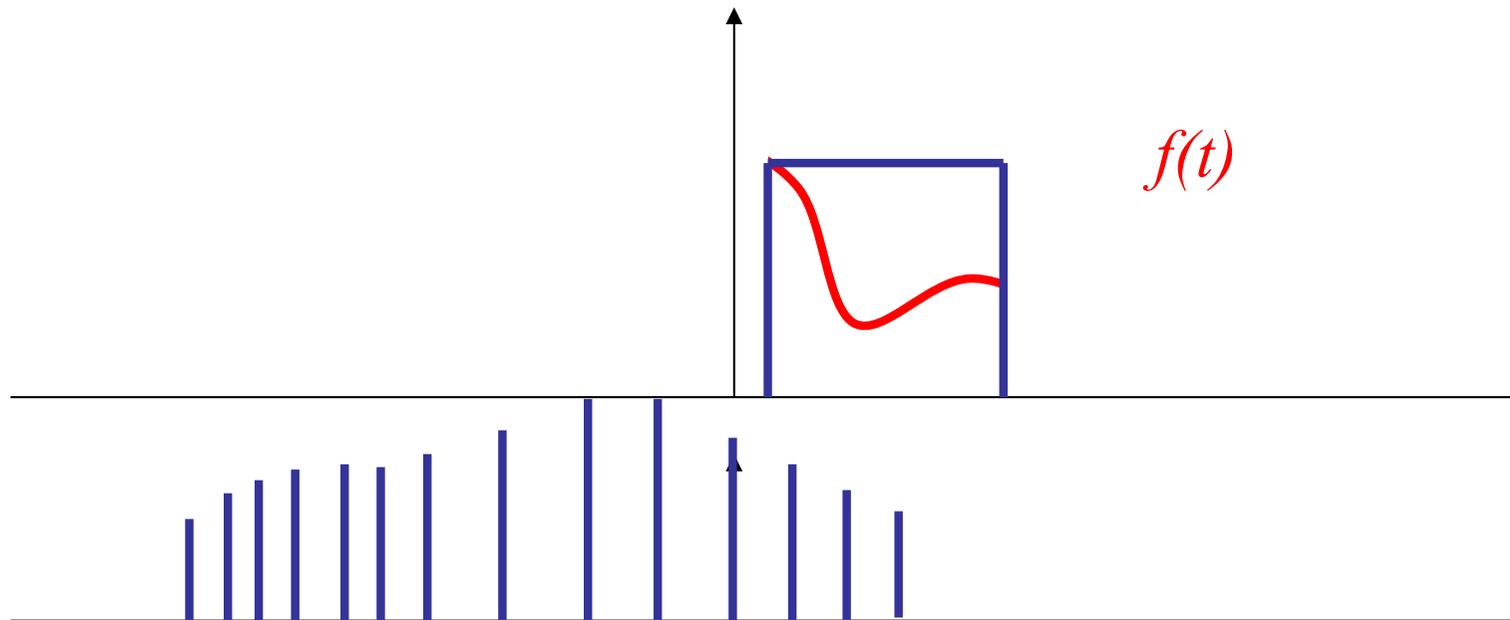
Convolution

- This function *windows* our function $f(x)$.



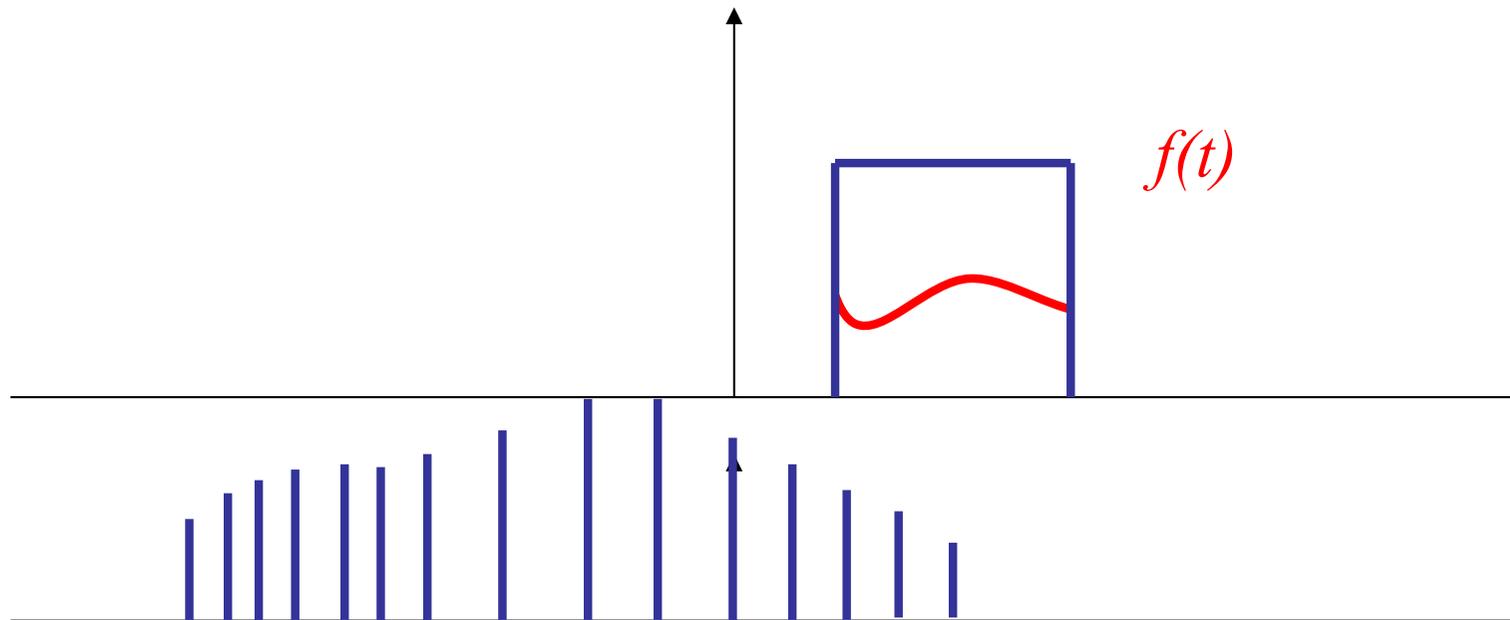
Convolution

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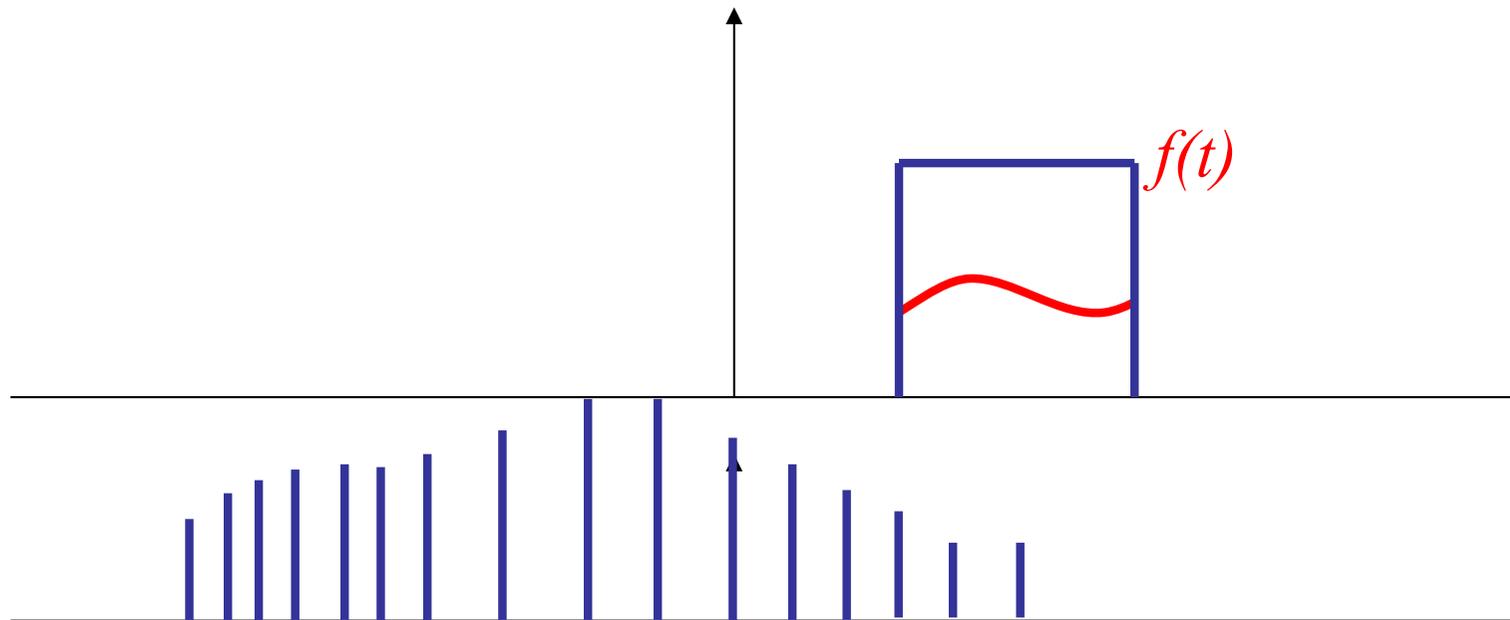
Convolution

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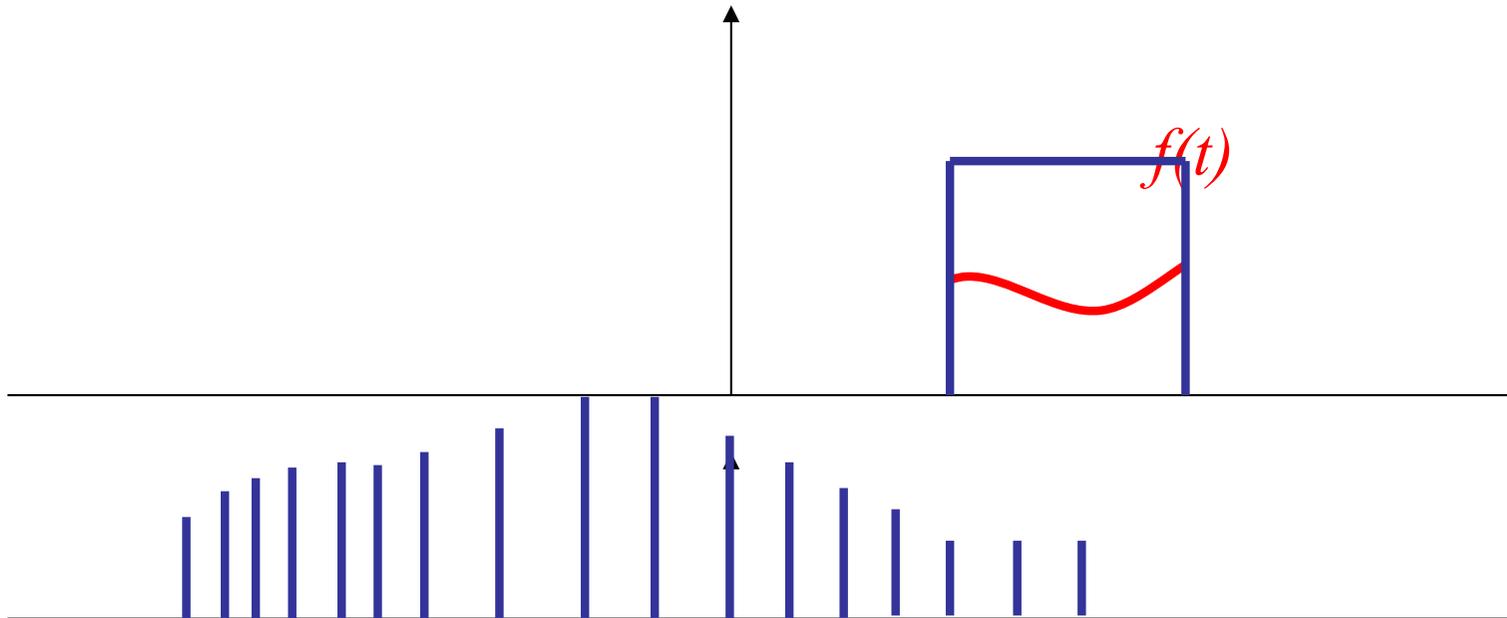
Convolution

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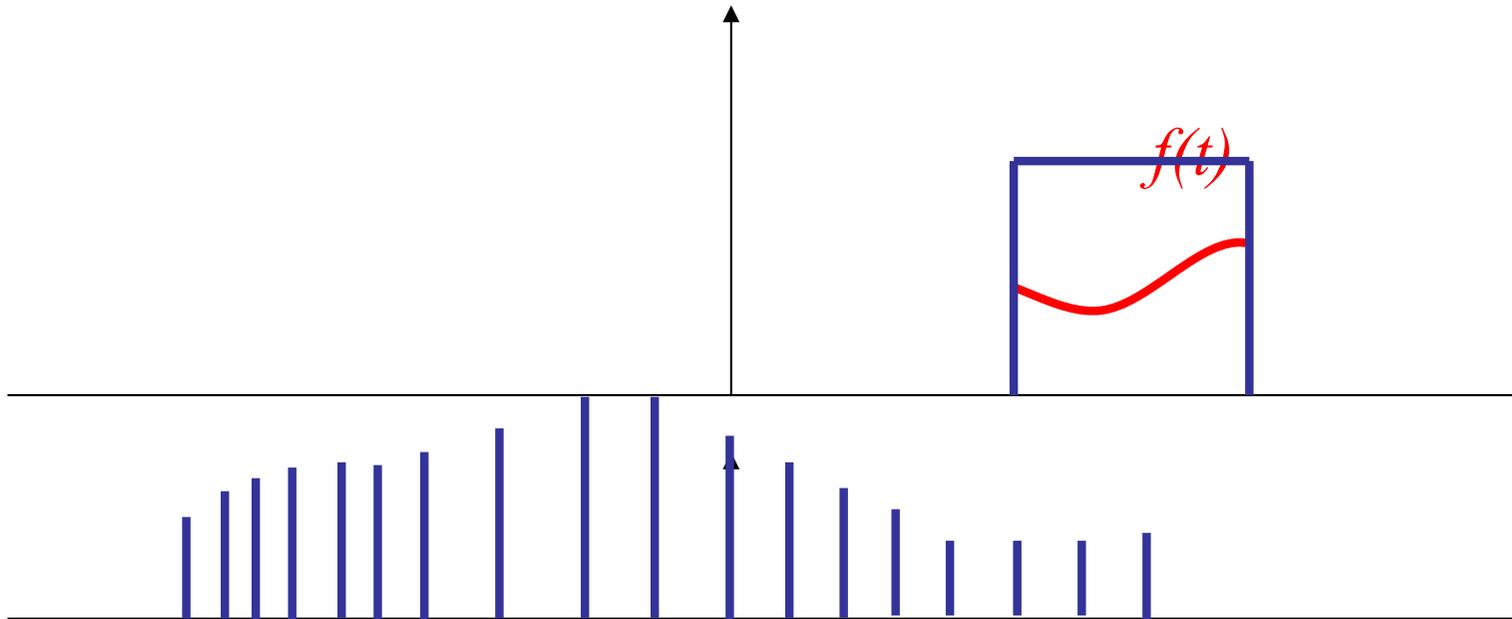
Convolution

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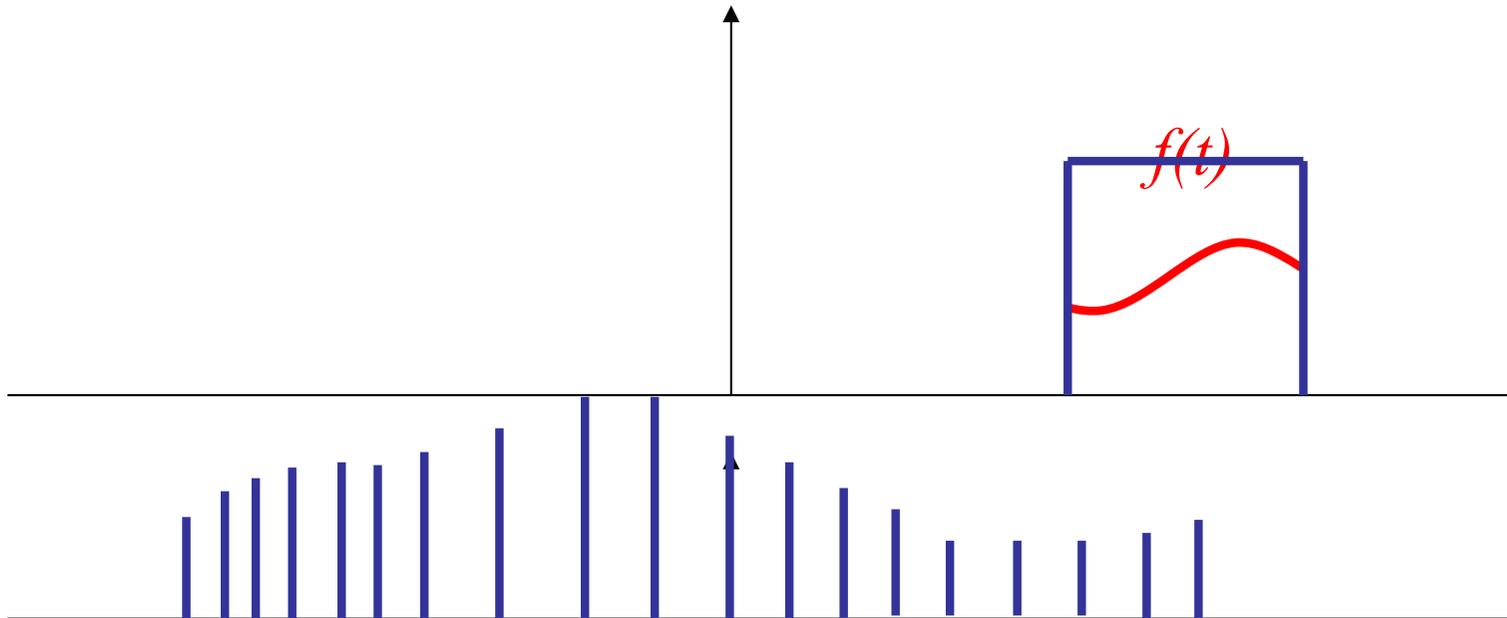
Convolution

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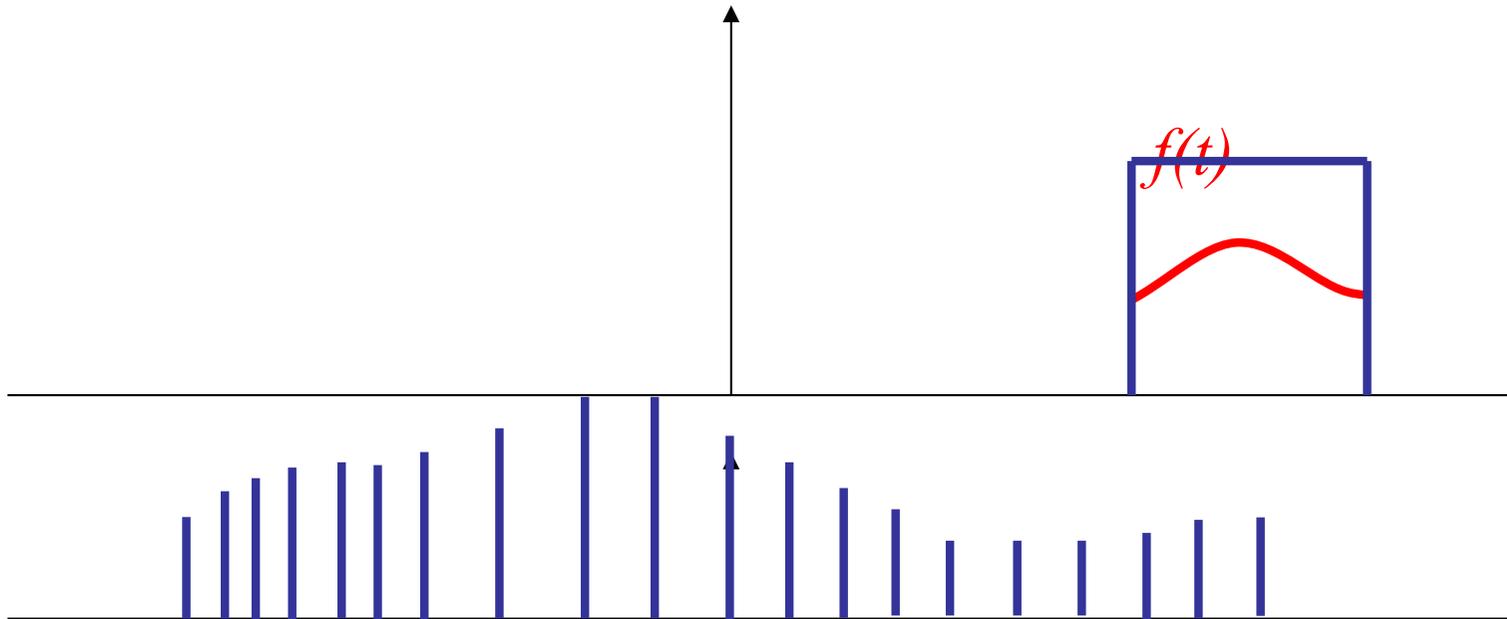
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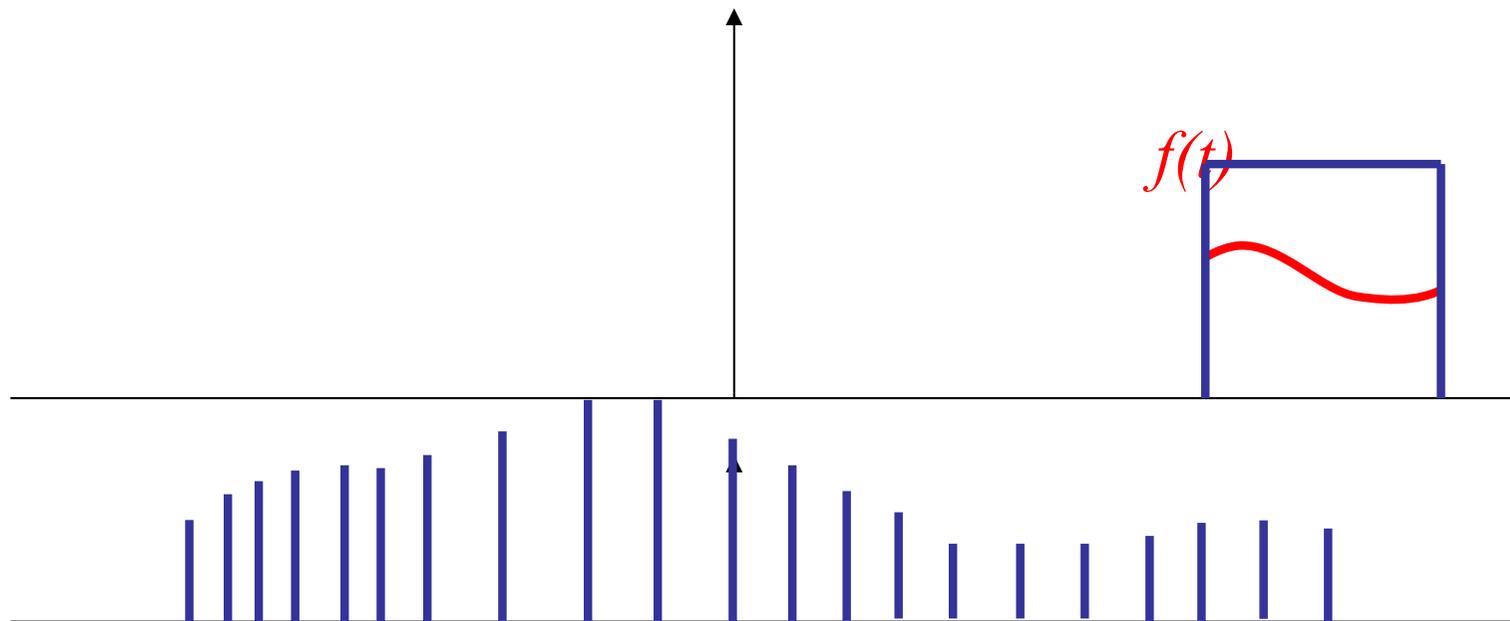
Convolution

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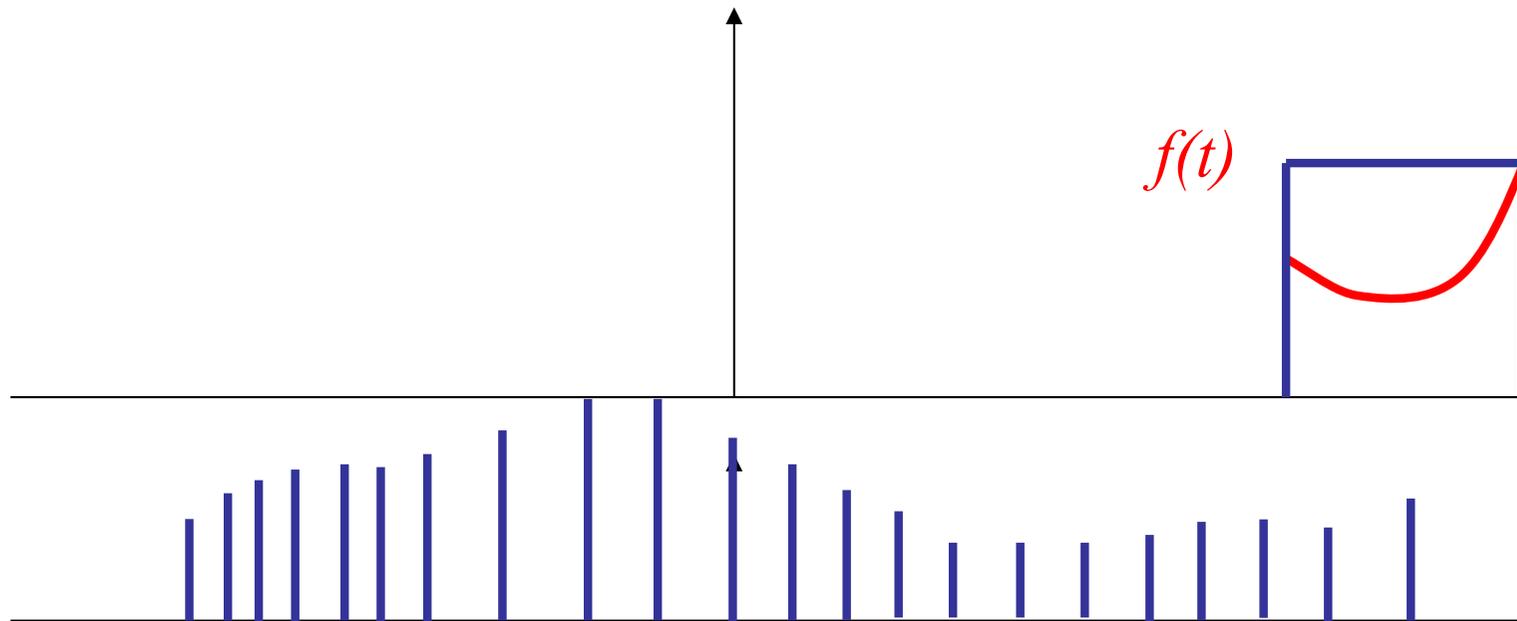
Convolution

- This function *windows* our function $f(x)$.



Convolution

- This function *windows* our function $f(x)$.



Convolution

- This particular convolution smooths out some of the high frequencies in $f(x)$.

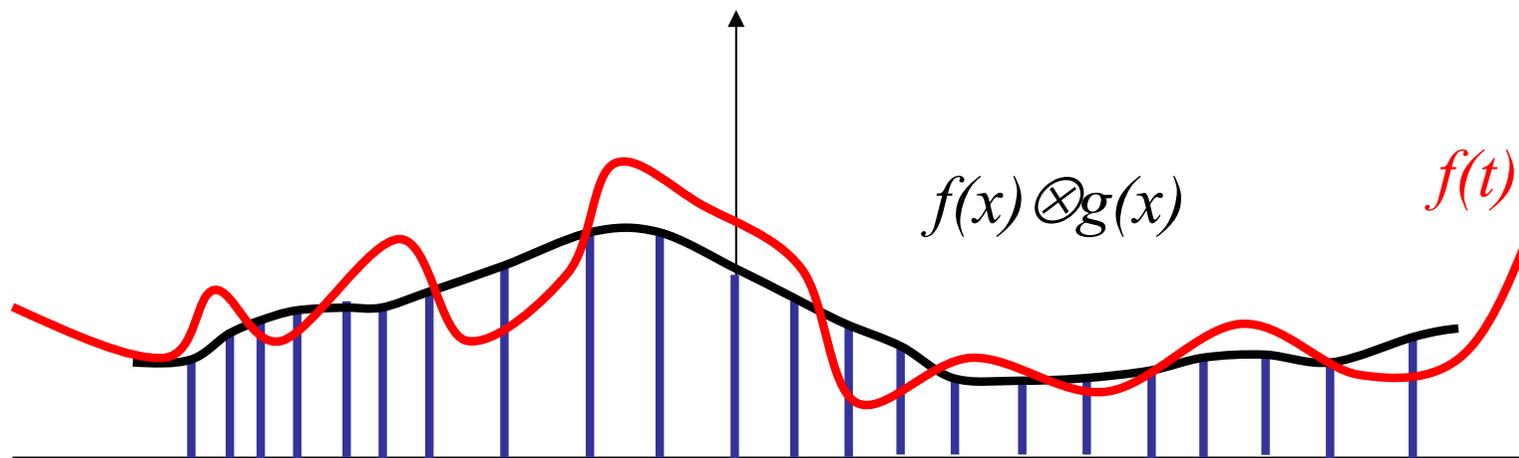


Ilustração da convolução

$$h(x) = \int_{t=-\infty}^{t=\infty} g(t-x) f(x) dt$$

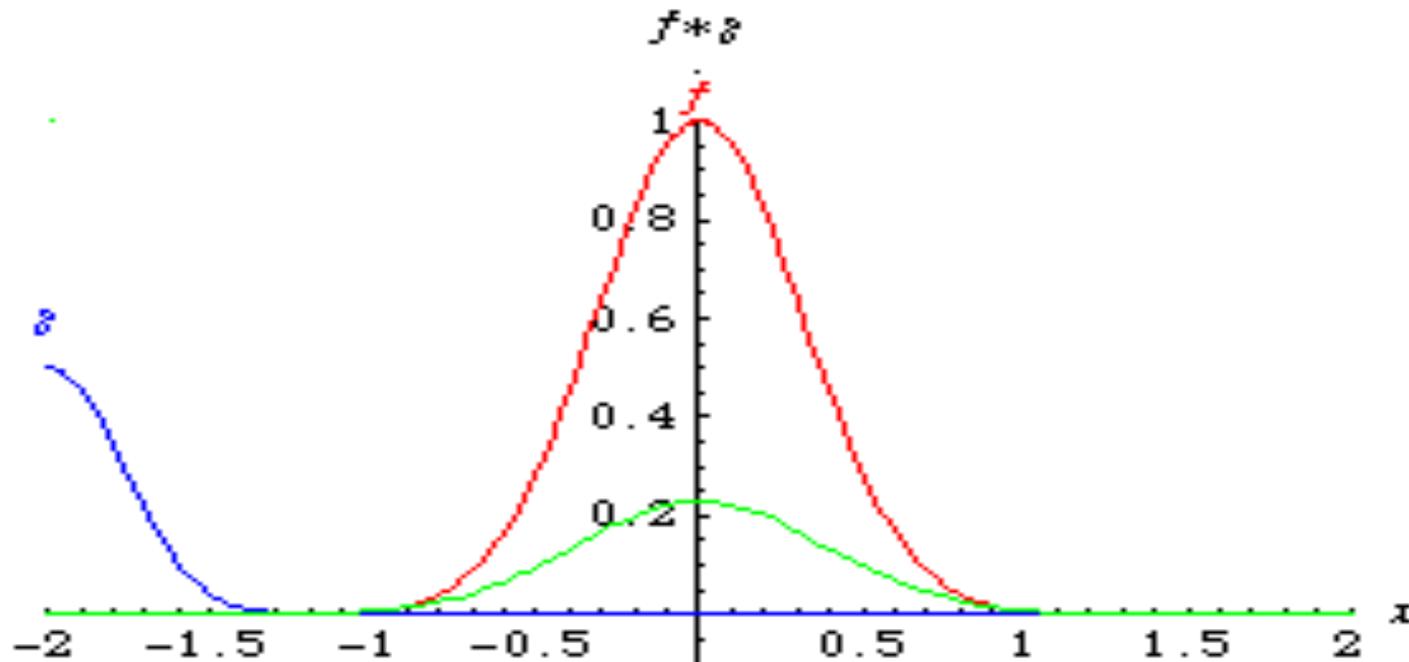
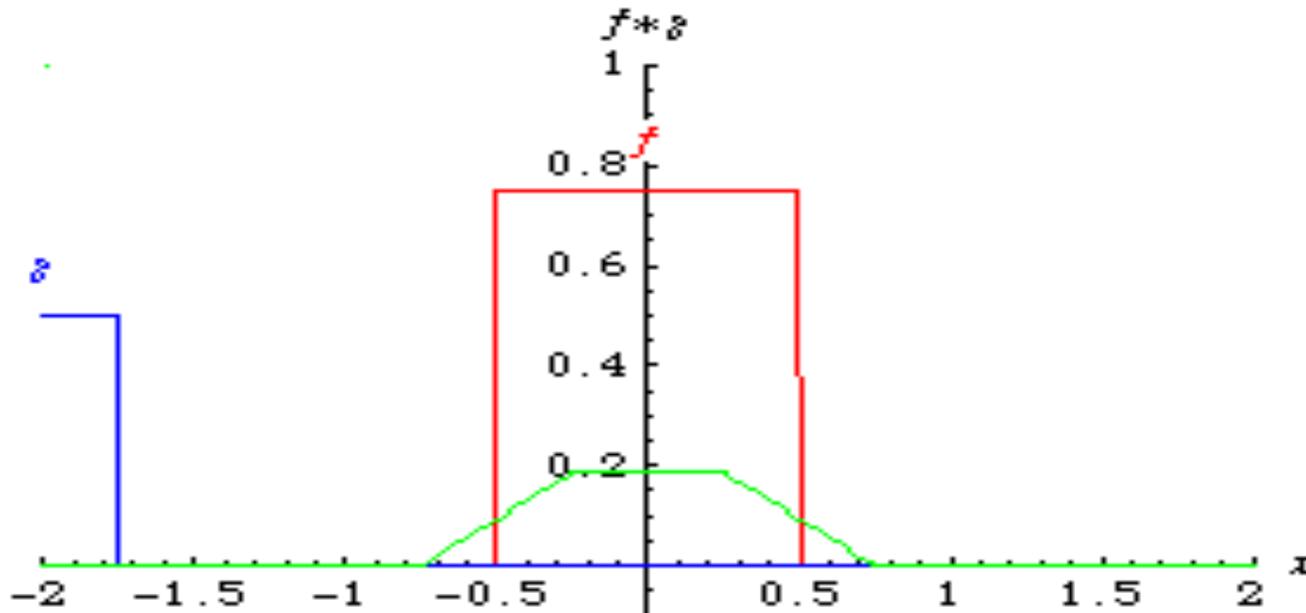


Ilustração da convolução

$$h(x) = \int_{t=-\infty}^{t=\infty} g(t-x) f(x) dt$$



Fourier

Jean Baptiste Joseph Fourier (1768-1830)
Paper de 1807 para o *Institut de France*



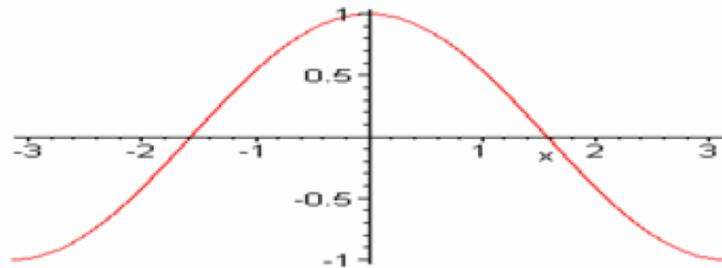
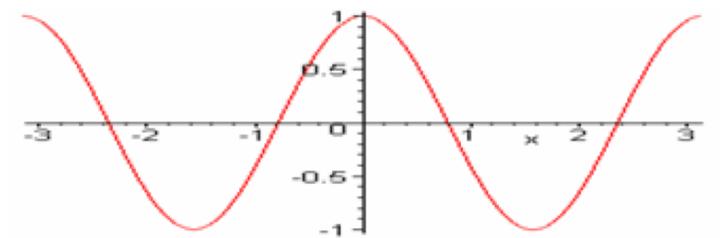
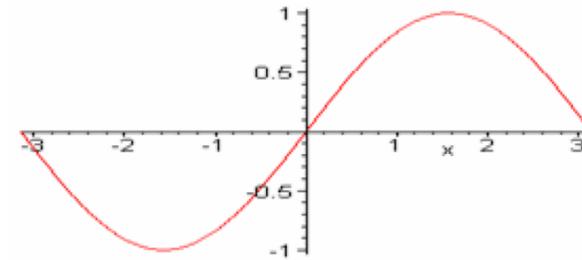
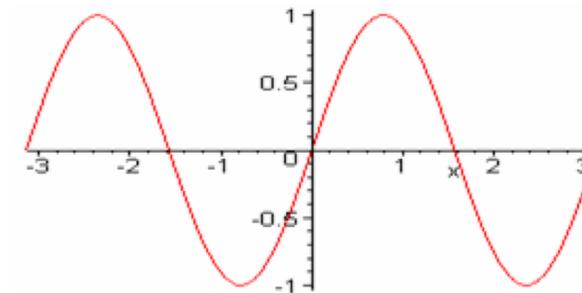
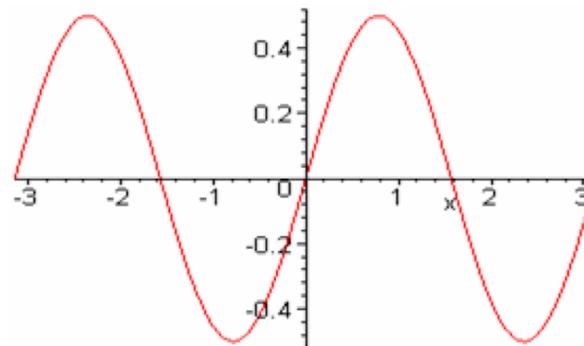
$$f(x) = a_0 + \sum_{k=1}^{\infty} \left(a_k \cos\left(\frac{2\pi kx}{n}\right) + b_k \sin\left(\frac{2\pi kx}{n}\right) \right)$$

onde:

$$a_0 = \frac{1}{n} \int f(x) dx$$

$$a_k = \frac{1}{n} \int f(x) \cos\left(\frac{2\pi kx}{n}\right) dx$$

$$b_k = \frac{1}{n} \int f(x) \sin\left(\frac{2\pi kx}{n}\right) dx$$

Integrais de senos e cosenos em $[-\pi, \pi]$ $\cos(nx)$  $n = 1$ $\sin(nx)$  $n = 2$  $\sin(nx)\cos(nx)$ 

Áreas se compensam.
Integrais resultam em 0.

Integrais de senos e cosenos em $[-\pi, \pi]$

$$\int_{-\pi}^{\pi} \sin(mx) \sin(nx) dx = \pi \delta_{mn} \quad \text{for } n, m \neq 0$$

$$\int_{-\pi}^{\pi} \cos(mx) \cos(nx) dx = \pi \delta_{mn} \quad \text{for } n, m \neq 0$$

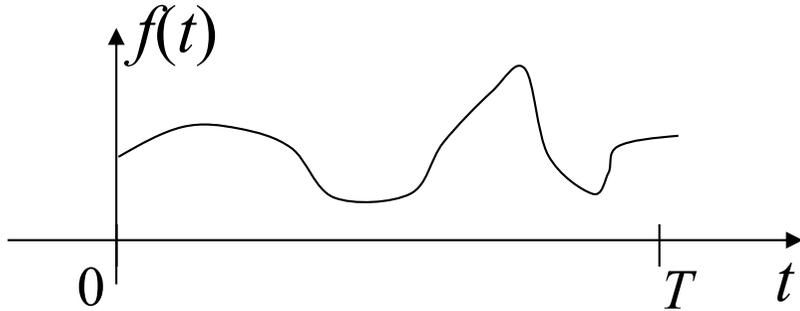
$$\int_{-\pi}^{\pi} \sin(mx) \cos(nx) dx = 0$$

$$\int_{-\pi}^{\pi} \sin(mx) dx = 0$$

$$\int_{-\pi}^{\pi} \cos(mx) dx = 0$$

Funções ortogonais

Série de Fourier: cálculo de a_0



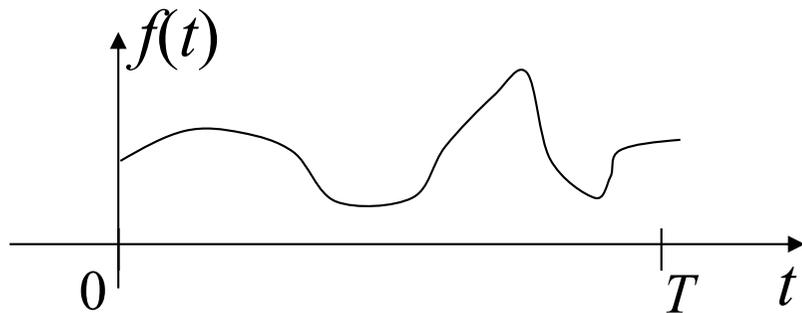
$$f(t) = a_0 + 2 \sum_{k=1}^{\infty} \left(a_k \cos \frac{2\pi kt}{T} + b_k \sin \frac{2\pi kt}{T} \right)$$

$$\int_0^T f(t) dt = \int_0^T a_0 dt + \sum_{k=1}^{\infty} \left(a_k \int_0^T \cos\left(\frac{2\pi nkt}{T}\right) dt + b_k \int_0^T \sin\left(\frac{2\pi kt}{T}\right) dt \right)$$

$$\int_0^T f(t) dt = a_0 T + 0 + 0$$

$$a_0 = \frac{1}{T} \int_0^T f(t) dt$$

Série de Fourier: a_n e b_n



$$f(t) = a_0 + 2 \sum_{k=1}^{\infty} \left(a_k \cos \frac{2\pi k t}{T} + b_k \sin \frac{2\pi k t}{T} \right)$$

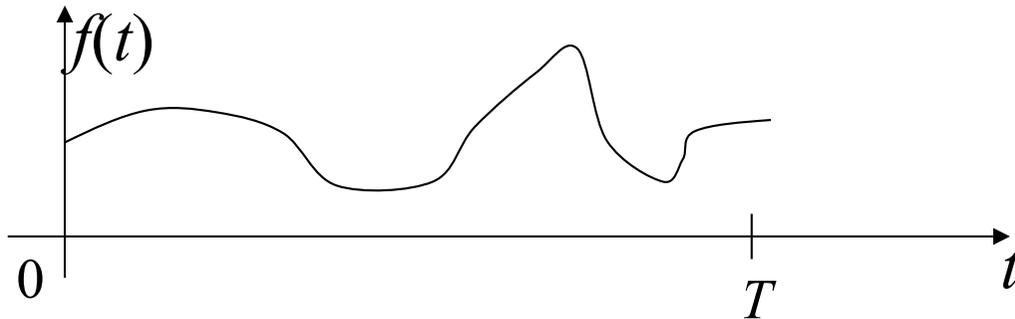
$$\begin{aligned} \int_0^T \cos\left(\frac{2\pi n t}{T}\right) f(t) dt &= 0 + 2 \sum_{k=1}^{\infty} a_n \int_0^T \cos\left(\frac{2\pi n t}{T}\right) \cos\left(\frac{2\pi k t}{T}\right) dt + 0 \\ &= T a_n \end{aligned}$$

$$a_n = \frac{1}{T} \int_0^T f(t) \cos\left(\frac{2\pi n t}{T}\right) dt$$

...

$$b_n = \frac{1}{T} \int_0^T f(t) \sin\left(\frac{2\pi n t}{T}\right) dt$$

Resumindo



$$f(t) = a_0 + 2 \sum_{k=1}^{\infty} \left(a_k \cos \frac{2\pi kt}{T} + b_k \sin \frac{2\pi kt}{T} \right)$$

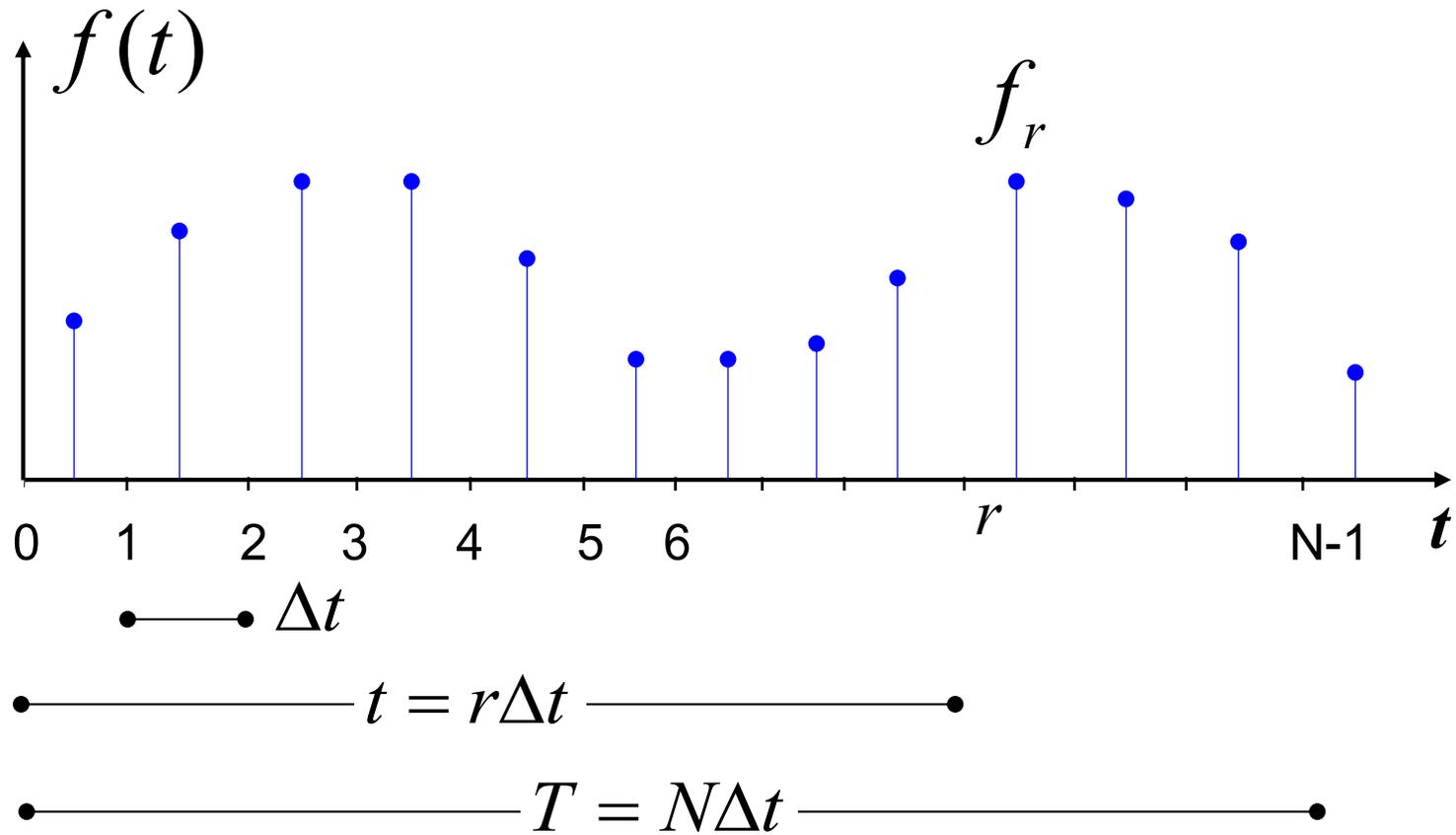
$$a_k = \frac{1}{T} \int_0^T f(t) \cos\left(\frac{2\pi kt}{T}\right) dt \quad k = 0, 1, 2, 3, \dots$$

$$b_k = \frac{1}{T} \int_0^T f(t) \sin\left(\frac{2\pi kt}{T}\right) dt \quad k = 1, 2, 3, \dots$$

$$\omega_k = \frac{2\pi k}{T}$$

$$\Delta\omega = \frac{2\pi}{T}$$

Transformada de Fourier



$$(f_0, f_1, f_2, \dots, f_r, \dots, f_{N-2}, f_{N-1},)$$

$$(f_0, f_1, f_2, \dots, f_r, \dots, f_{N-2}, f_{N-1},)$$

$$a_k \cong \frac{1}{N} \sum_{r=0}^{N-1} f_r \cos\left(\frac{2\pi kr}{N}\right) \quad b_k \cong \frac{1}{N} \sum_{r=0}^{N-1} f_r \sin\left(\frac{2\pi kr}{N}\right)$$

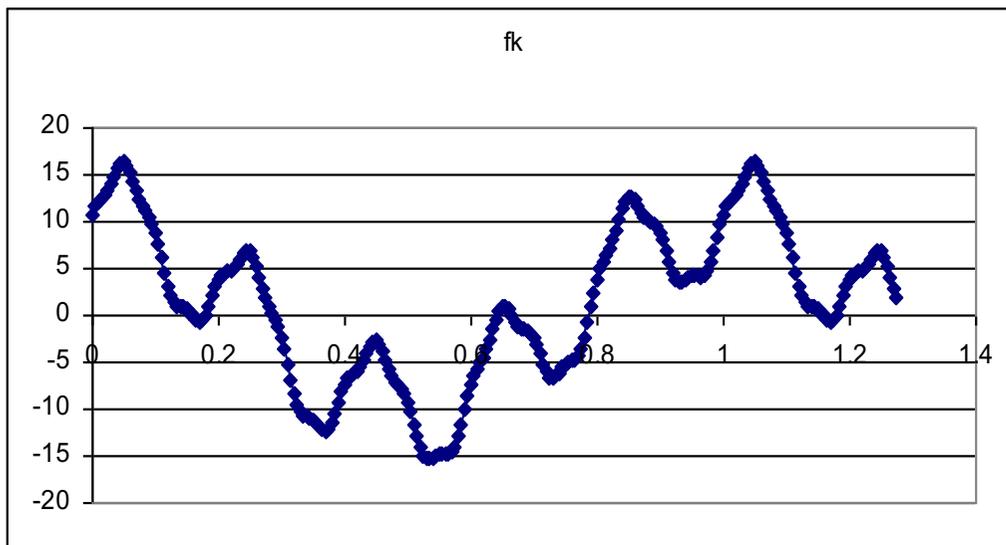
$$\begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_{N-1} \end{bmatrix} = \frac{1}{N} \begin{bmatrix} c_{00} & c_{01} & \cdots & c_{0(N-1)} \\ c_{10} & c_{11} & \cdots & c_{1(N-1)} \\ \vdots & \vdots & \ddots & \vdots \\ c_{(N-1)0} & c_{(N-1)1} & \cdots & c_{(N-1)(N-1)} \end{bmatrix} \begin{bmatrix} f_0 \\ f_1 \\ \vdots \\ f_{N-1} \end{bmatrix} \quad \text{onde:}$$

$$c_{kr} = \cos\left(\frac{2\pi kr}{N}\right)$$

$$\begin{bmatrix} b_0 \\ b_1 \\ \vdots \\ b_{N-1} \end{bmatrix} = \frac{1}{N} \begin{bmatrix} s_{00} & s_{01} & \cdots & s_{0(N-1)} \\ s_{10} & s_{11} & \cdots & s_{1(N-1)} \\ \vdots & \vdots & \ddots & \vdots \\ s_{(N-1)0} & s_{(N-1)1} & \cdots & s_{(N-1)(N-1)} \end{bmatrix} \begin{bmatrix} f_0 \\ f_1 \\ \vdots \\ f_{N-1} \end{bmatrix} \quad \text{onde:}$$

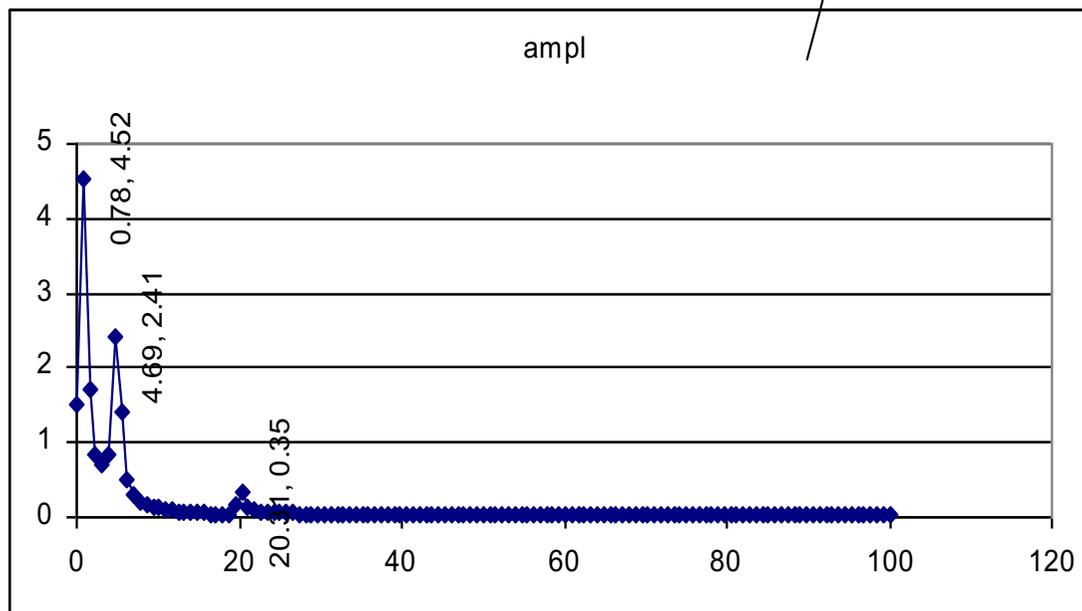
$$s_{kr} = \sin\left(\frac{2\pi kr}{N}\right)$$

Transformada

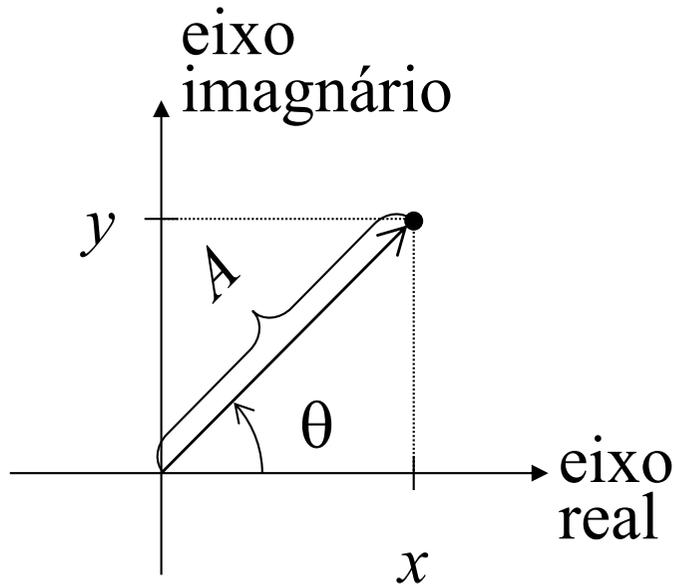


$$f_k \equiv \sum_{r=0}^{N-1} F_r e^{i\left(\frac{2\pi kr}{N}\right)}$$

$$F_k \equiv \frac{1}{N} \sum_{s=0}^{N-1} f_s e^{-i\left(\frac{2\pi ks}{N}\right)}$$



Números complexos



- x é a parte real
- y é a parte imaginária
- A é a magnitude
- θ é a fase

$$z = x + iy = A(\cos \theta + i \sin \theta)$$

$$i = \sqrt{-1}$$

Operação básicas com complexos

$$(x_1 + iy_1) + (x_2 + iy_2) = (x_1 + x_2) + i(y_1 + y_2) \quad a(x + iy) = ax + iay$$

$$i^2 = -1$$

$$(x_1 + iy_1)(x_2 + iy_2) = (x_1x_2 + i^2y_1y_2) + i(x_2y_1 + x_1y_2) = (x_1x_2 - y_1y_2) + i(x_2y_1 + x_1y_2)$$

$$(x + iy)(x - iy) = (x^2 + y^2) + i(xy - xy) = x^2 + y^2$$

$$\frac{x_1 + iy_1}{x_2 + iy_2} = \frac{(x_1 + iy_1)(x_2 - iy_2)}{(x_2 + iy_2)(x_2 - iy_2)} = \frac{1}{x_2^2 + y_2^2} (x_1 + iy_1)(x_2 - iy_2)$$

$$e^{i\theta} = \cos \theta + i \sin \theta$$

Derivada de $e^{i\omega t}$

$$\frac{d}{dt} e^{i\omega t} = i\omega e^{i\omega t}$$

$$\frac{d}{dt} (\cos \omega t + i \sin \omega t) = -\omega \sin \omega t + i \omega \cos \omega t$$

$$= i\omega \left(-\frac{1}{i} \sin \omega t + \cos \omega t \right)$$

$$\frac{-1}{i} = \frac{-i}{i^2} = \frac{-i}{-1} = i$$

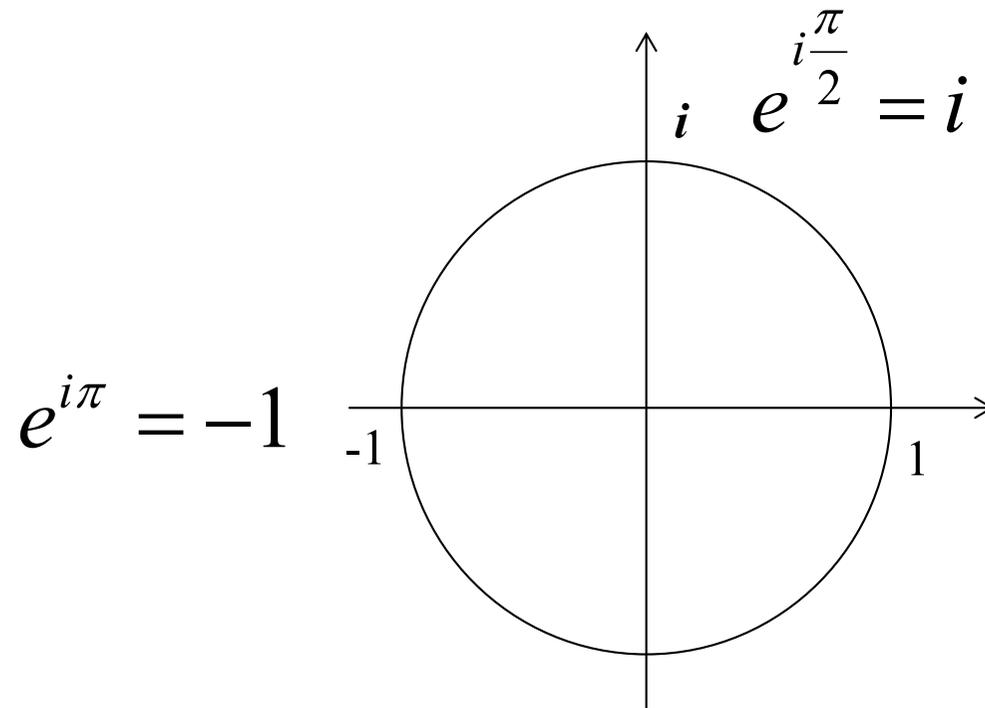
$$= i\omega (i \sin \omega t + \cos \omega t)$$

C.Q.D.

Outras propriedades úteis

revisão

$$e^{i\theta} = \cos \theta + i \sin \theta$$



Outras propriedades úteis (2)

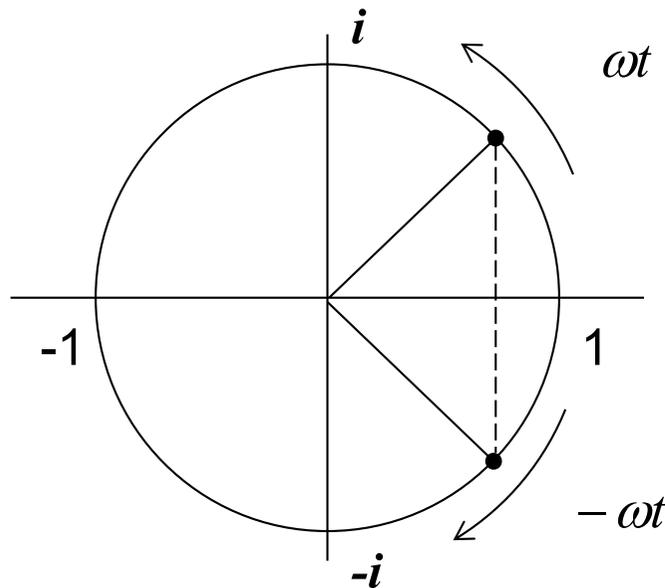
revisão

$$e^{i\theta} = \cos \theta + i \sin \theta$$

$$e^{-i\theta} = \cos \theta - i \sin \theta$$

$$\cos \theta = \frac{1}{2} (e^{i\theta} + e^{-i\theta})$$

$$\cos \omega t = \frac{1}{2} (e^{i\omega t} + e^{-i\omega t})$$



o cosseno
corresponde a
média de
dois
harmônicos de
frequências
 w e $-w$

Outras propriedades úteis (2)

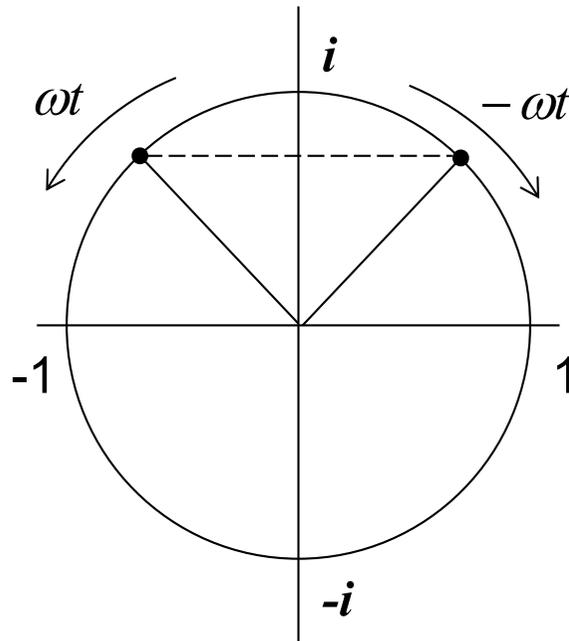
revisão

$$e^{i\theta} = \cos \theta + i \sin \theta$$

$$e^{-i\theta} = \cos \theta - i \sin \theta$$

$$\frac{1}{i} = \frac{i}{i^2} = \frac{i}{-1} = -i$$

$$\sin \theta = \frac{1}{2i} (e^{i\theta} - e^{-i\theta}) = \frac{i}{2} (e^{-i\theta} - e^{i\theta})$$



o seno também
corresponde a
dois harmônicos:
 w e $-w$

Outras propriedades úteis (3)

revisão

$$z_1 = A_1 e^{i\theta_1} = A_1 (\cos \theta_1 + i \sin \theta_1)$$

$$z_2 = A_2 e^{i\theta_2} = A_2 (\cos \theta_2 + i \sin \theta_2)$$

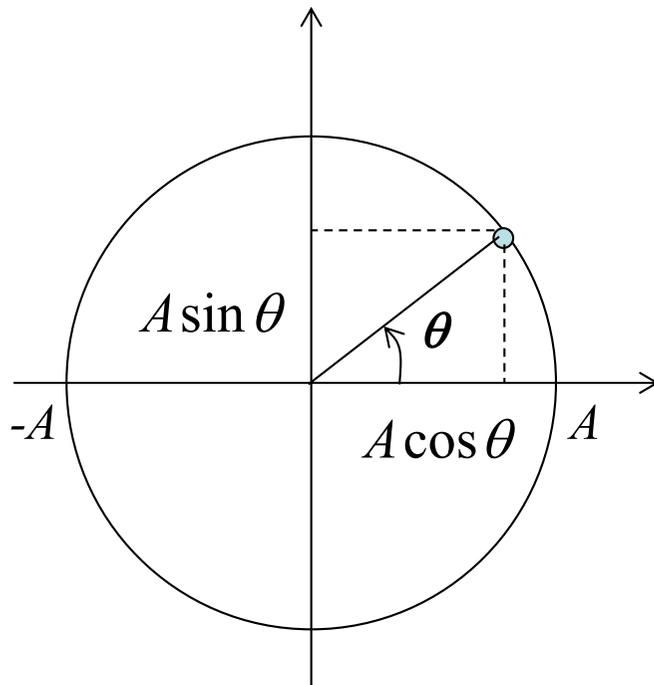
$$z_1 z_2 = A_1 A_2 e^{i(\theta_1 + \theta_2)}$$

$$\frac{z_1}{z_2} = \frac{A_1}{A_2} e^{i(\theta_1 - \theta_2)}$$

Amplitude e fase de complexos

Dado um valor:

$$z = A(\cos \theta + i \sin \theta) = x + iy$$



$$A^2 = x^2 + y^2 = z\bar{z}$$

Amplitude

$$\tan \theta = \frac{y}{x}$$

Fase

$$F_k = a_k - ib_k = \frac{1}{N} \sum_{s=0}^{N-1} f_s e^{-i\left(\frac{2\pi ks}{N}\right)}$$

$$\begin{bmatrix} F_0 \\ F_1 \\ \vdots \\ F_{N-1} \end{bmatrix} = \frac{1}{N} \begin{bmatrix} E_{00} & E_{01} & \cdots & E_{0(N-1)} \\ E_{10} & E_{11} & \cdots & E_{1(N-1)} \\ \vdots & \vdots & \ddots & \vdots \\ E_{(N-1)0} & E_{(N-1)1} & \cdots & E_{(N-1)(N-1)} \end{bmatrix} \begin{bmatrix} f_0 \\ f_1 \\ \vdots \\ f_{N-1} \end{bmatrix}$$

onde:

$$E_{kr} = e^{-i\frac{2\pi kr}{N}}$$

$$f_k = \sum_{r=0}^{N-1} F_r e^{i\left(\frac{2\pi kr}{N}\right)}$$

$$\begin{bmatrix} f_0 \\ f_1 \\ \vdots \\ f_{N-1} \end{bmatrix} = \begin{bmatrix} E'_{00} & E'_{01} & \cdots & E'_{0(N-1)} \\ E'_{10} & E'_{11} & \cdots & E'_{1(N-1)} \\ \vdots & \vdots & \ddots & \vdots \\ E'_{(N-1)0} & E'_{(N-1)1} & \cdots & E'_{(N-1)(N-1)} \end{bmatrix} \begin{bmatrix} F_0 \\ F_1 \\ \vdots \\ F_{N-1} \end{bmatrix}$$

onde:

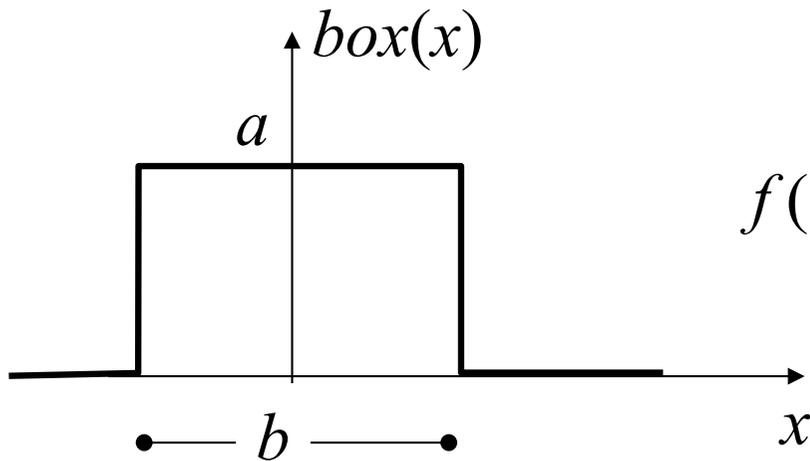
$$E'_{kr} = e^{i\frac{2\pi kr}{N}}$$

Transformada de Fourier

$$F(\omega) = \int_{-\infty}^{+\infty} f(x) e^{-i2\pi\omega x} dx$$

$$f(x) = \int_{-\infty}^{+\infty} F(\omega) e^{+i2\pi\omega x} d\omega$$

Exemplo 1: Função caixa (box)

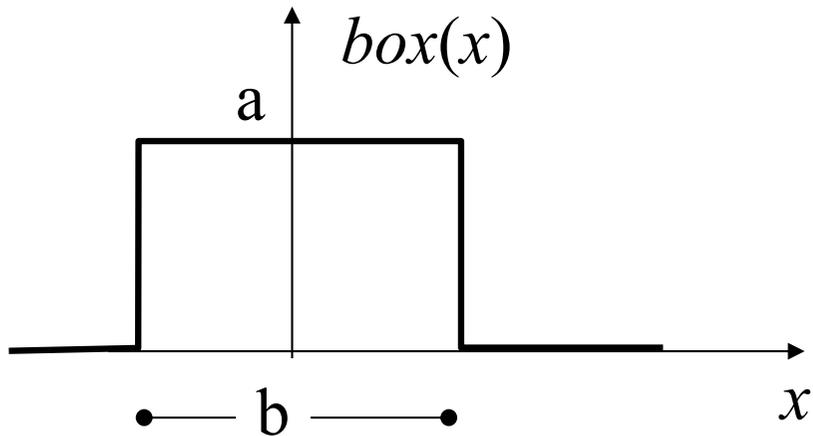


$$f(x) = box(x) = \begin{cases} 0 & \text{se } x < -b/2 \\ a & \text{se } x \in [-b/2, b/2] \\ 0 & \text{se } x > b/2 \end{cases}$$

$$\begin{aligned} F(\omega) &= \int_{-\infty}^{+\infty} box(x) e^{-i2\pi\omega x} dx = a \int_{-b/2}^{b/2} e^{-i2\pi\omega x} dx \\ &= \frac{a}{-i2\pi\omega} e^{-i2\pi\omega x} \Big|_{-b/2}^{b/2} = \frac{a}{-i2\pi\omega} (e^{-i\pi\omega b} - e^{i\pi\omega b}) \\ &= \frac{a}{\pi\omega} \frac{(e^{i\pi\omega b} - e^{-i\pi\omega b})}{2i} = \frac{a}{\pi\omega} \sin(b\pi\omega) \end{aligned}$$

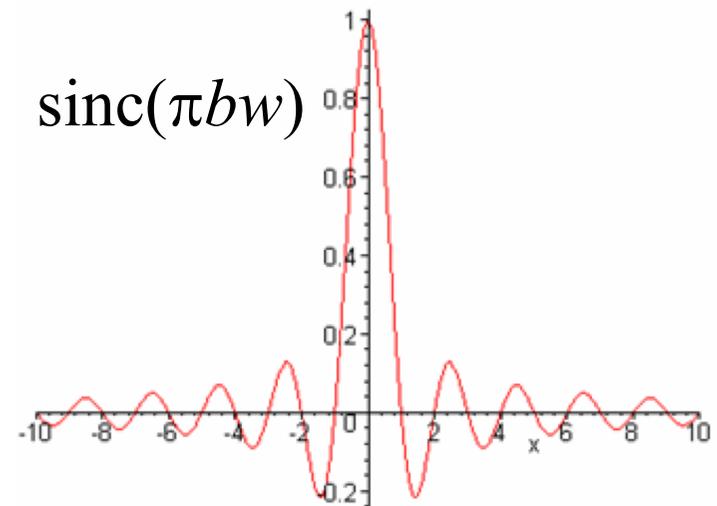
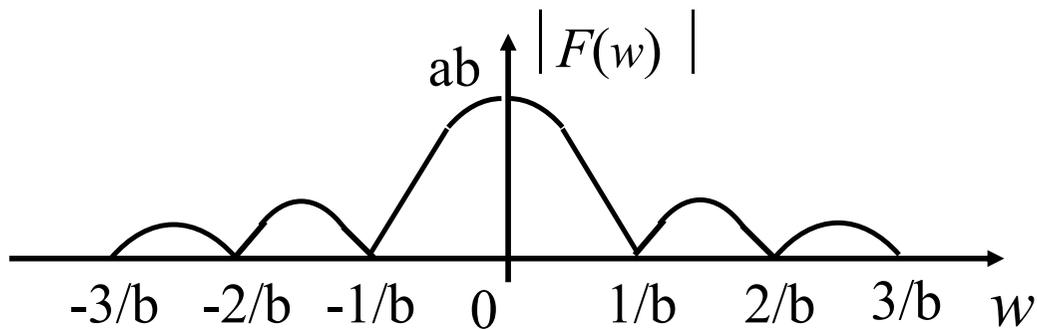
$$F(\omega) = ab \frac{\sin(b\pi\omega)}{\pi\omega b}$$

Transformada da função *box*



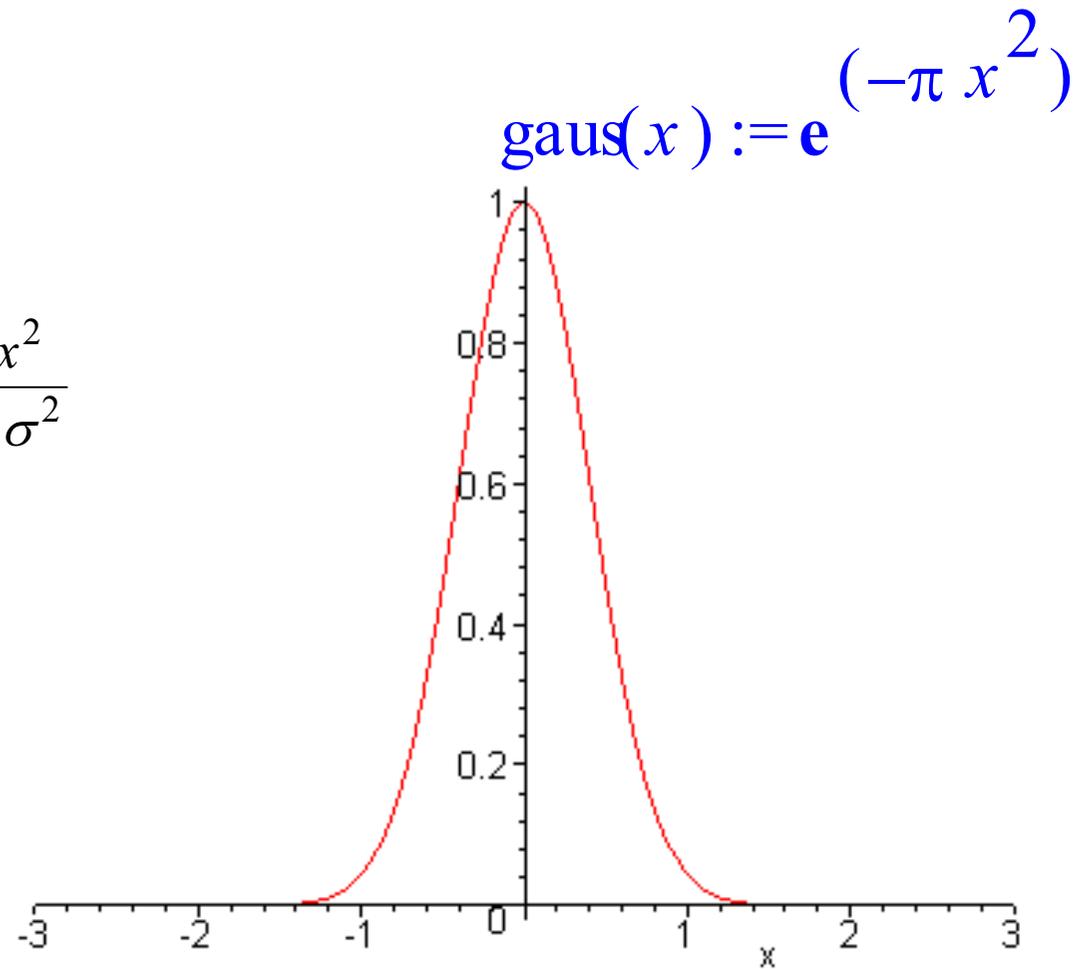
$$F(w) = ab \frac{\sin(\pi w b)}{\pi w b}$$

$$\|F(w)\| = |ab| \left| \frac{\sin(\pi b w)}{\pi b w} \right|$$

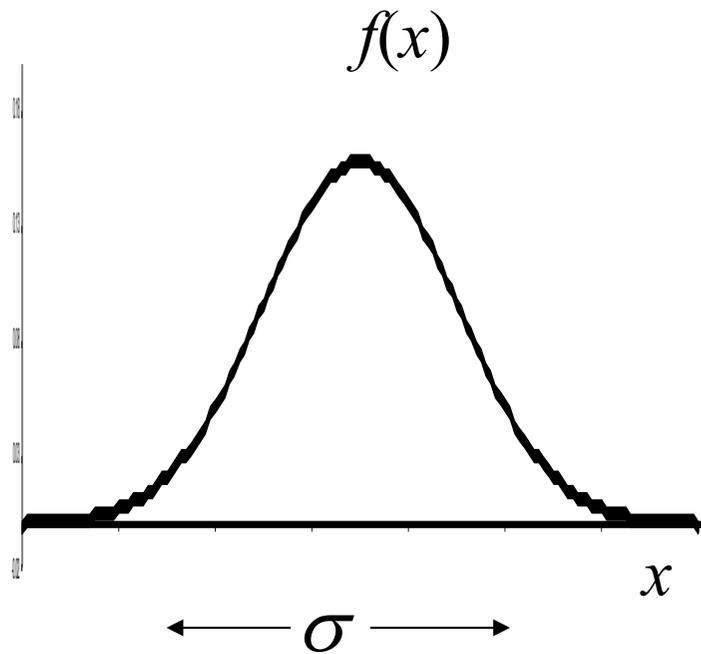


Distribuição normal: Gaussiana

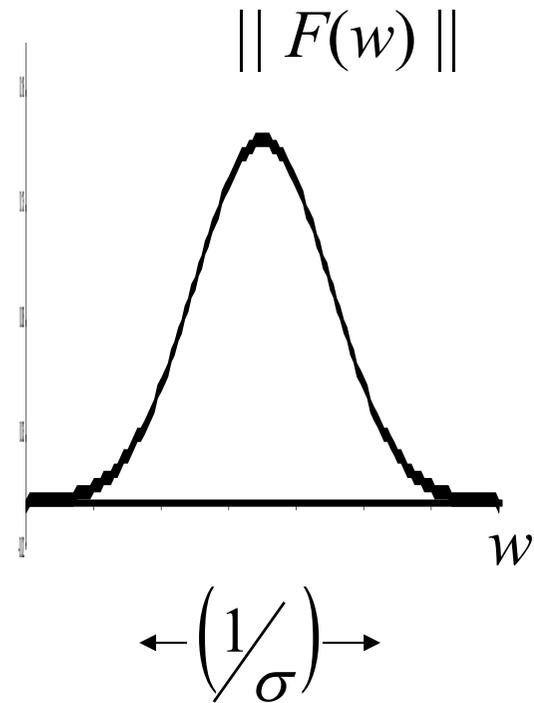
$$Gauss(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2}}$$



Exemplo 2: Gaussiana



$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2}}$$

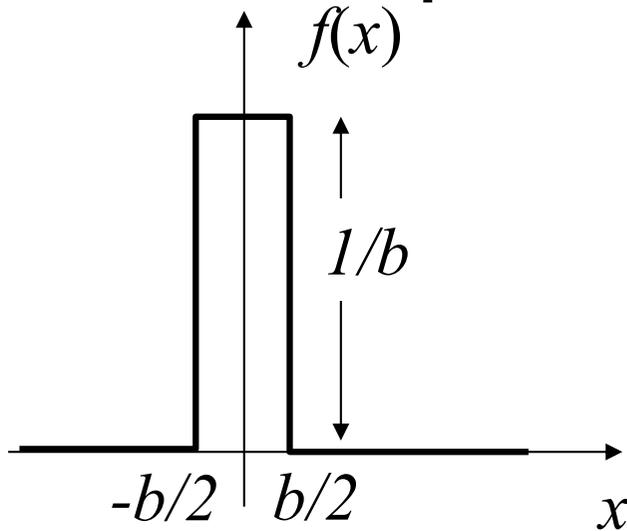


$$F(w) = e^{-\frac{w^2}{2\left(\frac{1}{\sigma^2}\right)}}$$

Transformada da Gaussiana

$$\begin{aligned} F(w) &= \int_{-\infty}^{\infty} \left(\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2}} \right) e^{-i2\pi wx} dx \\ &= \int_{-\infty}^{\infty} \left(\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2}} \right) (\cos(2\pi wx) + i \sin(2\pi wx)) dx \\ &= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{x^2}{2\sigma^2}} \cos(2\pi wx) dx = e^{-2\pi\sigma^2 w^2} \\ &= \nu\sqrt{2\pi} \left(\frac{1}{\nu\sqrt{2\pi}} e^{-\frac{w^2}{2\nu^2}} \right), \quad \nu = \frac{1}{2\pi\sigma} \end{aligned}$$

Exemplo 3: Delta de Dirac



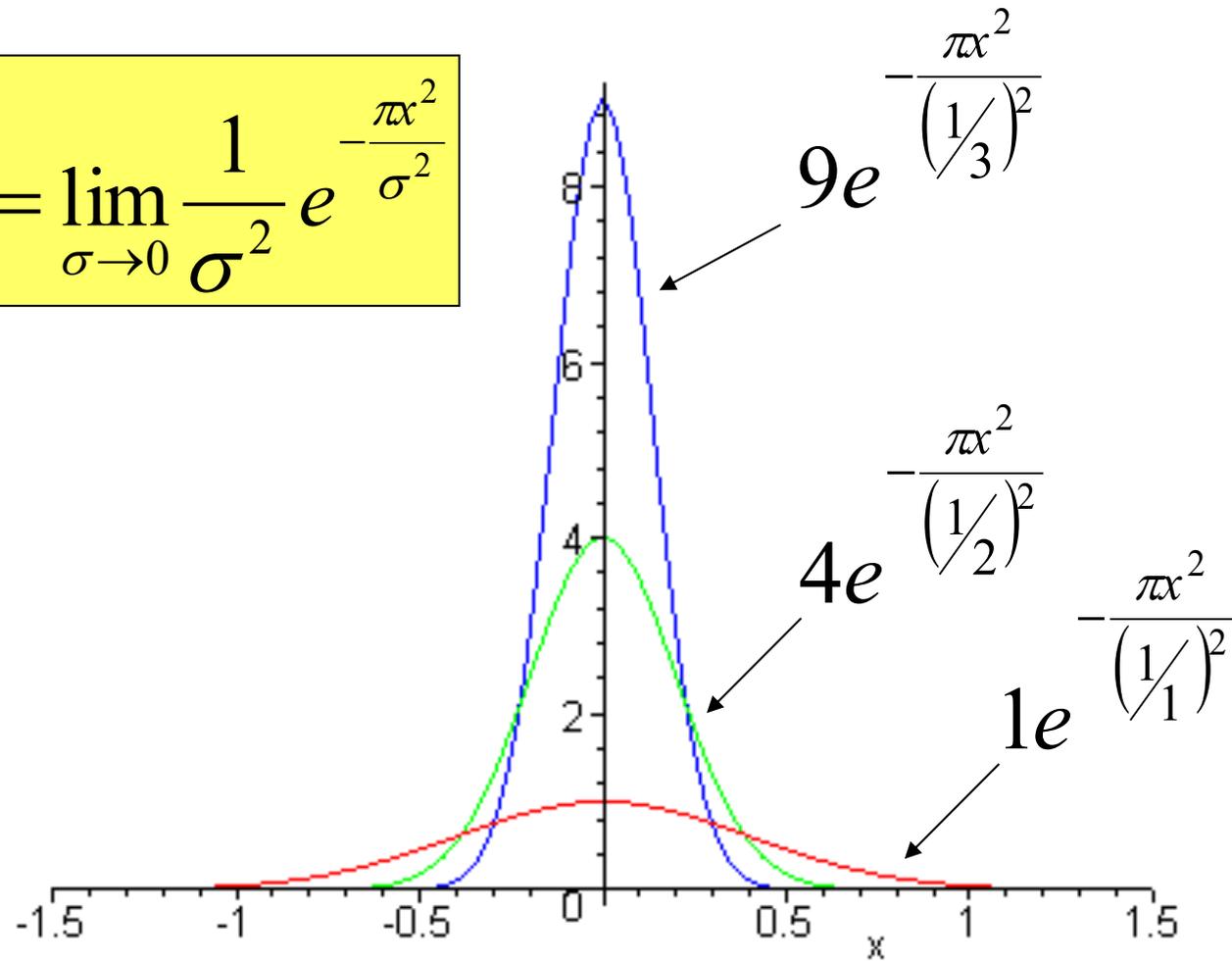
$$\delta(x) = \lim_{b \rightarrow 0} \begin{cases} 0 & \text{se } x < -b/2 \\ 1/b & \text{se } x \in [-b/2, b/2] \\ 0 & \text{se } x > b/2 \end{cases}$$

$$\int_{-\infty}^{\infty} f(x)\delta(x)dx = \lim_{b \rightarrow 0} \int_{-b/2}^{b/2} \frac{1}{b} f(x)dx = \lim_{b \rightarrow 0} \frac{(b/2 + b/2)}{b} f(\xi), \xi \in \left[-\frac{b}{2}, \frac{b}{2}\right]$$

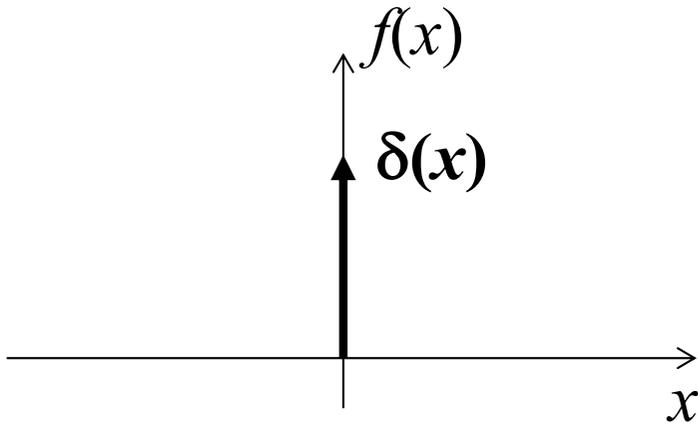
$$\int_{-\infty}^{\infty} f(x)\delta(x)dx = f(0)$$

Delta de Dirac de Gaussianas

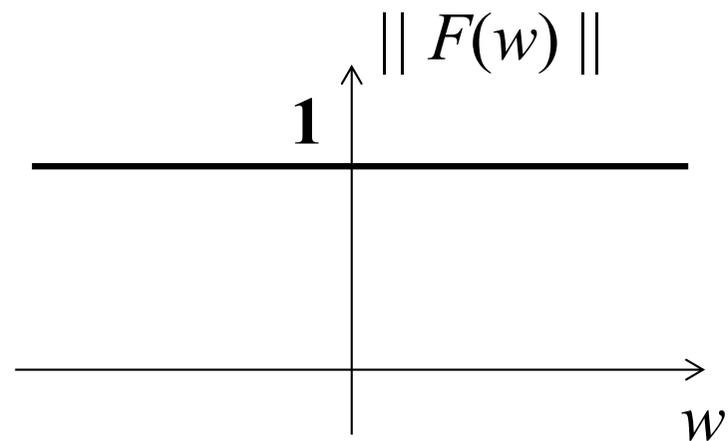
$$\delta(x) = \lim_{\sigma \rightarrow 0} \frac{1}{\sigma^2} e^{-\frac{\pi x^2}{\sigma^2}}$$



Transformada do Delta de Dirac

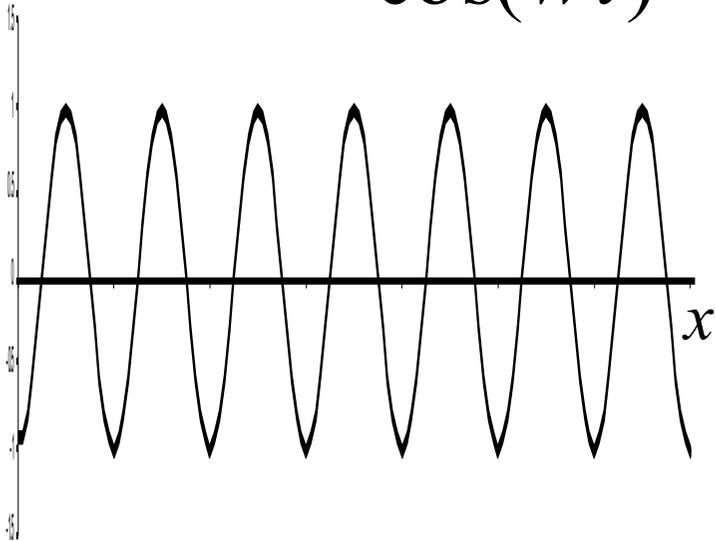


$$F(w) = \int_{-\infty}^{+\infty} \delta(x) e^{-i2\pi wx} dx = e^0 = 1$$



Transformada do cosseno

$\cos(\bar{w}t)$

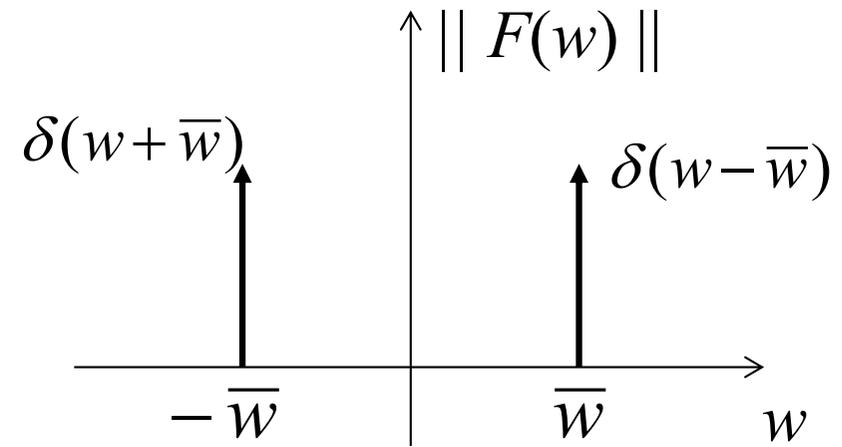
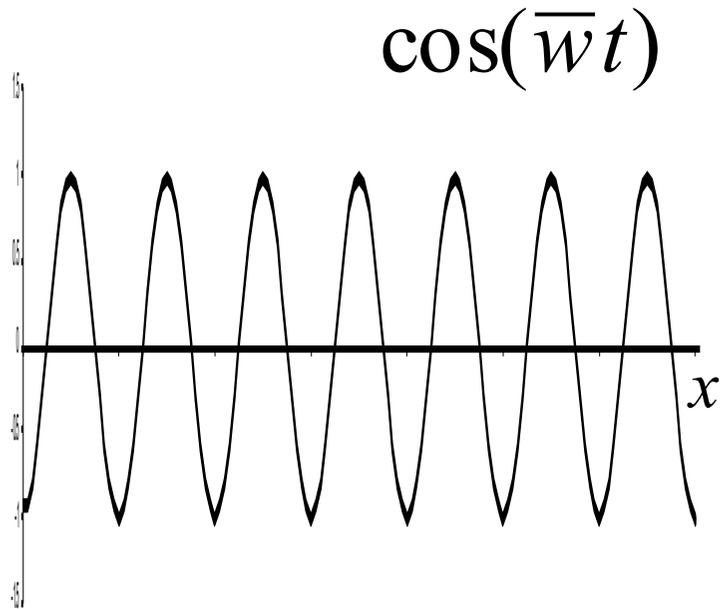


$$F(w) = \int_{-\infty}^{\infty} (\cos(\bar{w}x) e^{-i2\pi wx}) dx$$

$$= \int_{-\infty}^{\infty} \cos(\bar{w}x) (\cos(2\pi wx) + i \sin(2\pi wx)) dx$$

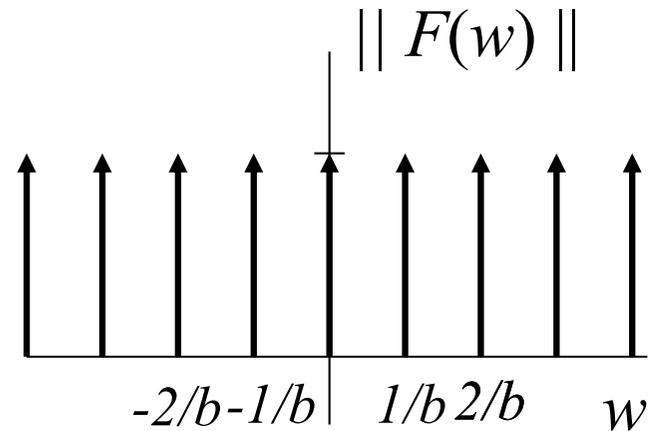
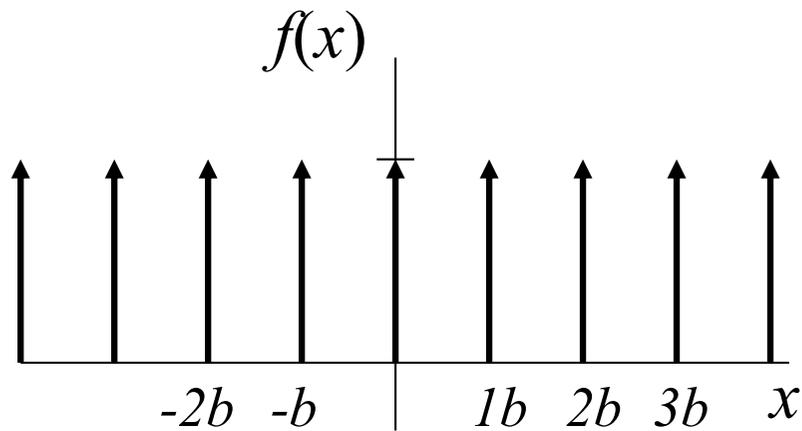
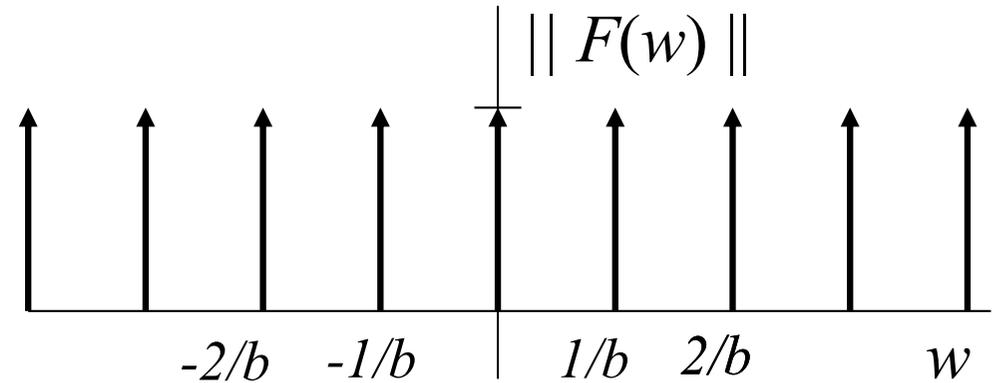
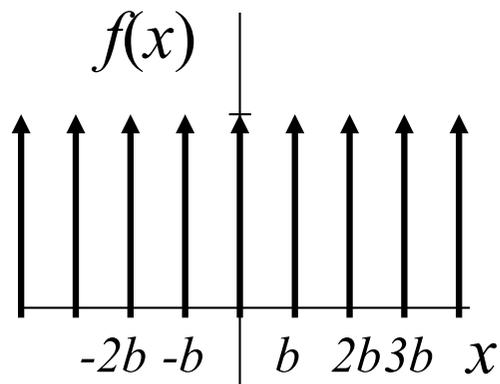
$$= \int_{-\infty}^{\infty} \cos(\bar{w}x) \cos(2\pi wx) dx = \begin{cases} 0 & \text{se } w \neq \frac{\bar{w}}{2\pi} \\ \infty & \text{se } w = \frac{\bar{w}}{2\pi} \end{cases}$$

Exemplo 4: Cosseno



$$F(\omega) = \frac{1}{2} \left[\delta\left(\omega + \frac{\bar{\omega}}{2\pi}\right) + \delta\left(\omega - \frac{\bar{\omega}}{2\pi}\right) \right]$$

Exemplo 5: Sequência de impulsos



$$\text{III}(t) = \sum_k \delta(t + kT_0)$$

$$\text{III}(w) = \sum_k \delta\left(t + k \frac{2\pi}{T_0}\right)$$

Pares importantes

$f(x)$	$F(u)$
1	$\delta(u)$
$\delta(x)$	1
$\cos \pi x$	$\frac{1}{2} \delta(u + \frac{1}{2}) + \frac{1}{2} \delta(u - \frac{1}{2})$
$\sin \pi x$	$\frac{j}{2} \delta(u + \frac{1}{2}) - \frac{j}{2} \delta(u - \frac{1}{2})$
$e^{-\pi x^2}$	$e^{-\pi u^2}$
$\text{rect}(x) = \begin{cases} 1 & x < \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$	$\text{sinc}(u) = \frac{\sin \pi u}{\pi u}$

Propriedades da transformada

$$f(ax)$$

$$f(x-a)$$

$$F(x)$$

$$\frac{d}{dx} f(x)$$

$$f(x)+h(x)$$

$$f(x)*h(x)$$

$$\frac{1}{|a|} F\left(\frac{u}{a}\right)$$

$$e^{-j2\pi au} F(u)$$

$$f(-u)$$

$$j2\pi u F(u)$$

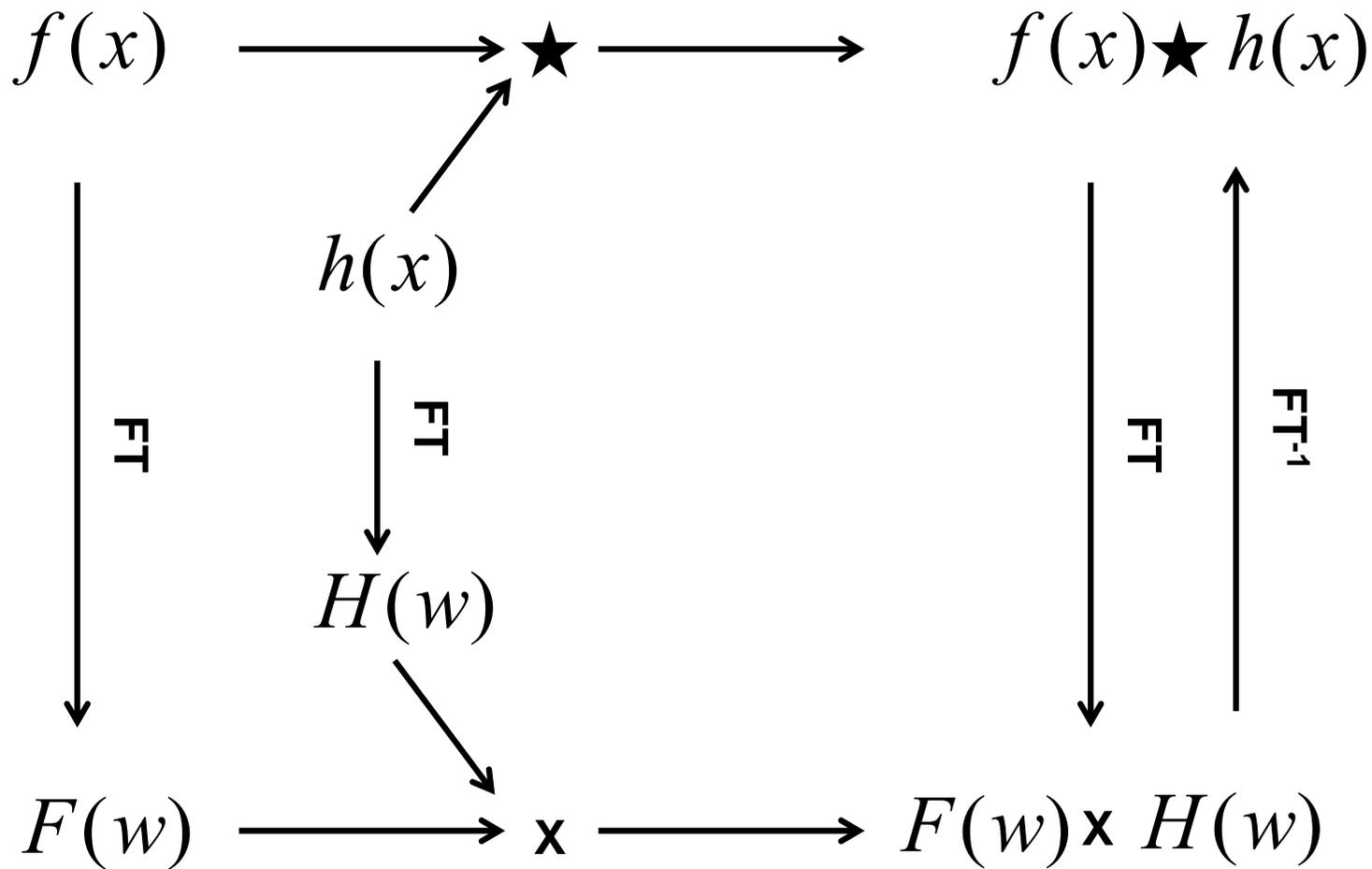
$$F(u)+H(u)$$

$$F(u)H(u)$$



convolução

Filtragem com Transformada de Fourier

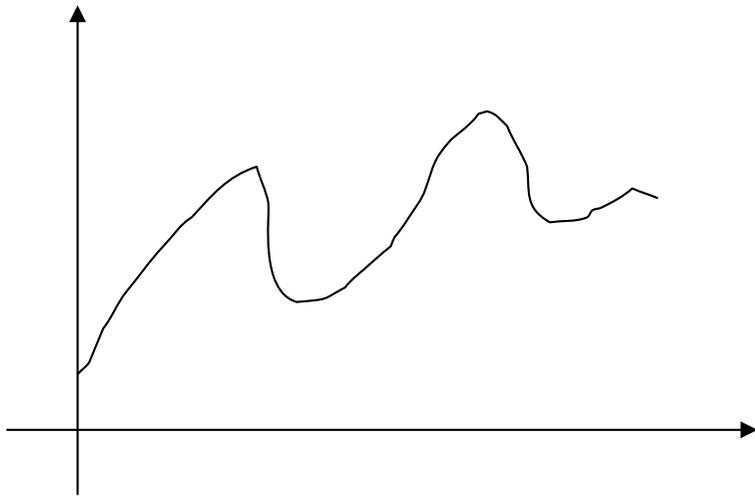


Amostragem e Reconstrução

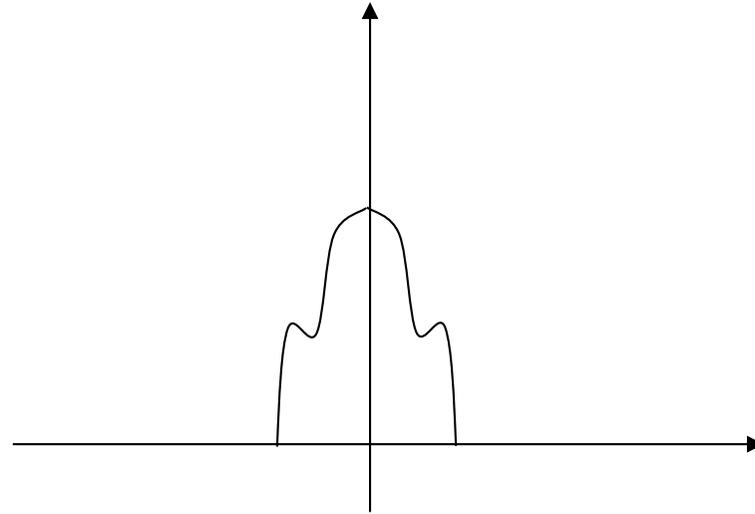
Observando os domínio do
espaço e das frequências

Sinal original

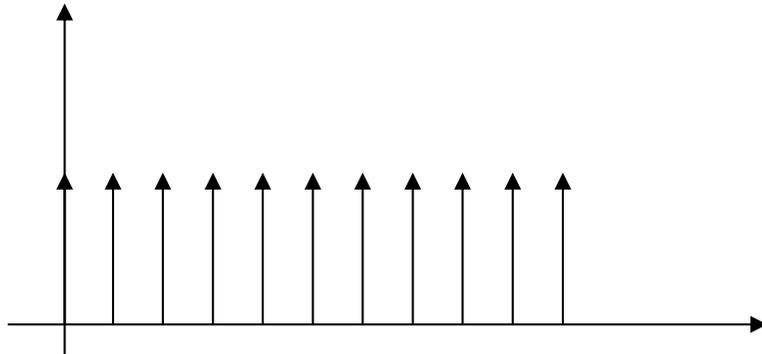
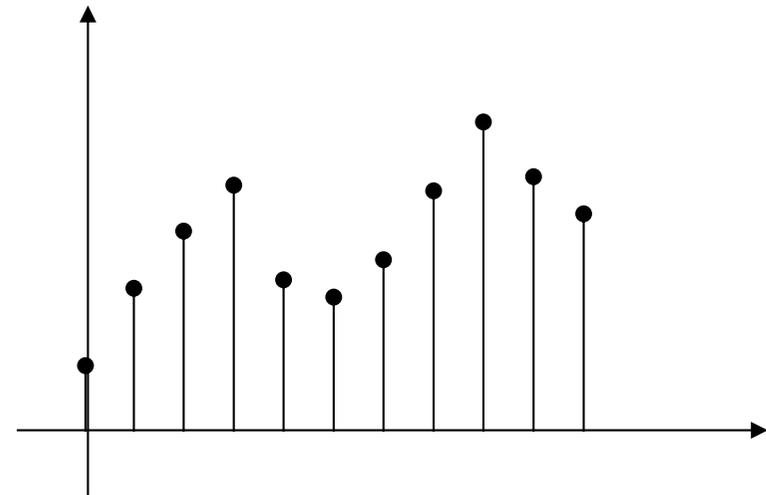
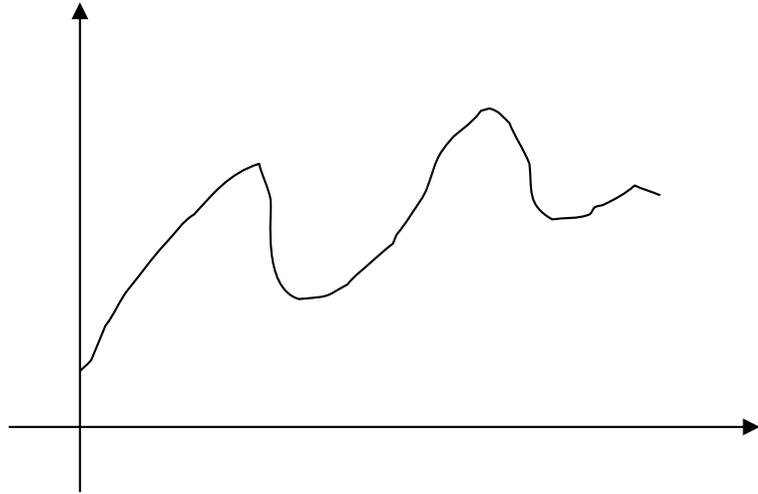
domínio do espaço



domínio das frequências



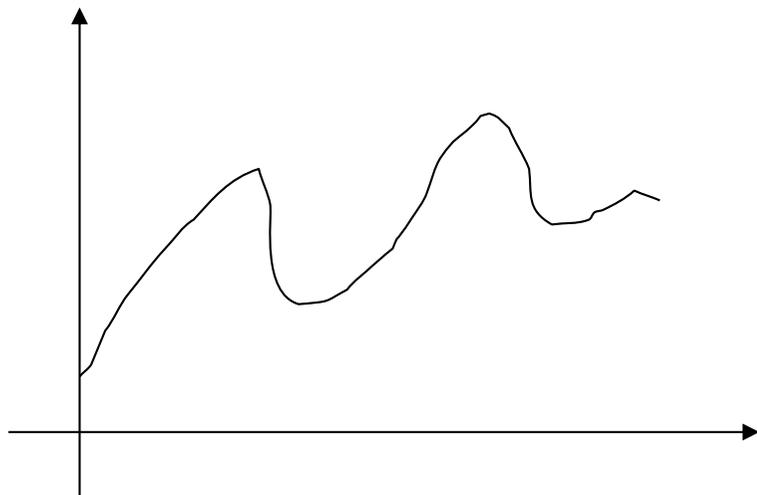
Sinal discretizado



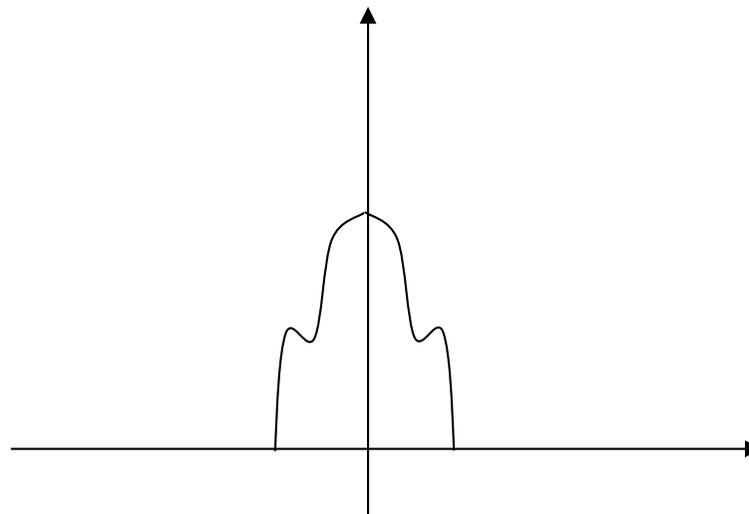
$$f_k = \int f(t) \sum_k \bar{\delta}(t - kT_0) dt$$

Amostragem

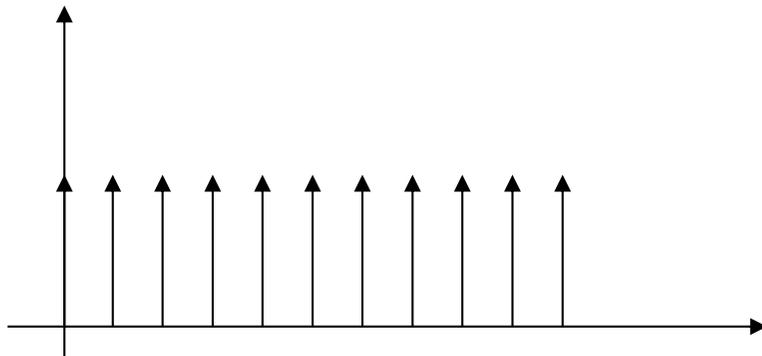
domínio do espaço



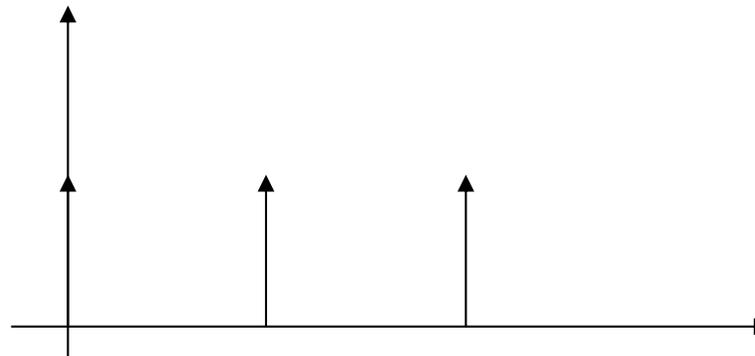
domínio das freqüências



produto

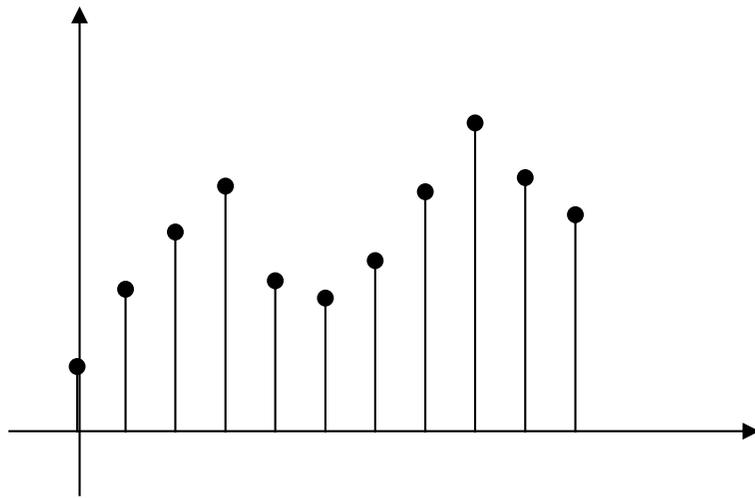


convolução

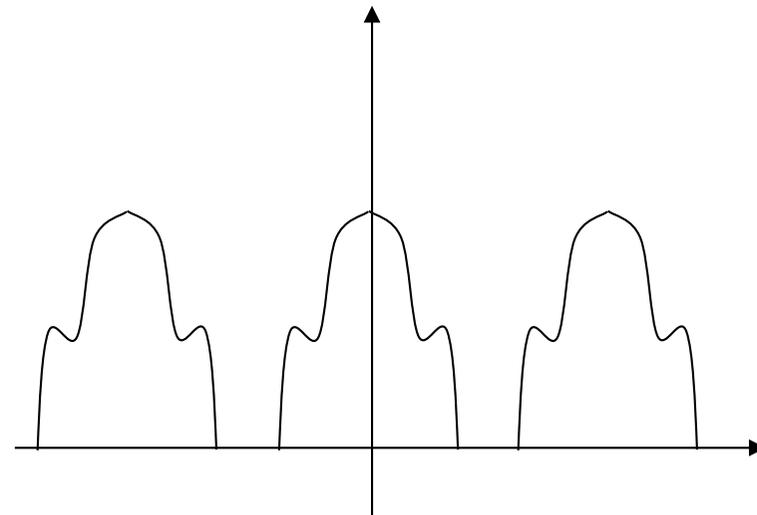


Sinal discretizado

domínio do espaço

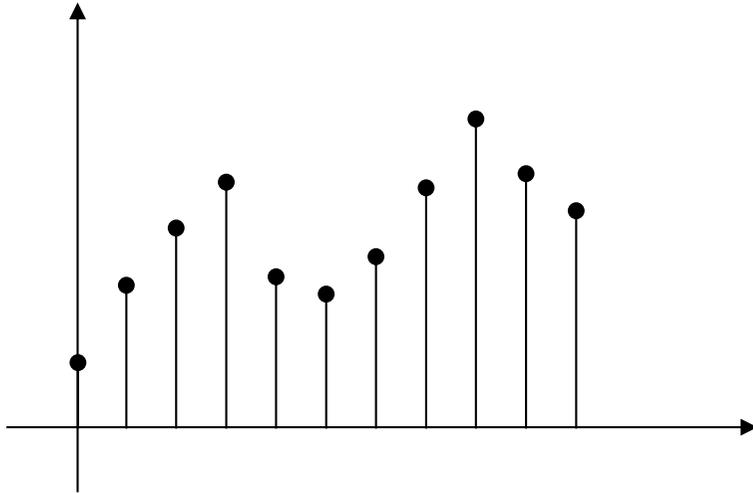


domínio das frequências

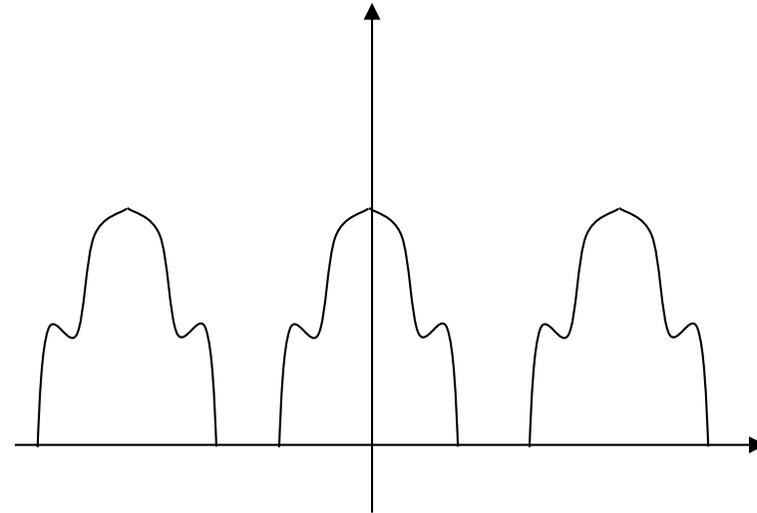


Reconstrução

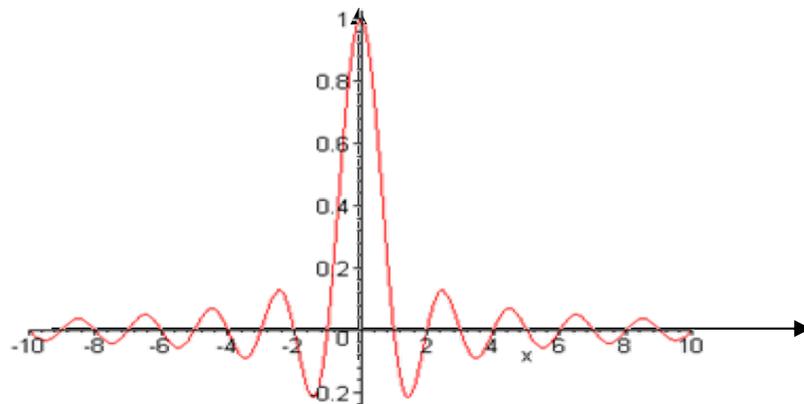
domínio do espaço



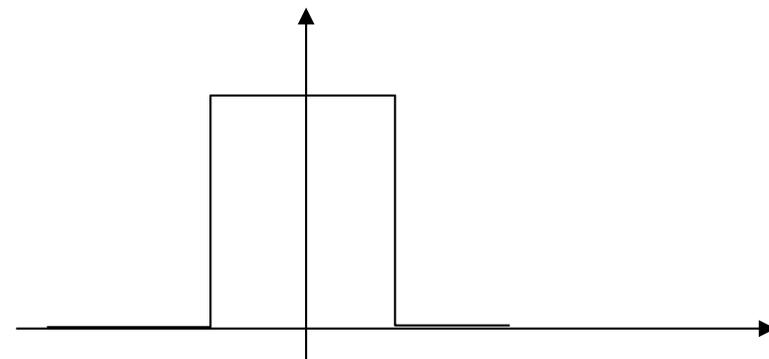
domínio das freqüências



convolução

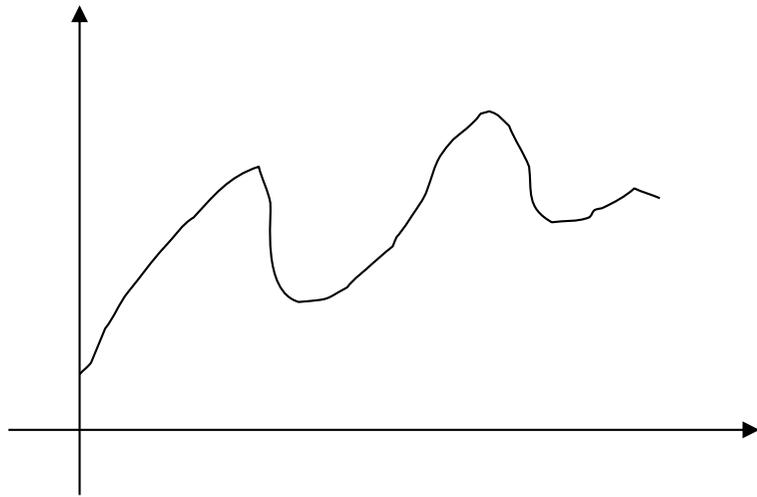


produto

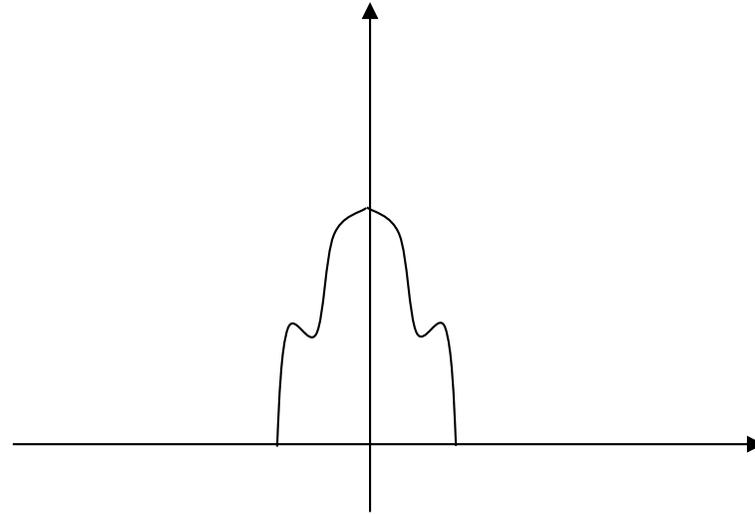


Retorno ao sinal original

domínio do espaço

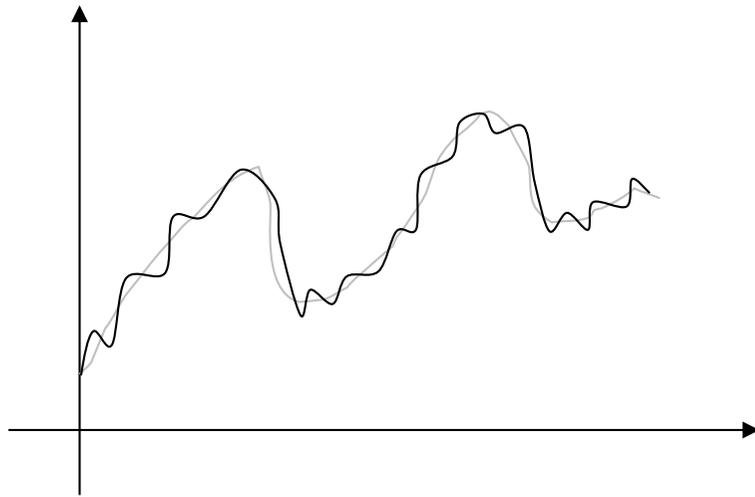


domínio das frequências

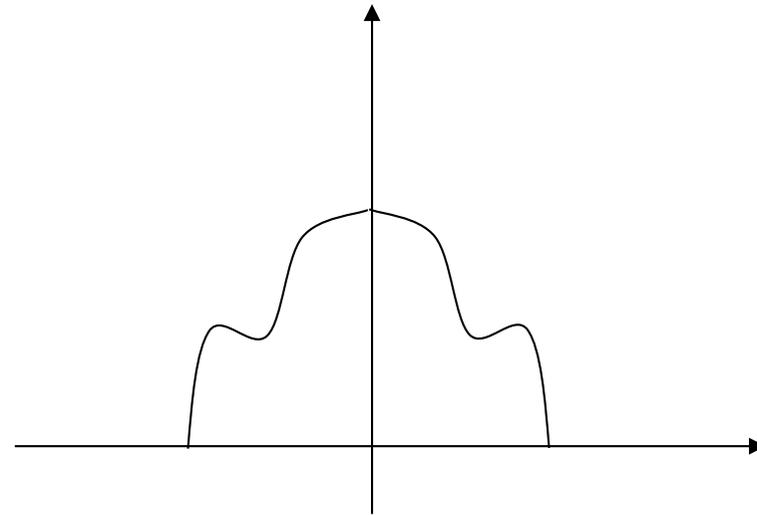


Sinal original com mais altas freqüências

domínio do espaço



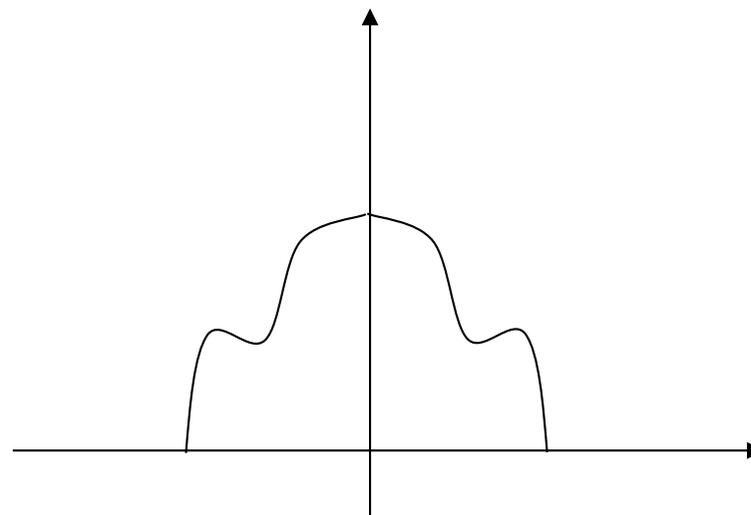
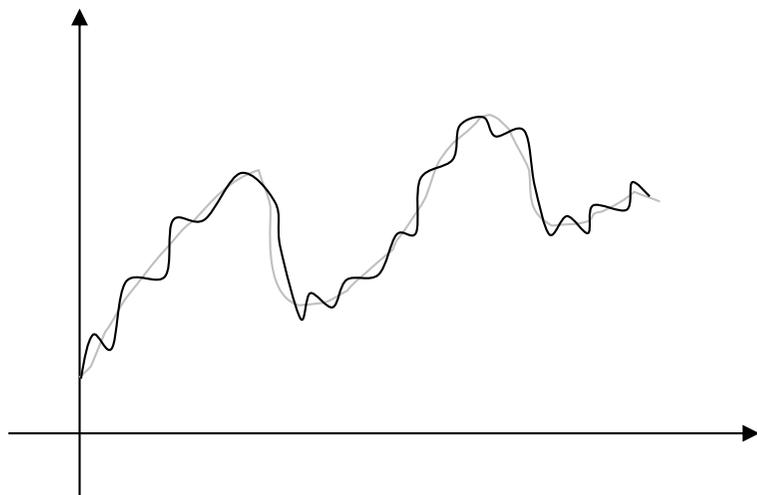
domínio das freqüências



Mesma taxa de amostragem

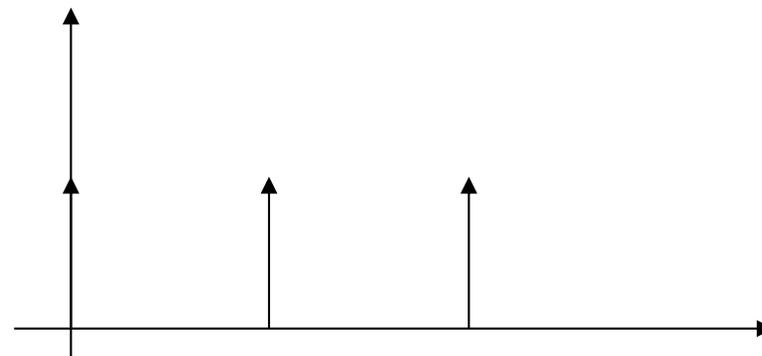
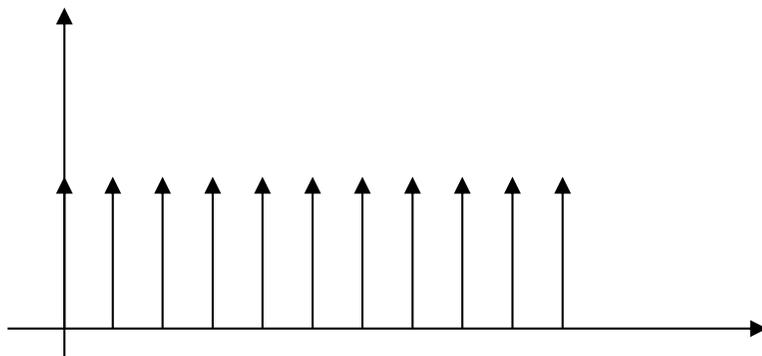
domínio do espaço

domínio das frequências



produto

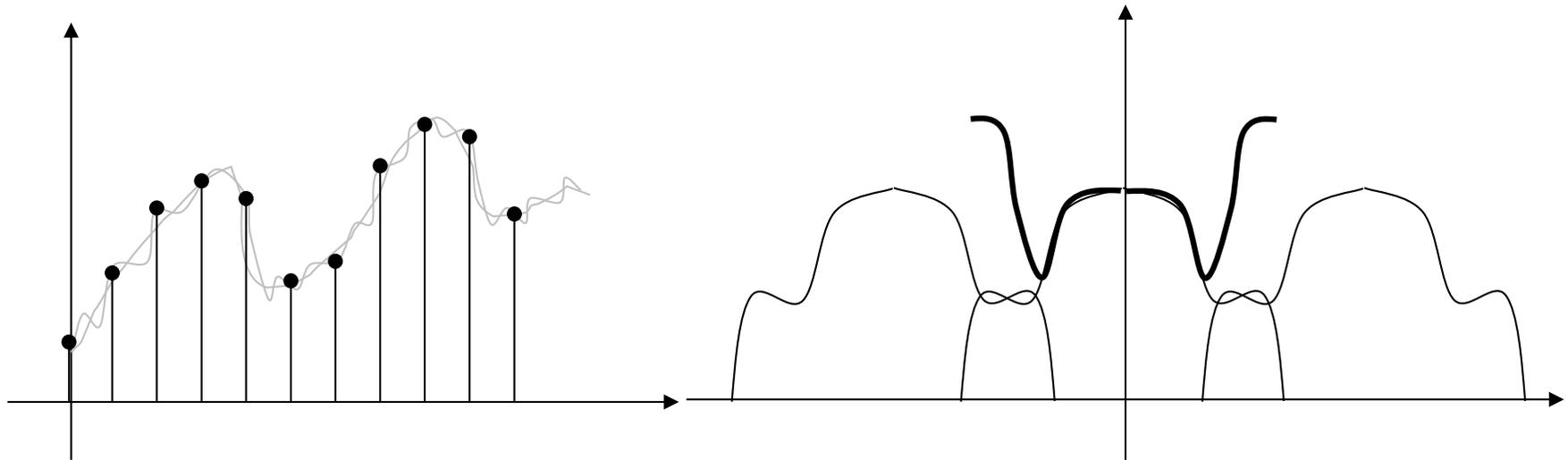
convolução



Sinal amostrado

domínio do espaço

domínio das frequências



Não temos como reconstruir sem introduzir artefatos!

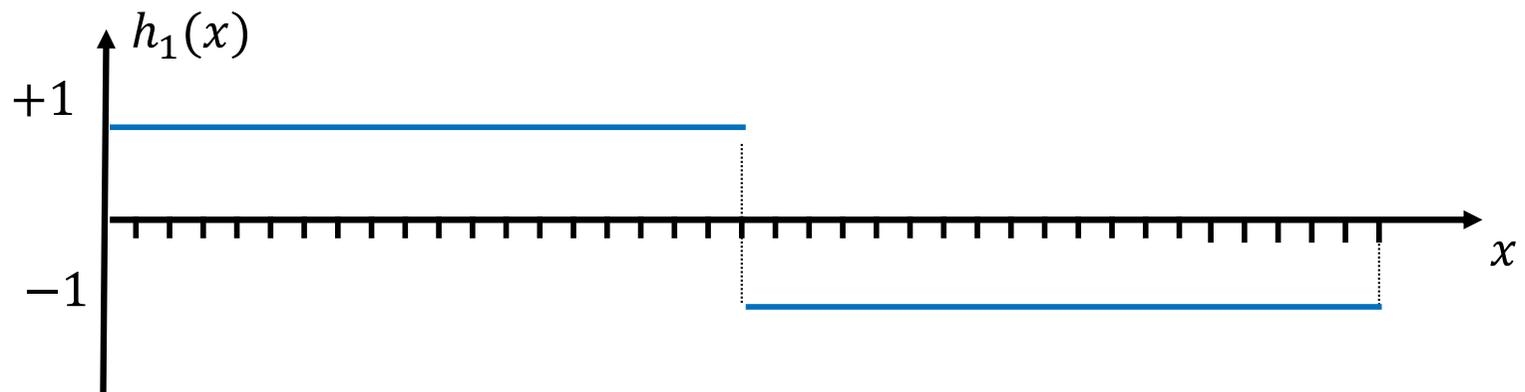
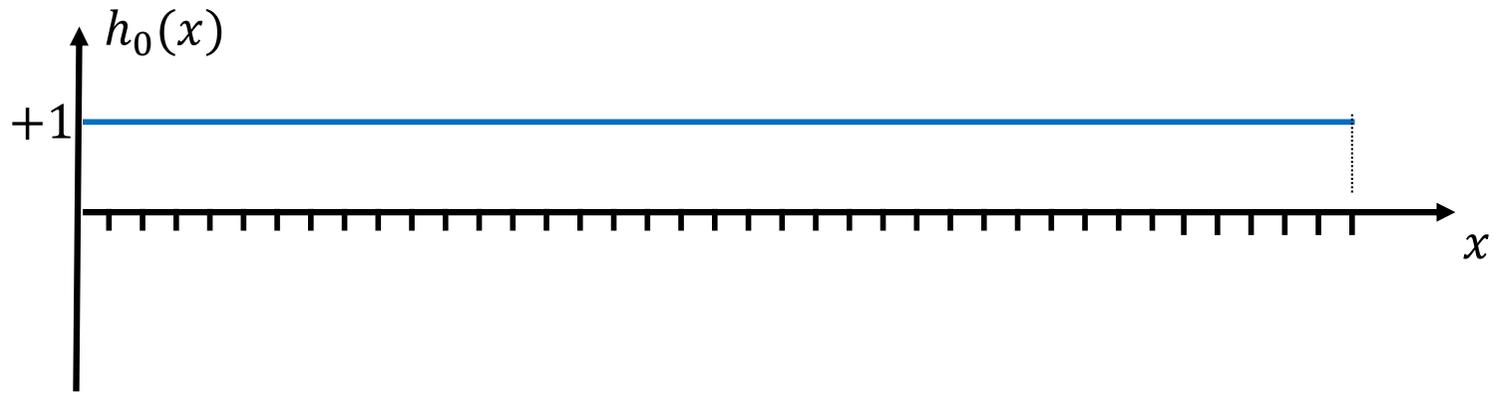
Teorema de Nyquist

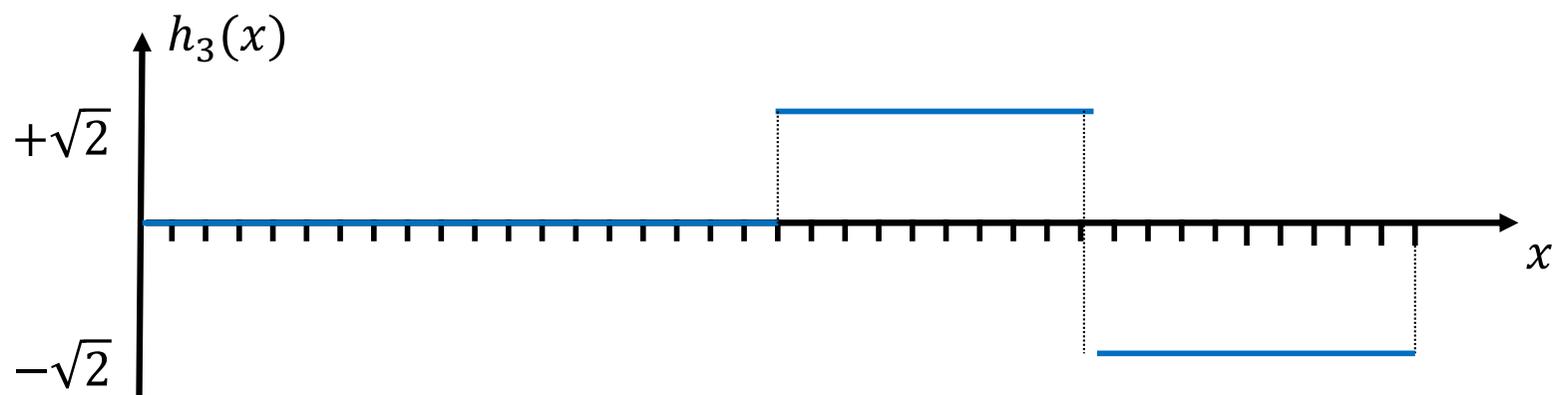
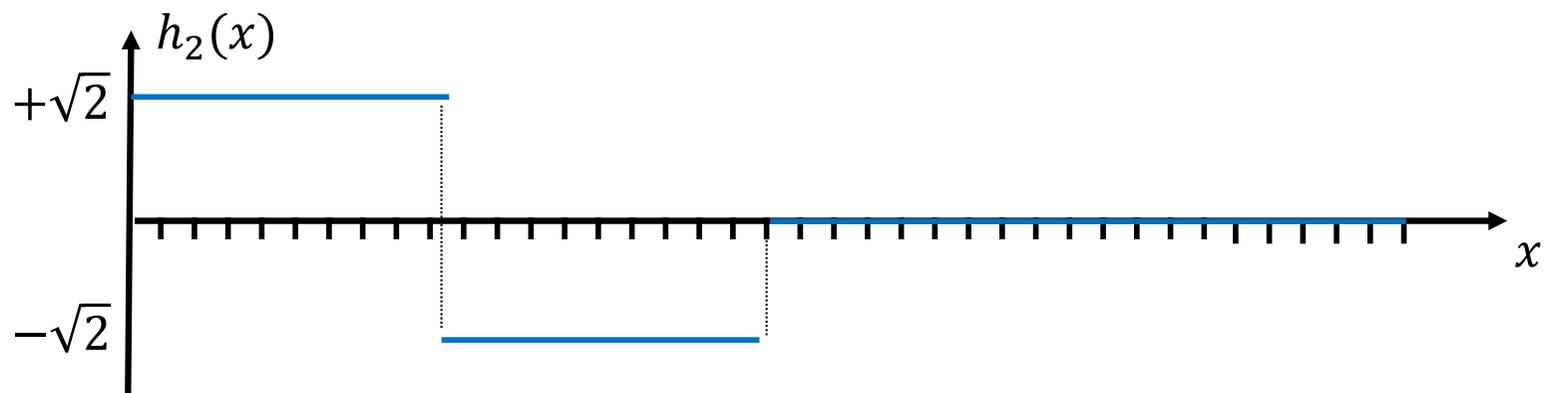
Para que um sinal de banda limitada (i.e. aqueles cuja a transformada resultam em zero para frequências $f > B$) seja reconstruído plenamente ele precisa ser amostrado numa frequência $f \geq 2B$.

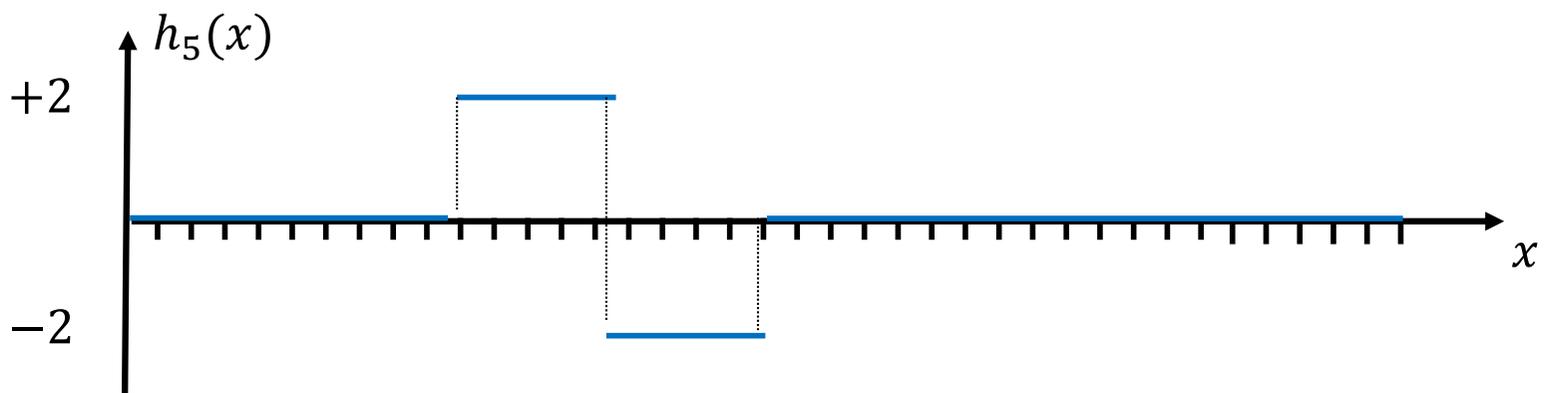
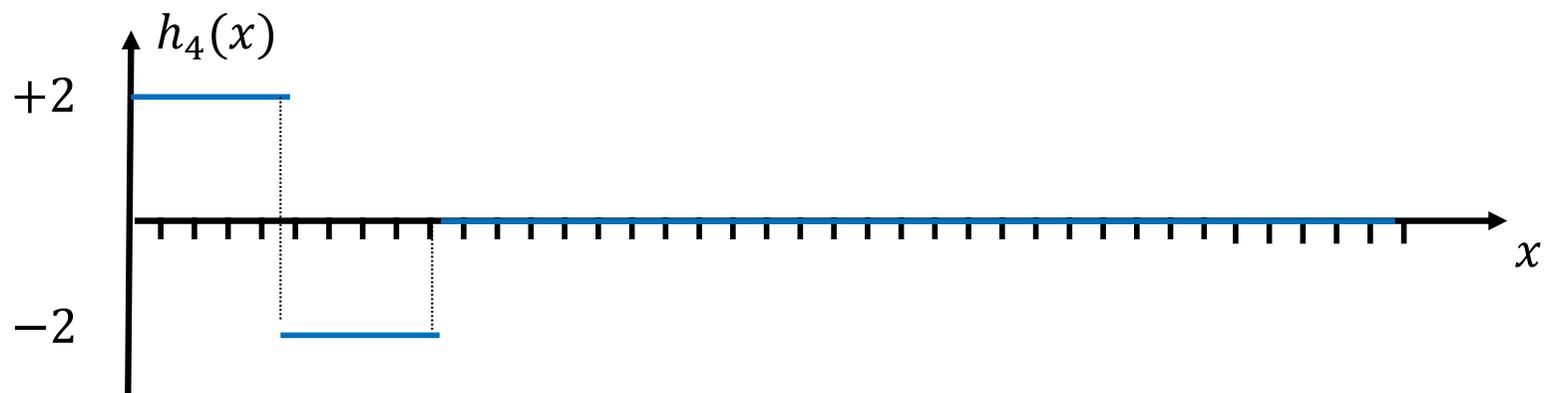
Um sinal amostrado na frequência ($f=2B$) é dito amostrado por Nyquist e $f=2B$ é a frequência de Nyquist.

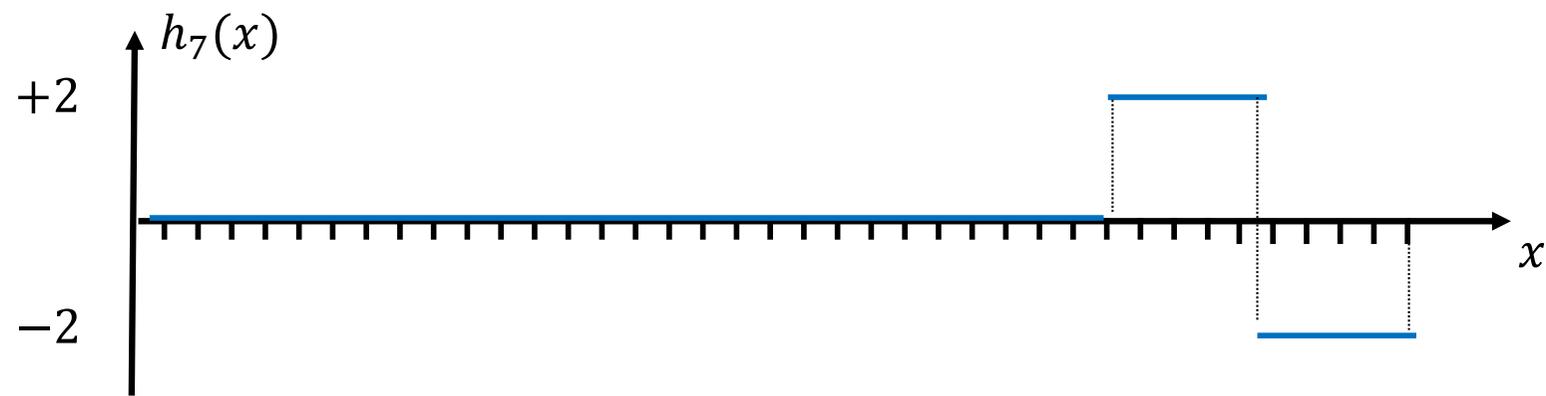
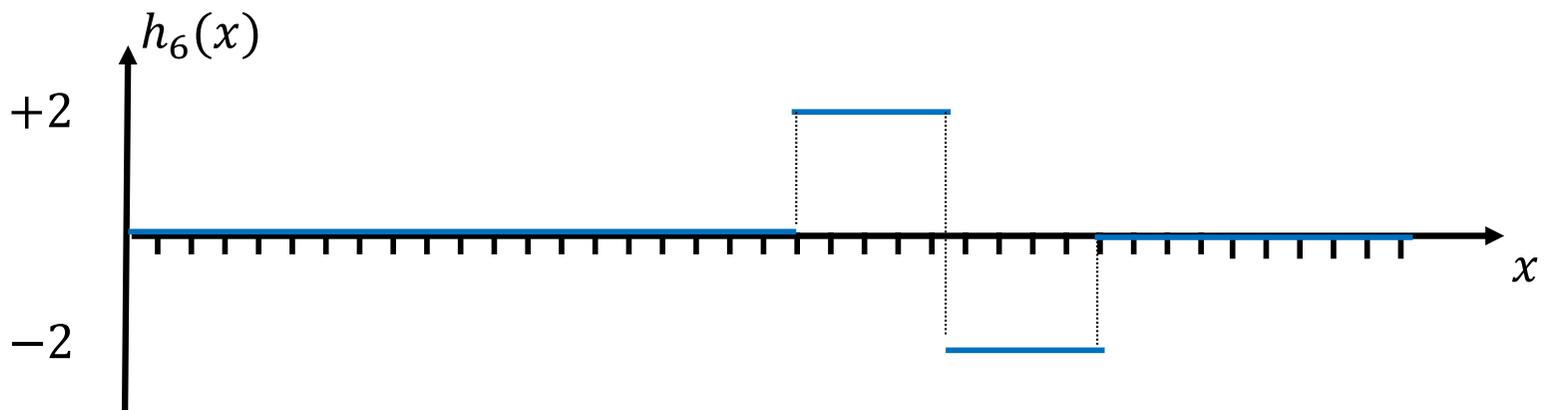
Não há perda de informação nos sinais amostrados na frequência de Nyquist, e não adicionamos nenhuma informação se amostrarmos numa frequência maior.

Base de Haar

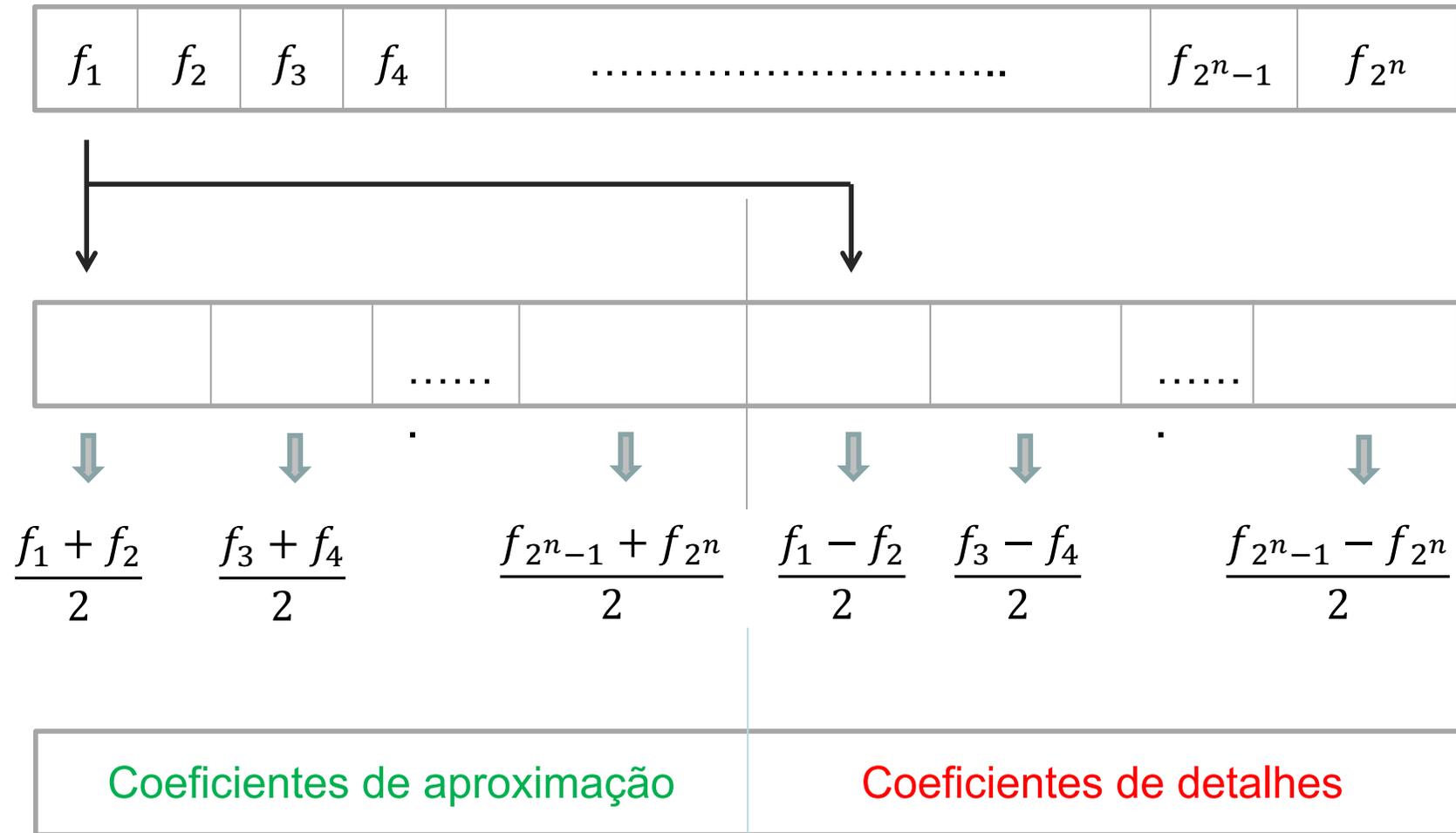








Transformada Haar



Exemplo de transformada Haar

10	4	6	12	8	4	2	6
----	---	---	----	---	---	---	---

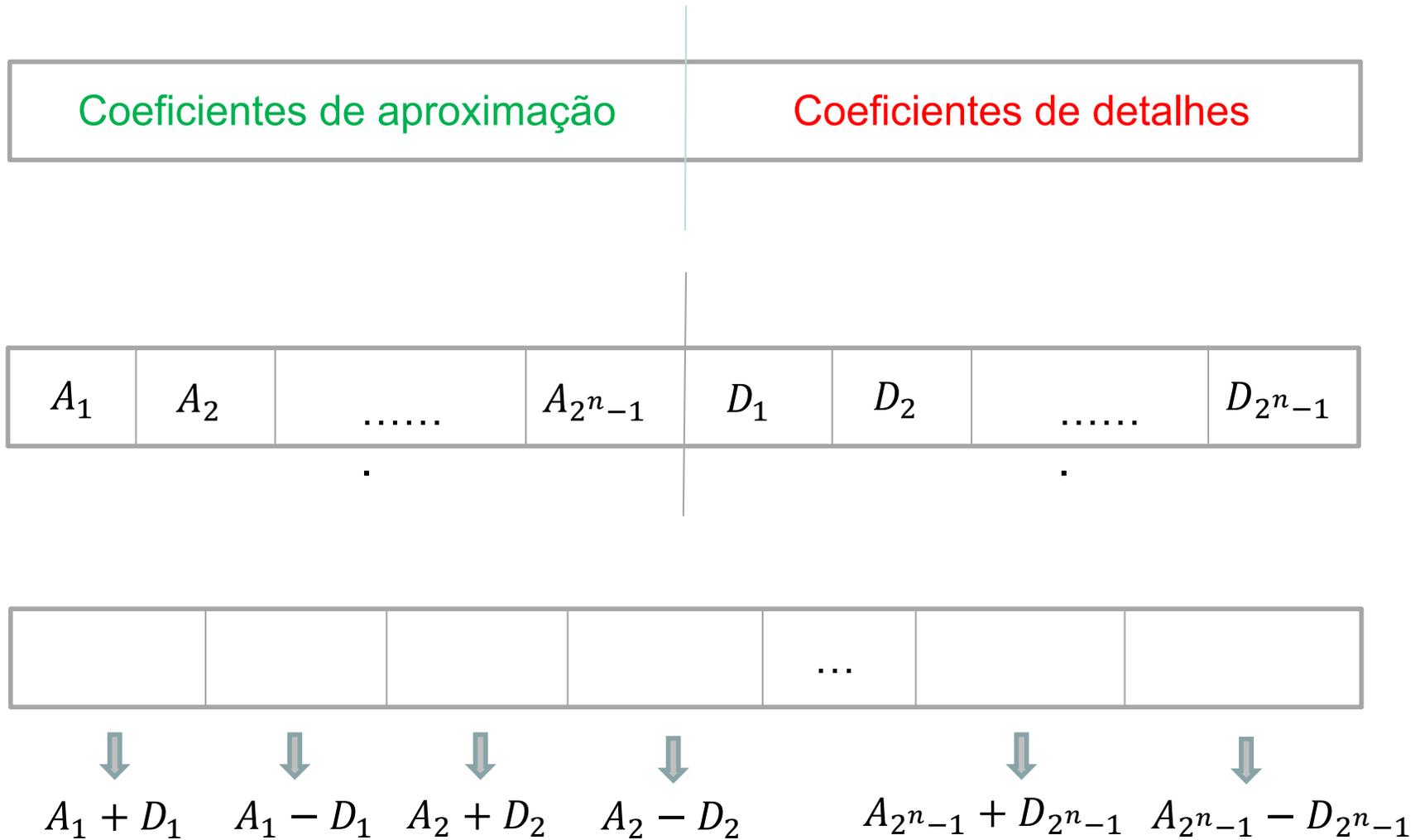
7	9	6	4	3	-3	2	-2
---	---	---	---	---	----	---	----

8	5	-1	1	3	-3	2	-2
---	---	----	---	---	----	---	----

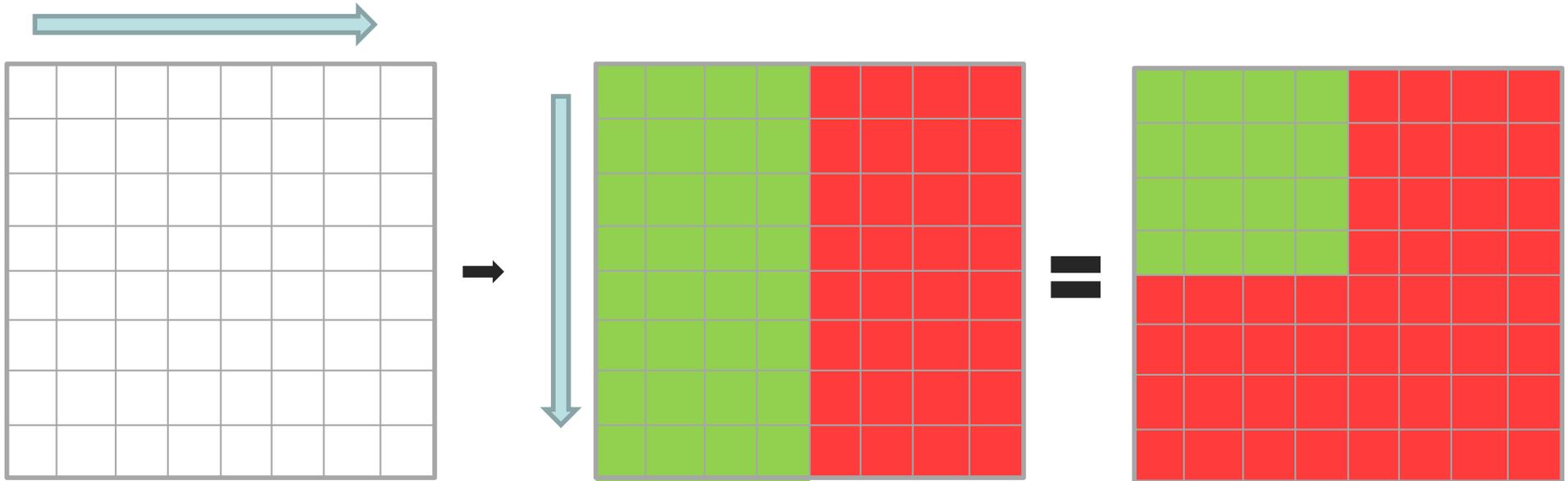
6,5	1,5	-1	1	3	-3	2	-2
-----	-----	----	---	---	----	---	----

três iterações de Haar sobre um vetor

Inversa Transformada de Haar



Transformada Haar



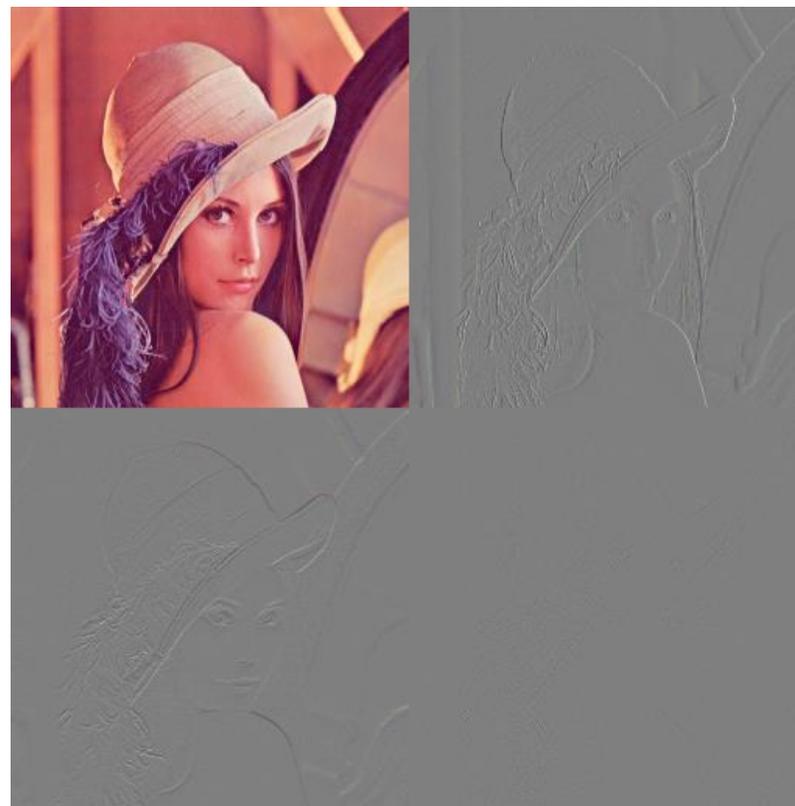
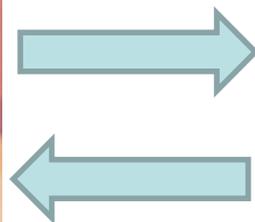
Aplicando a transformada de Haar na horizontal

Aplicando a transformada de Haar na vertical

Matriz resultante

Uma iteração da transformada de Haar sobre uma matriz

Duas iterações da Transformada de Haar



Ruídos gaussianos e impulsivo (sal e pimenta)

REDUÇÃO DE RUÍDOS

O problema

- Dada uma imagem $I(x, y)$ com um ruído $N(x, y)$, reduza $N(x, y)$ o máximo que puder sem alterar significativamente $I(x, y)$.

Modelo aditivo de ruído: $\tilde{I}(x, y) = I(x, y) + n(x, y)$

$$SNR = \frac{\sigma_s}{\sigma_n} \quad SNR_{dB} = 10 \log_{10} \frac{\sigma_s}{\sigma_n}$$

20 dB significam $\frac{\sigma_s}{\sigma_n} = 100$

Dois tipos básicos de ruídos

- Ruído impulsivo: causado por erro de transmissão, CCDs defeituosos, etc...
Também chamado de pico e de sal e pimenta.

$$n_{sp}(i, j) = \begin{cases} 0 & x < l \\ i_{\min} + y(i_{\max} - i_{\min}) & x \geq l \end{cases}$$

$x, y \in [0,1]$ são v.a. uniformemente distribuídas

i_{\min} , i_{\max} e l são parâmetros de controle da quantidade de ruídos.

Dois tipos básicos de ruídos

- Ruído Gaussiano branco : processo estocástico de média zero, independente do tempo e do espaço.

$\bar{n}(i, j) \sim \bar{n}(i + i_0, j + j_0)$ é o mesmo processo estocástico que não varia no tempo.

$$\bar{n}(i, j) = 0$$

$\bar{n}(i, j)$ é uma variável aleatória com a distribuição:

$$G(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma^2}}$$

Exemplo de ruído Gaussiano ($\sigma=5$) e Impulsivo ($\ell=0.99$)

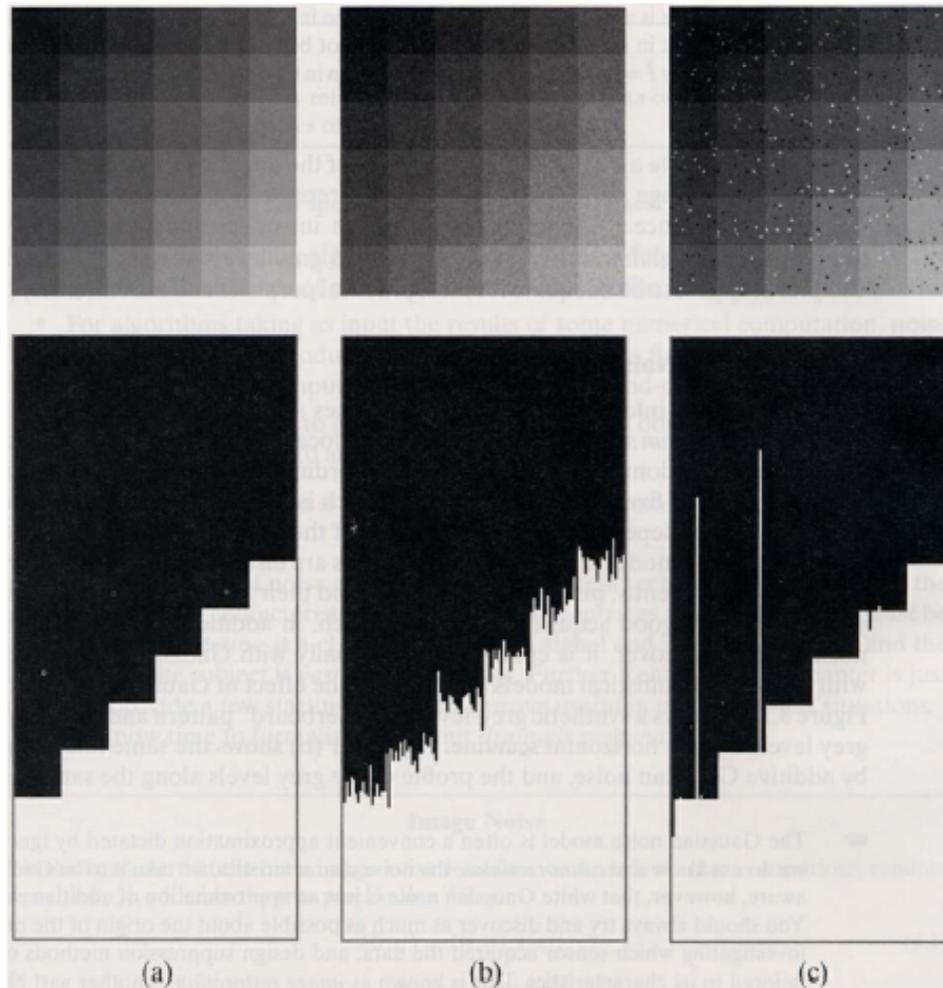


Figure 3.1 (a) Synthetic image of a 120×120 grey-level “checkerboard” and grey-level profile along a row. (b) After adding zero-mean Gaussian noise ($\sigma = 5$). (c) After adding salt and pepper noise (see text for parameters).

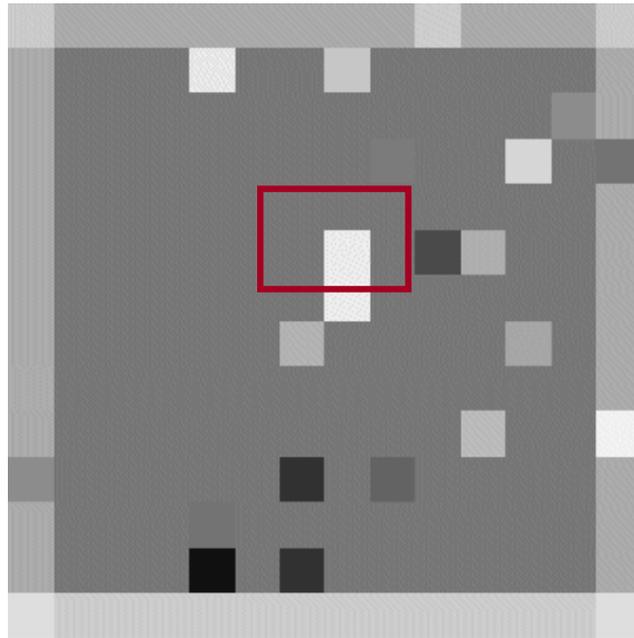
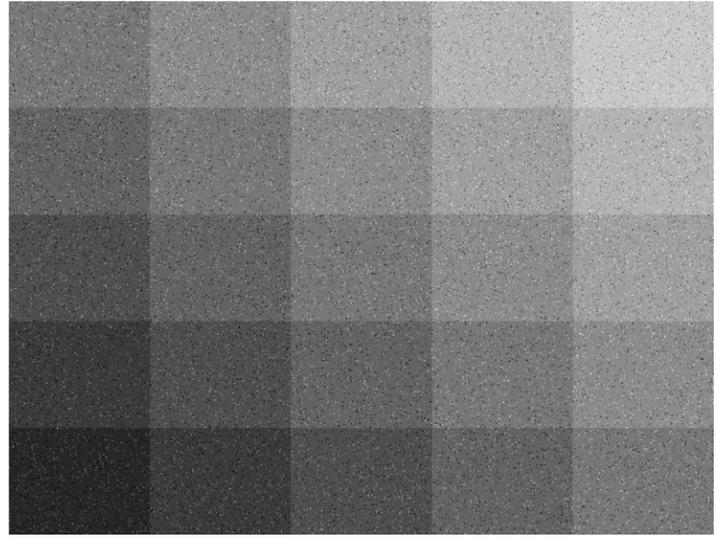
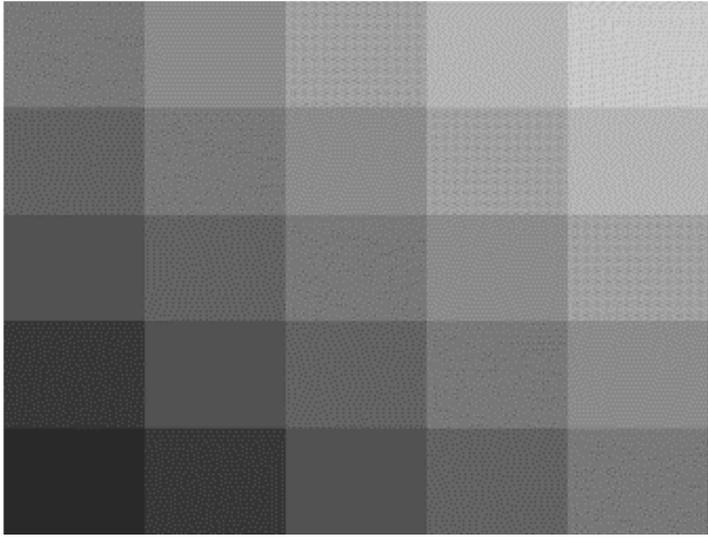


Imagem com ruído impulsivo

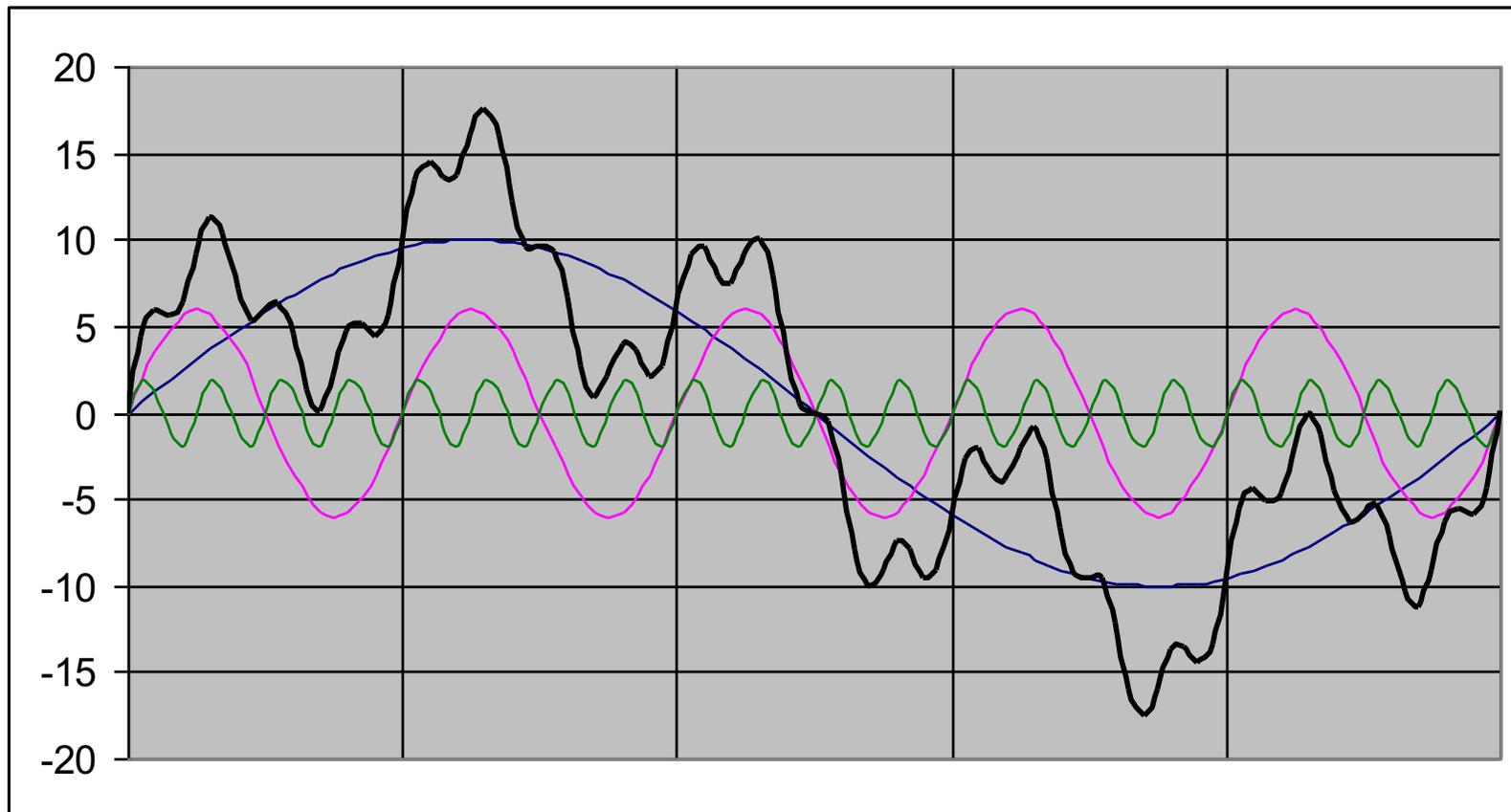
Uso da mediana para reduzir o ruído

223	204	204	204	204	204	204	204	204	204	204	204	204	223
171	120	120	120	18	120	50	120	120	120	120	120	120	171
171	120	120	120	116	120	120	120	120	120	120	120	120	171
138	120	120	120	120	120	50	120	97	120	120	120	120	171
171	120	120	120	120	120	120	120	120	120	187	120	120	242
172	120	120	120	120	120	120	120	120	120	120	120	120	171
171	120	120	120	120	120	<u>179</u>	120	120	120	120	167	120	171
171	120	120	120	120	120	120	235	120	120	120	120	120	171
171	120	120	120	120	120	120	235	120	76	175	120	120	171
171	120	120	120	120	120	120	120	120	120	120	120	120	171
171	120	120	120	120	120	120	120	123	120	120	214	120	114
171	120	120	120	120	120	120	120	120	120	120	120	143	171
171	120	120	120	232	120	120	198	120	120	120	120	120	171
203	171	171	171	171	171	171	171	171	205	171	171	171	203

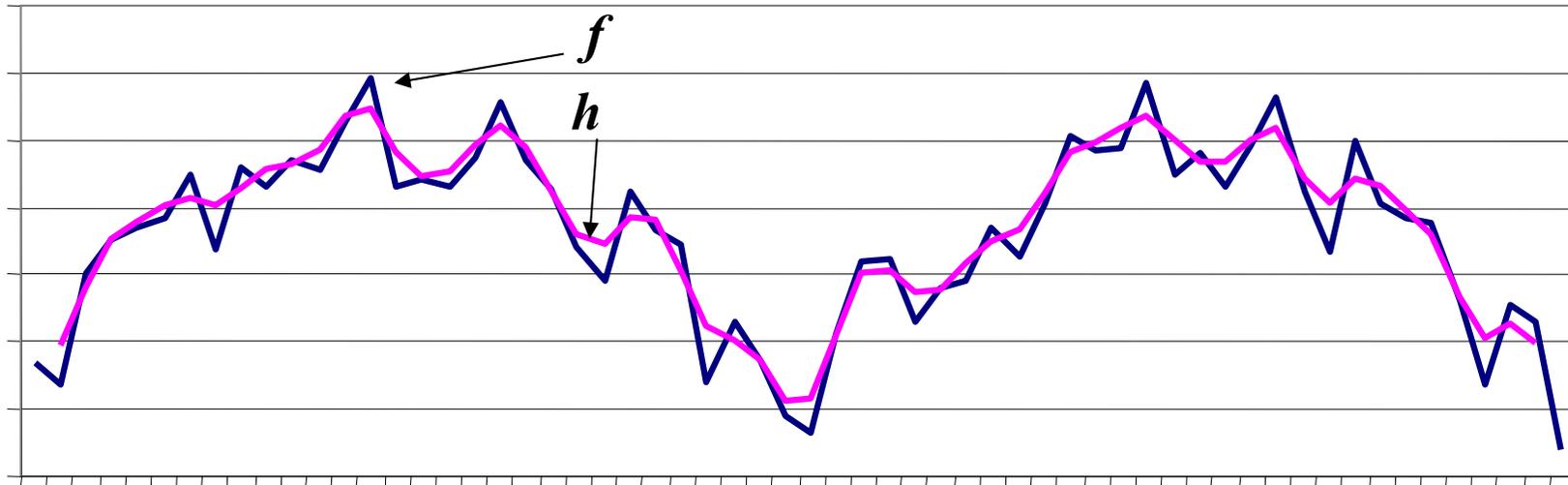
$$I_{ij} = \text{mediana } \Omega_{ij} = 120$$

Sinal com ruído gaussiano

$$f_3(x) := 10 \cos(2 \pi x) + 6 \sin(10 \pi x) + .8 \cos(40 \pi x)$$

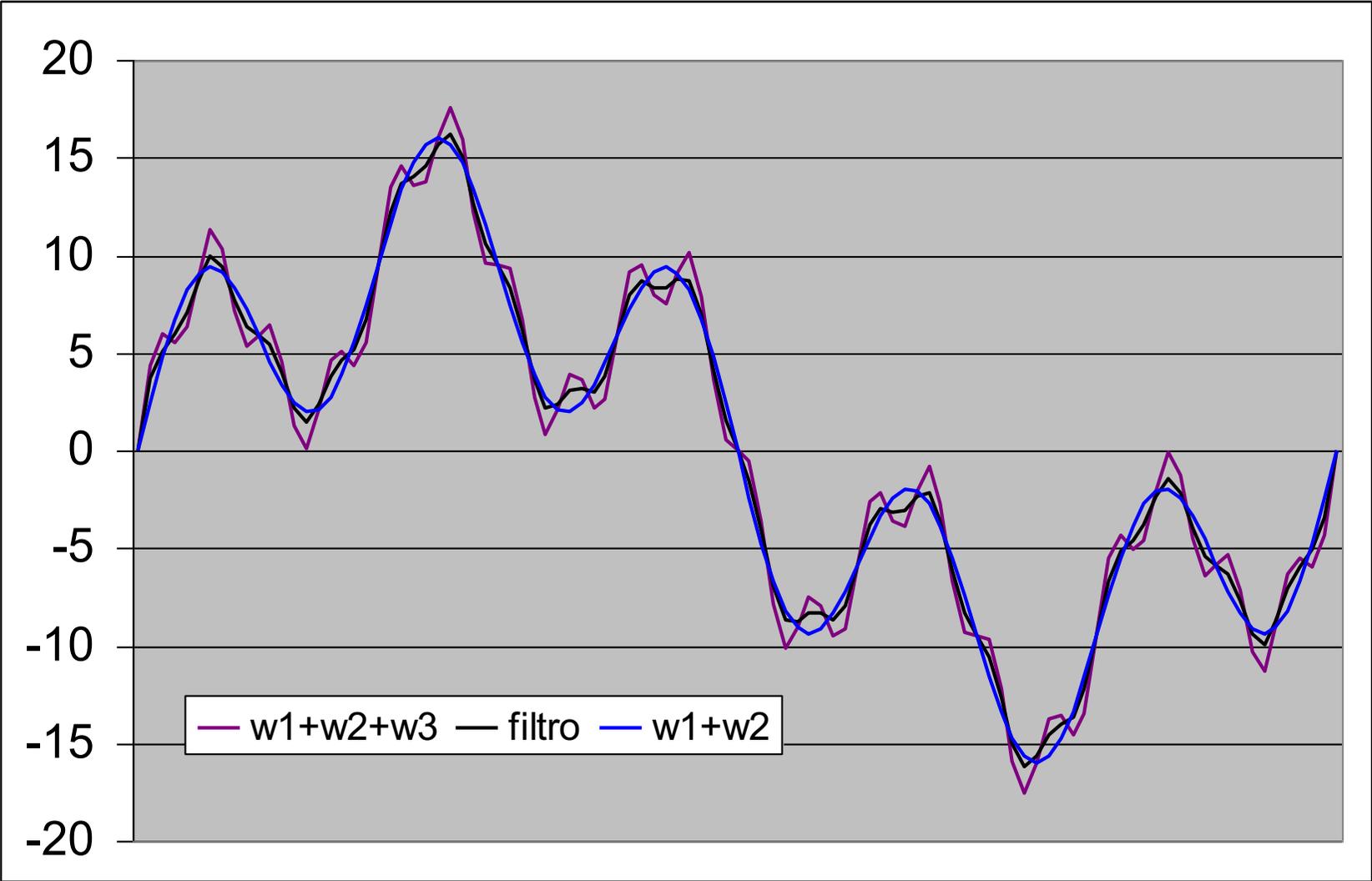


Suavização



$$h_i = \frac{f_{i-1} + 2f_i + f_{i+1}}{4}$$

Filtragem Gaussiana



Mascara ou Filtro

$$h_i = \frac{f_{i-1} + 2f_i + f_{i+1}}{4}$$

ou:

$$h_i = \sum_{k=0}^{n-1} g_{(k-i)} f_k$$

$$g_l = \begin{cases} 0 & \text{se } l < -1 \\ 1/4 & \text{se } l = -1 \\ 2/4 & \text{se } l = 0 \\ 1/4 & \text{se } l = +1 \\ 0 & \text{se } l > +1 \end{cases}$$

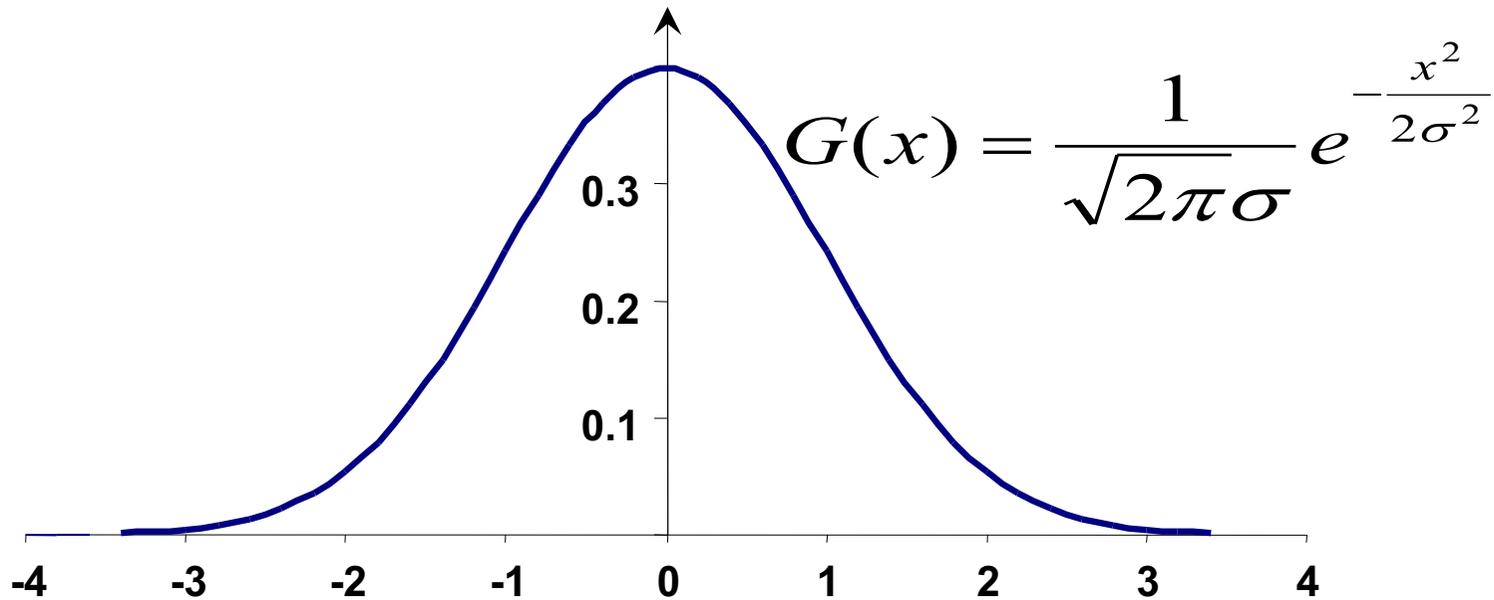
Convolução

$$h_i = \sum_{k=0}^{n-1} g_{(k-i)} f_k$$

$$h(x) = \int_{t=-\infty}^{t=\infty} g(t-x) f(t) dt$$

$$h(x) = f \otimes g = \int_{-\infty}^{\infty} f(u) g(x-u) du$$

Discretização da Gaussiana 1D



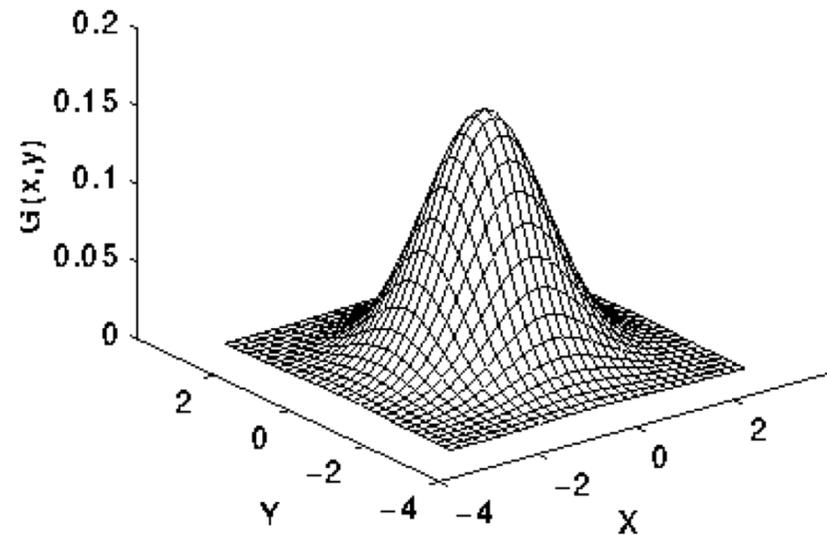
$$\frac{1}{4} [1 \quad 2 \quad 1]$$

$$\frac{1}{16} [1 \quad 4 \quad 6 \quad 4 \quad 1]$$

$$\frac{1}{64} [1 \quad 6 \quad 15 \quad 20 \quad 15 \quad 6 \quad 1]$$

Discretização da Gaussiana 2D

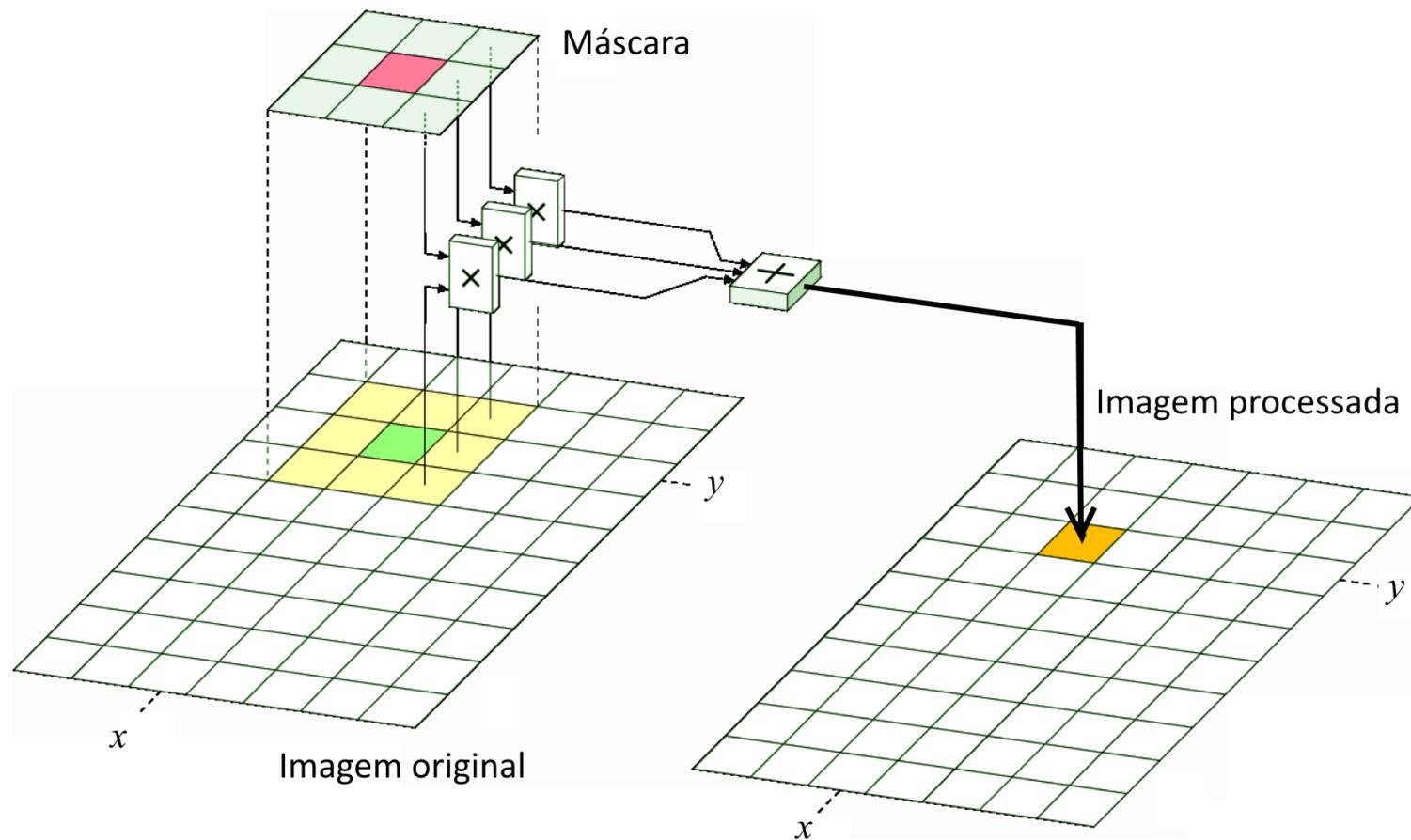
$$G(x, y) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{x^2 + y^2}{2\sigma^2}}$$



$$\frac{1}{16} \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

$$\frac{1}{273} \begin{bmatrix} 1 & 4 & 7 & 4 & 1 \\ 4 & 16 & 26 & 16 & 4 \\ 7 & 26 & 41 & 26 & 7 \\ 4 & 16 & 26 & 16 & 4 \\ 1 & 4 & 7 & 4 & 1 \end{bmatrix}$$

Convolução com máscara



Separabilidade do filtro gaussiano

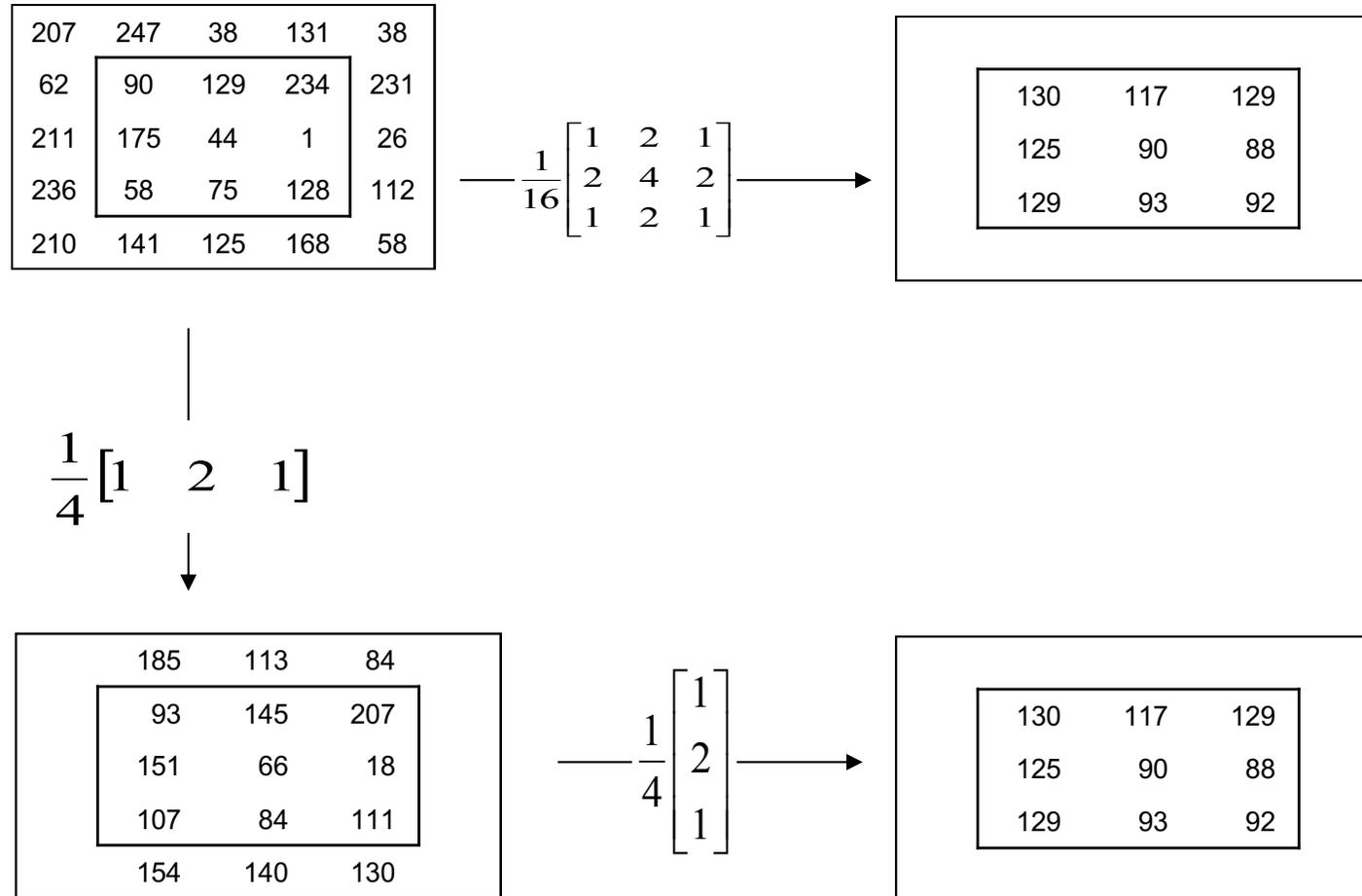
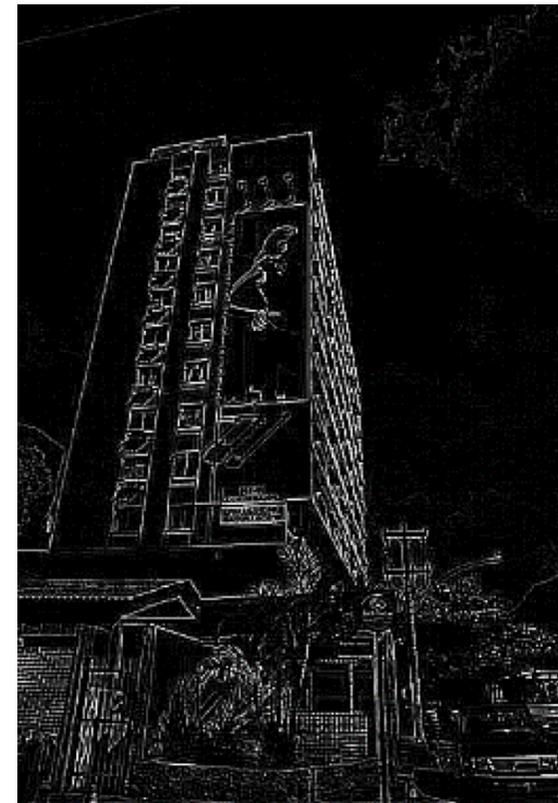


Imagem filtrada com um filtro passa baixa

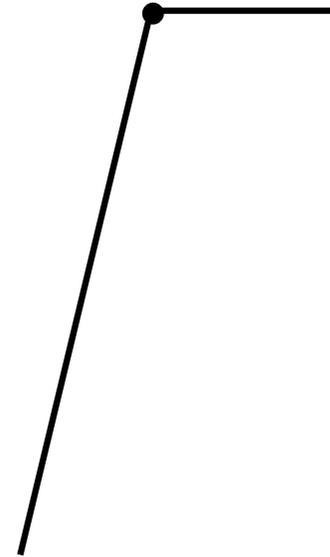
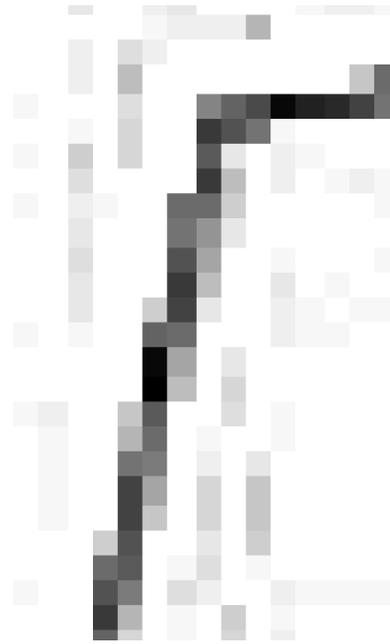
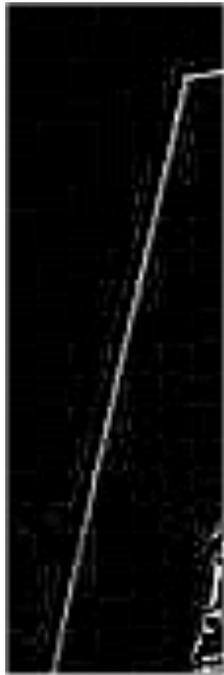


Arestas e cantos

- Locais de mudanças significativas na intensidade da imagem



Edgedels = edge elements



Tipos de arestas

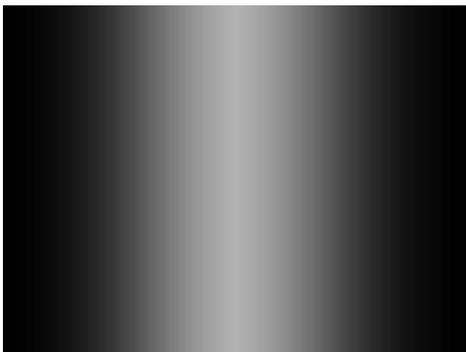
degrau (step)



rampa (ramp)



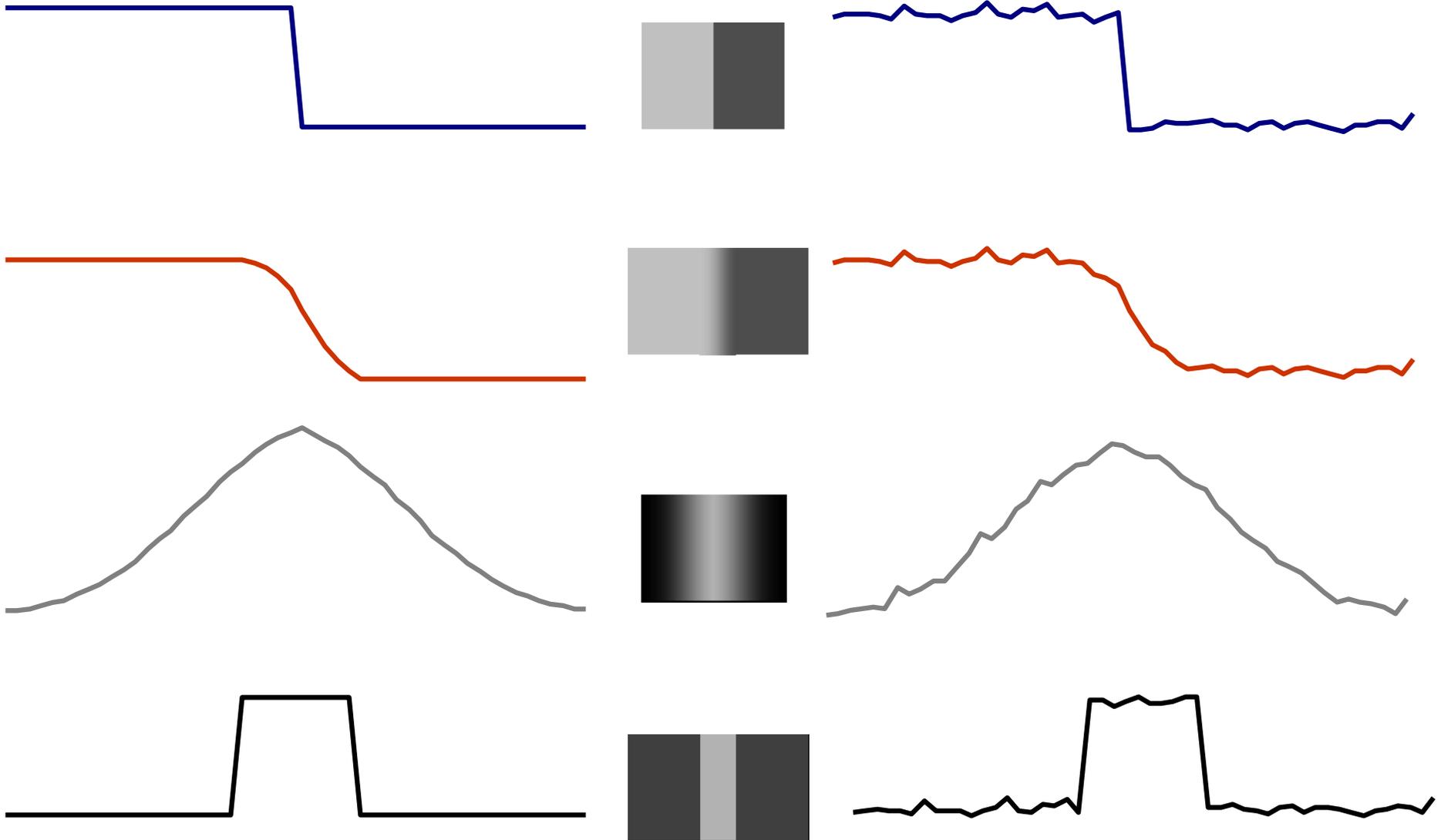
cume (roof)



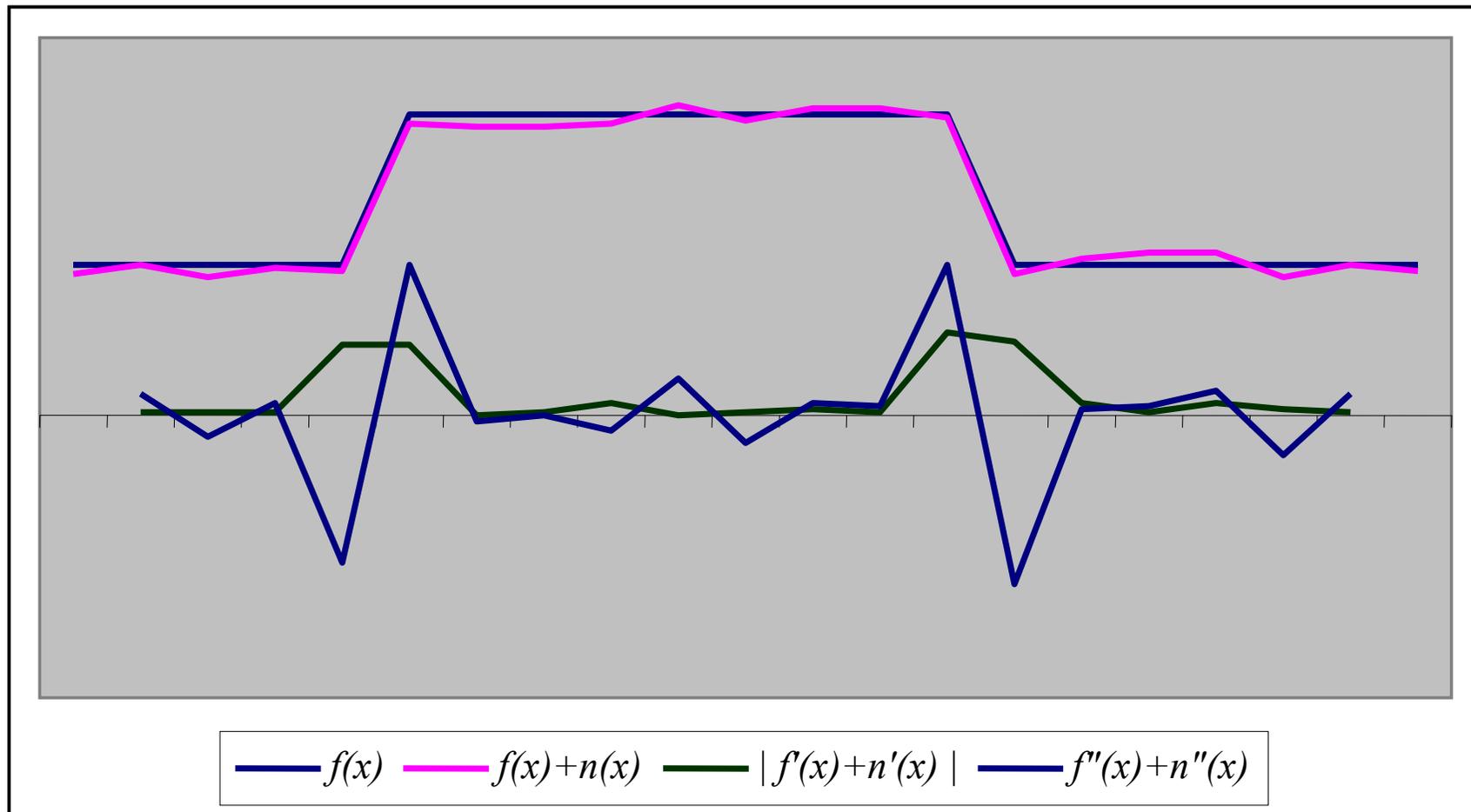
impulso (spike)



Gráfico sem e com ruído



Derivadas e arestas



Série de Taylor

$$f(x + \Delta x) = f(x) + (\Delta x)f'(x) + \frac{(\Delta x^2)}{2} f''(x) + O(\Delta x^3)$$

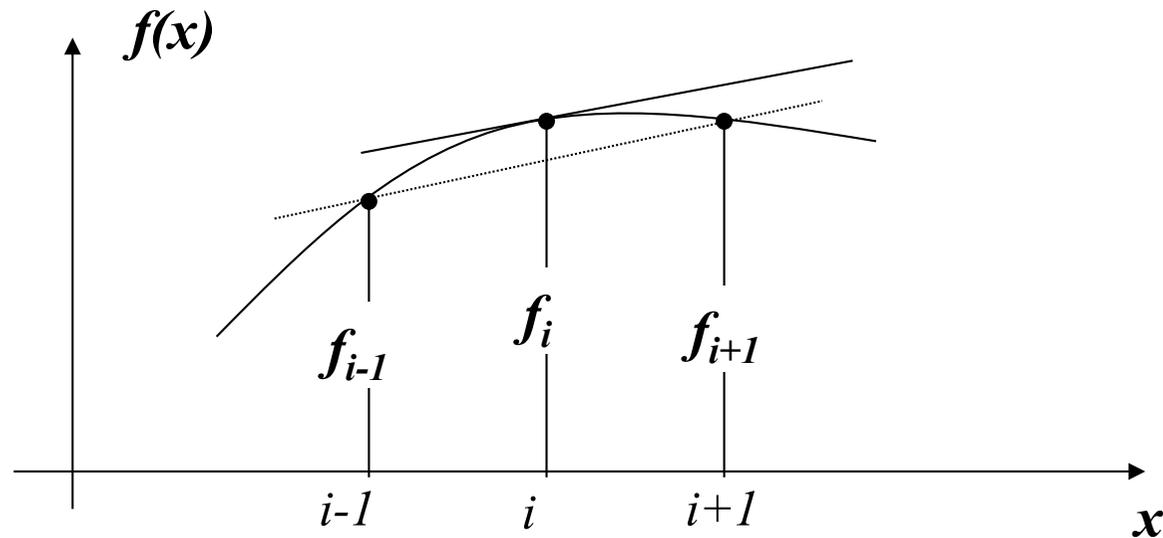
Com $\Delta x=1$, $f(x)=f_i$ e $f(x+\Delta x)=f_{i+1}$

$$f_{i+1} \cong f_i + f'_i + \frac{1}{2} f''_i \quad \text{(a)}$$

Com $\Delta x=-1$, $f(x)=f_i$ e $f(x+\Delta x)=f_{i-1}$

$$f_{i-1} \cong f_i - f'_i + \frac{1}{2} f''_i \quad \text{(b)}$$

Aproximações para derivadas



(a-b) \Rightarrow

$$f'_i \cong (f_{i+1} - f_{i-1}) / 2$$

(a+b) \Rightarrow

$$f''_i \cong -(-f_{i+1} + 2f_i - f_{i-1})$$

Em 2D

Gradiente

$$\nabla f(x, y) = \begin{pmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{pmatrix} \quad \begin{aligned} \frac{\partial f(x, y)}{\partial x} &\approx \frac{f(x_{n+1}, y_m) - f(x_n, y_m)}{\Delta x} & \begin{bmatrix} -1 & 1 \end{bmatrix} \\ \frac{\partial f(x, y)}{\partial y} &\approx \frac{f(x_n, y_{m+1}) - f(x_n, y_m)}{\Delta y} & \begin{bmatrix} 1 \\ -1 \end{bmatrix} \end{aligned}$$

Laplaciano

Núcleos de convolução

$$\nabla^2 f(x, y) = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} \quad \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

Derivação após suavização

Sobel:

$$\begin{bmatrix} -1 & 0 & +1 \\ -2 & 0 & +2 \\ -1 & 0 & +1 \end{bmatrix}$$

$$\begin{bmatrix} +1 & +2 & +1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

Laplaciano:

$$\begin{pmatrix} 1 & 4 & 1 \\ 4 & -20 & 4 \\ 1 & 4 & 1 \end{pmatrix}$$

Imagem filtrada com um filtro passa alta



TRABALHO 1: 28/3 - SLICO

Implemente o algoritmo de superpixel SLICO descrito em:

https://infoscience.epfl.ch/record/177415/files/Superpixel_PAMI2011-2.pdf

<https://infoscience.epfl.ch/record/177415>

Seguindo as adaptações indicadas nos próximos slides.

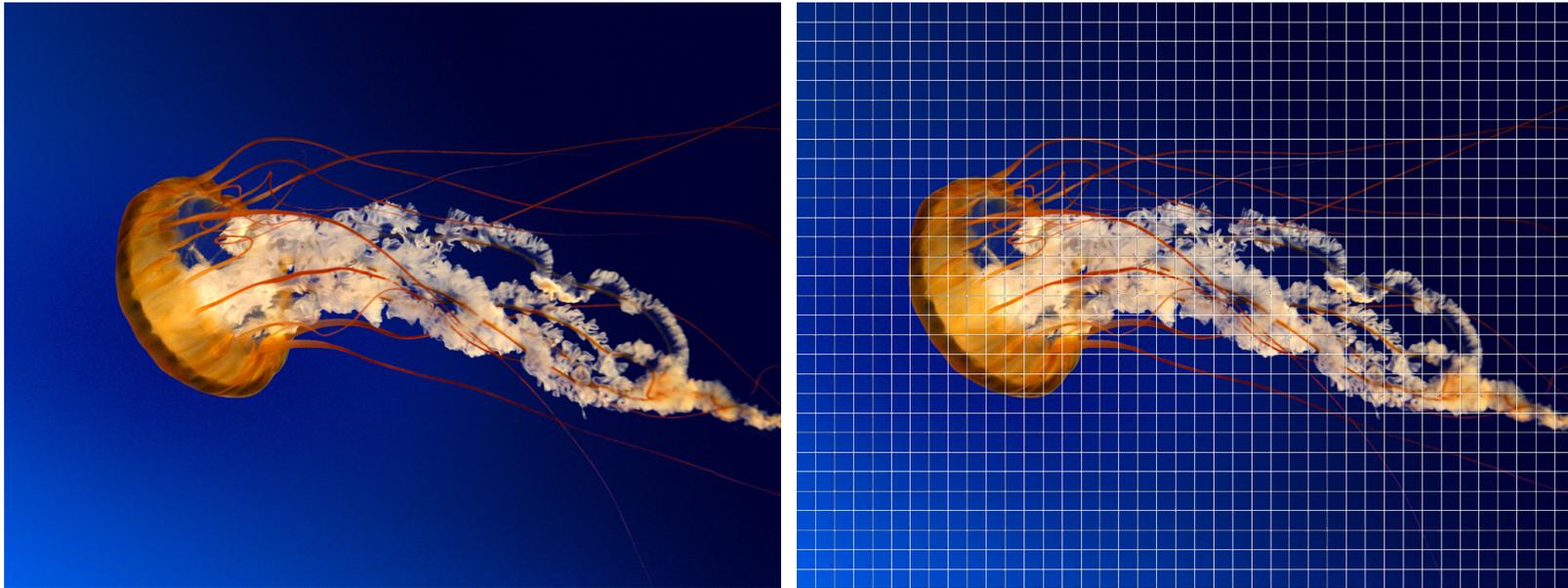
Superpixels



O algoritmo SLIC (*Simple Linear Iterative Clustering*)

Entrada: n_s (número aprox. de superpixels) e, opcionalmente, m (compacidade)

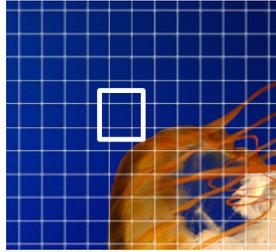
1. Calcule as coordenadas CIE Lab de todos os pixels da imagem;
2. Calcule o tamanho, s , da célula quadrada
$$s = \sqrt{\frac{w * h}{n_s}}$$
3. Inicialize os representantes das células $c_i = [l_i; a_i; b_i; x_i; y_i]^T$ amostrando em uma grade de tamanho s , $i=0, \dots, (n_s-1)$



evite as bordas e pontos ruidosos: escolha na vizinhança 3x3 do centro da célula o pixel que tenha o menor gradiente para c_i

O algoritmo SLIC (*cont.*)

4. Para cada superpixel c_k crie uma janela de tamanho $2s$ centrada em (x_k, y_k)



- a) *Para cada pixel neste janela que estiver atribuído a outro superpixel c_j , verifique se a distância dele ao c_k é menor e, se for, atribua este pixel ao c_k .*
5. *Quando todos os superpixels tiverem sido visitados, recalcule o sua cor e centro através da média de seus pixels. Calcule também o deslocamento de seu centro e acumule numa medida de erro E .*
6. *Se o erro acumulado (de todos os superpixels) for pequeno ou se o número de iterações for excessivo, maior que 10, por exemplo, pare. Caso contrário volte para o passo 4.*

Cálculo de distância

Distância entre o pixel $\mathbf{p}_j = [l_j; a_j; b_j; x_j; y_j]^T$ e o superpixel $\mathbf{c}_i = [l_i; a_i; b_i; x_i; y_i]^T$

$$d_c = \sqrt{(l_i - l_j)^2 + (a_i - a_j)^2 + (b_i - b_j)^2}$$

$$d_s = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$$

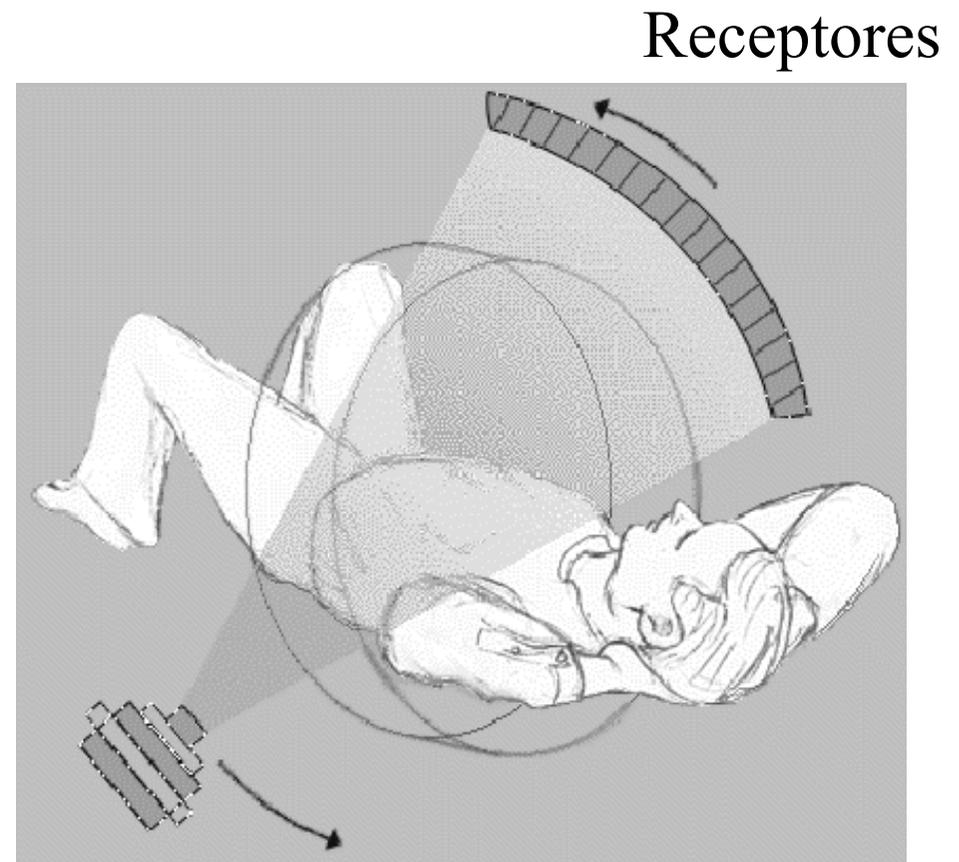
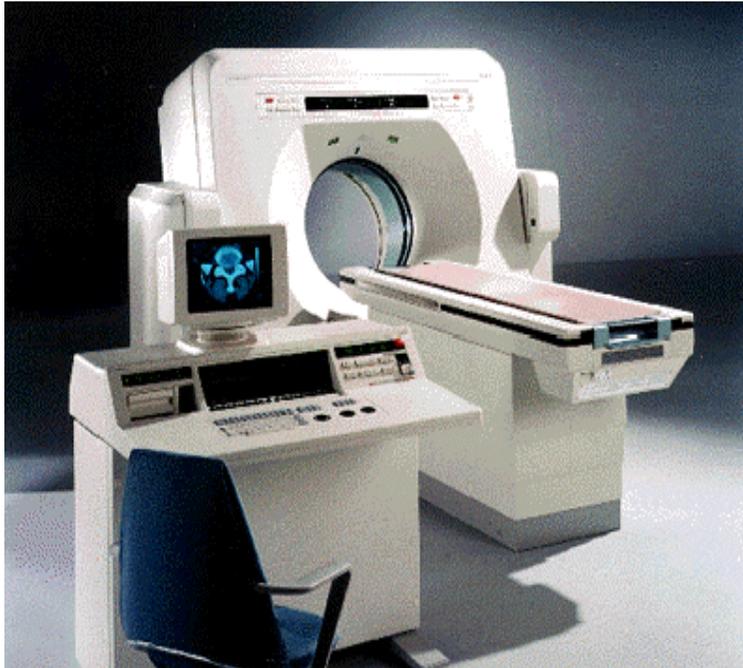
$$d_t = \sqrt{\left(\frac{d_c}{m_c}\right)^2 + \left(\frac{d_s}{m_s}\right)^2}$$

onde m_c e m_s são as máximas distâncias esparadas no superpixel.

Opções:

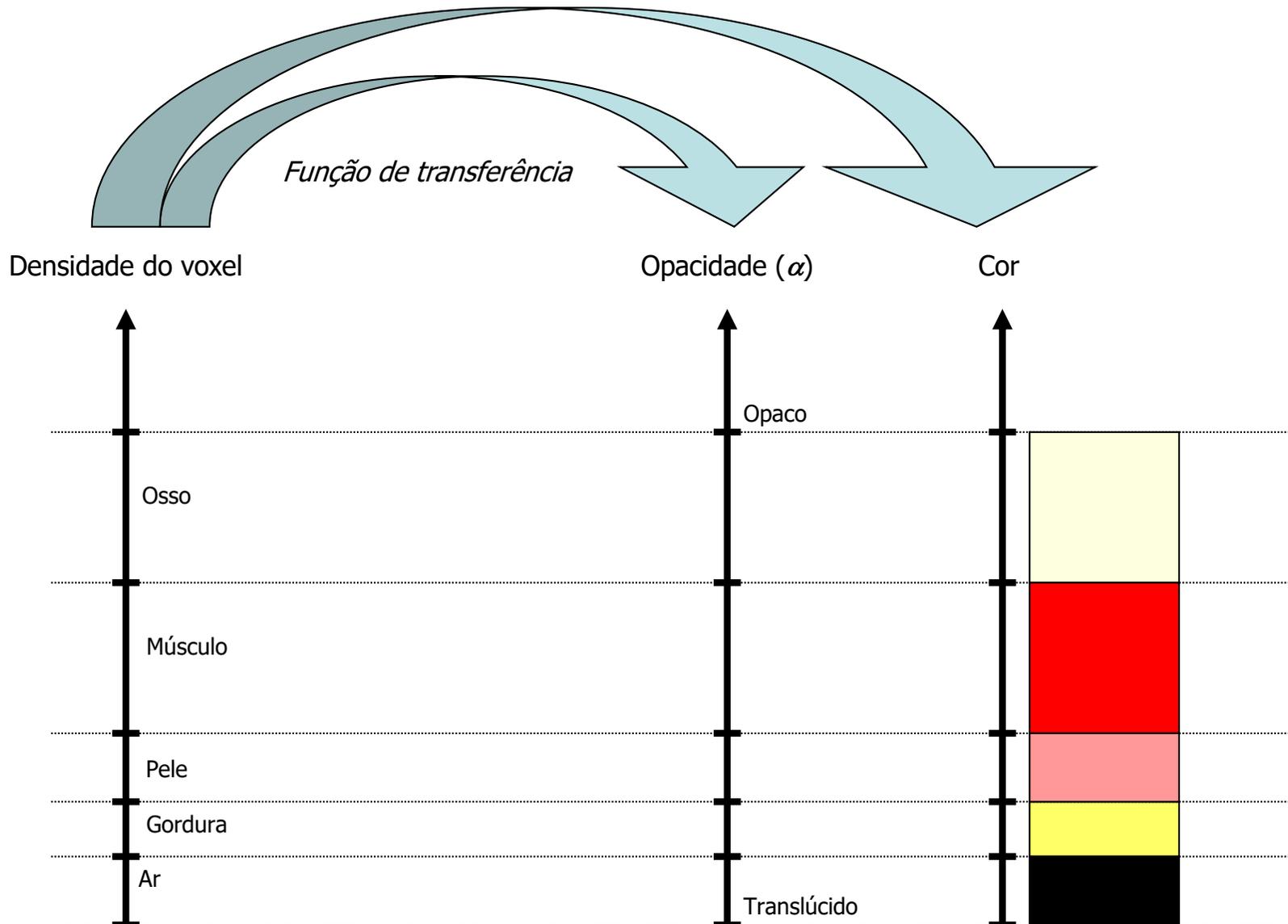
1. constantes: $m_s = s$ e m_c é *parâmetro de entrada*.
2. adaptativos: m_s e m_c são *calculados a partir da última iteração*.

Tomografia Computadorizada (TC)



Raio X

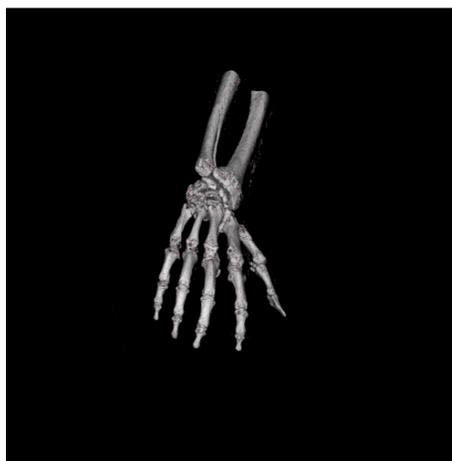
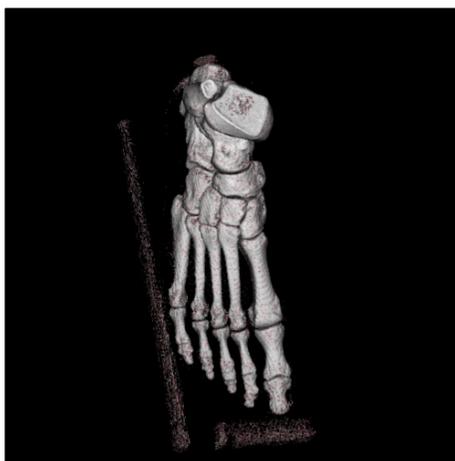
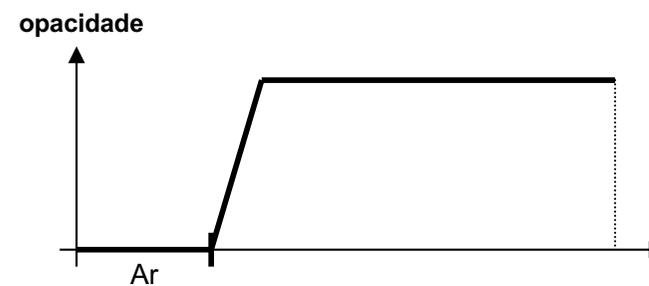
Visualização Volumétrica Direta: Função de Transferência



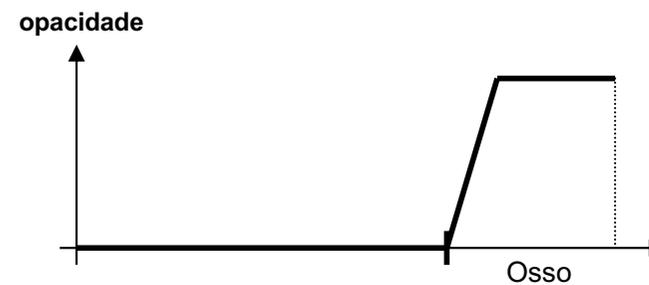
Visualização Volumétrica Direta



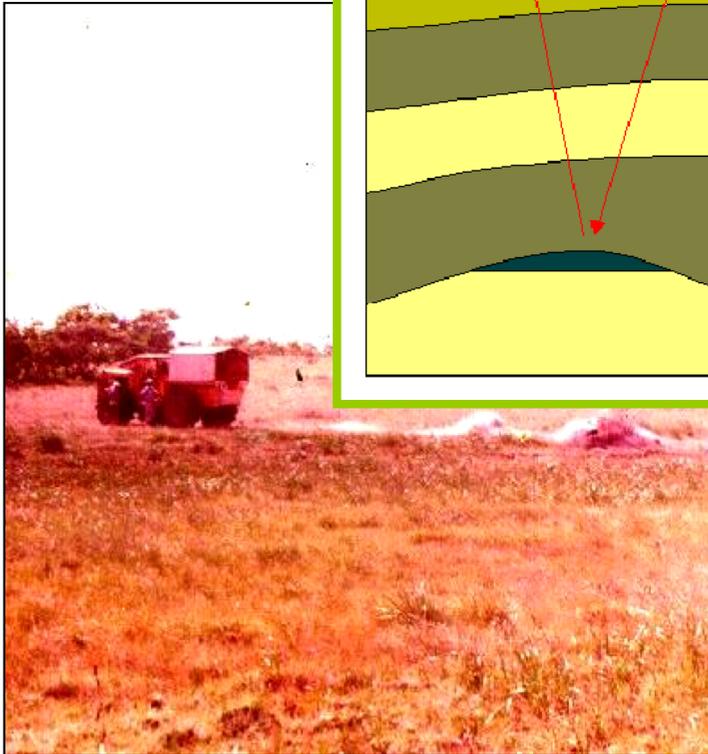
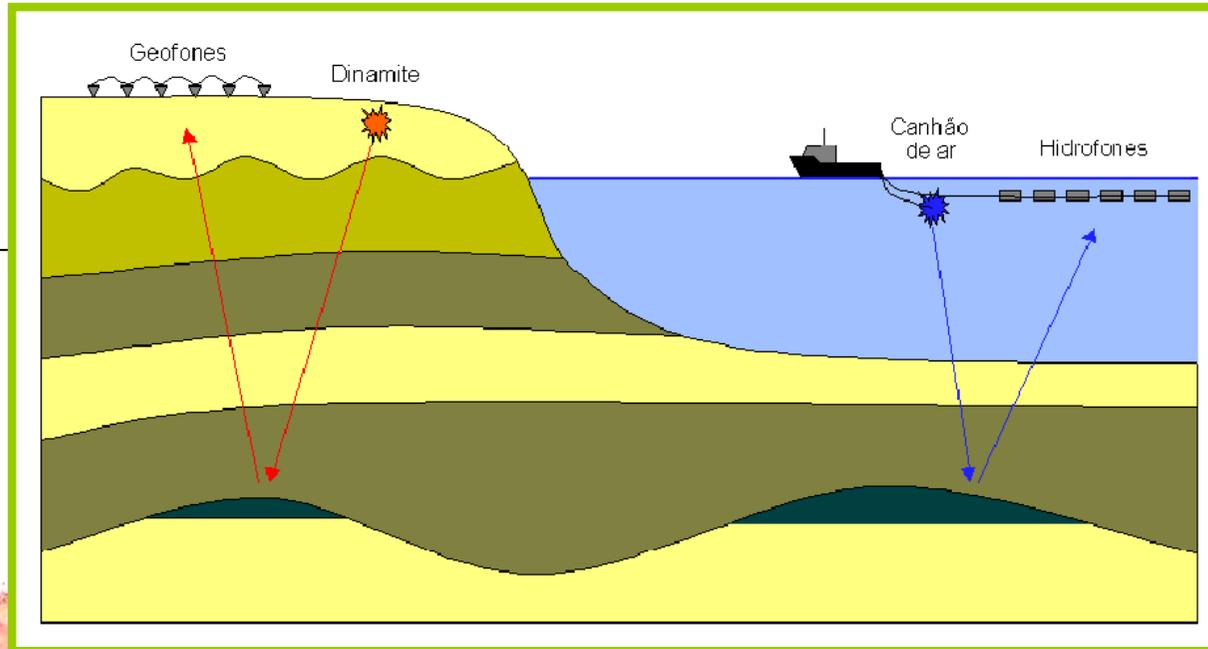
Apenas os valores de densidade baixa (ar) são considerados translúcidos.



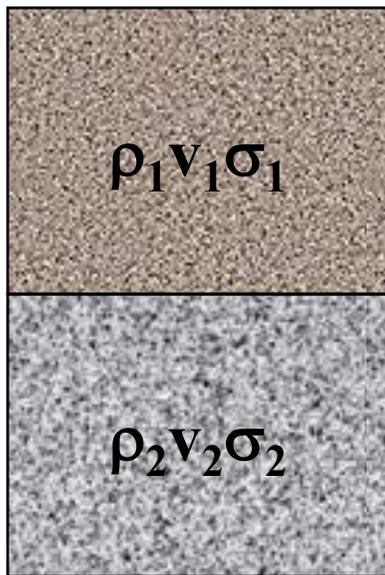
Apenas os valores de densidade alta (ossos) são considerados opacos.



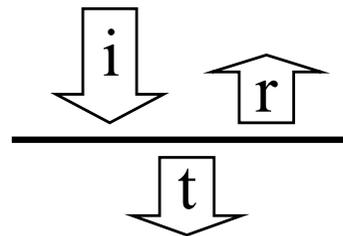
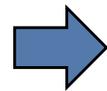
Seismic acquisitions



Refletividade



**Modelo
geológico**

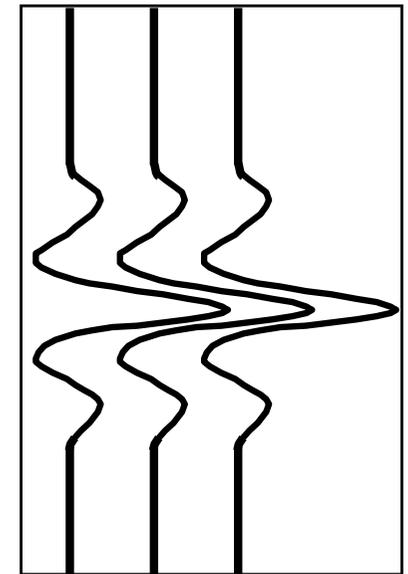
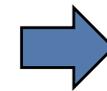


$$R_{n+1} = \frac{Z_{n+1} - Z_n}{Z_{n+1} + Z_n}$$

Refletividade

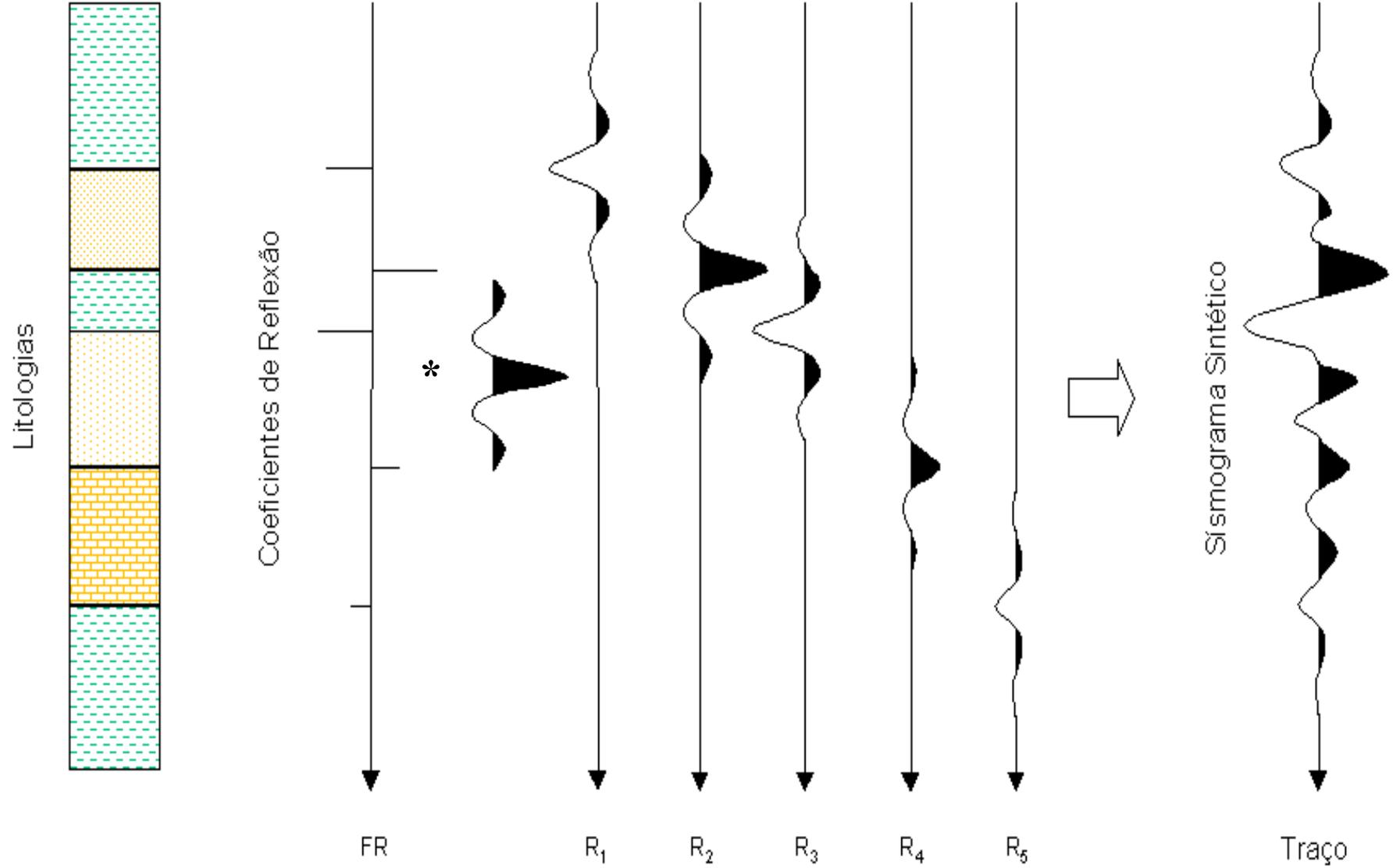


**Pulso
sísmico**

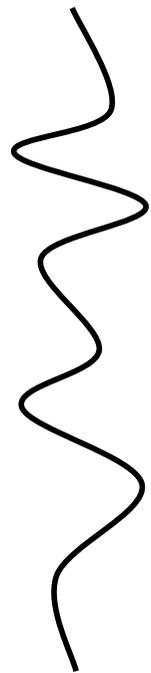


**Dado
sísmico**

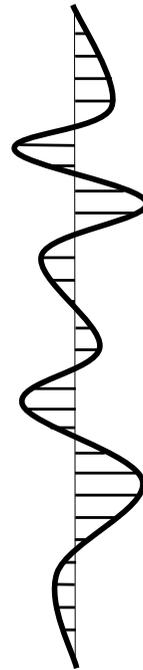
Processamento



Representação digital de dados



Amostragem



Quantização



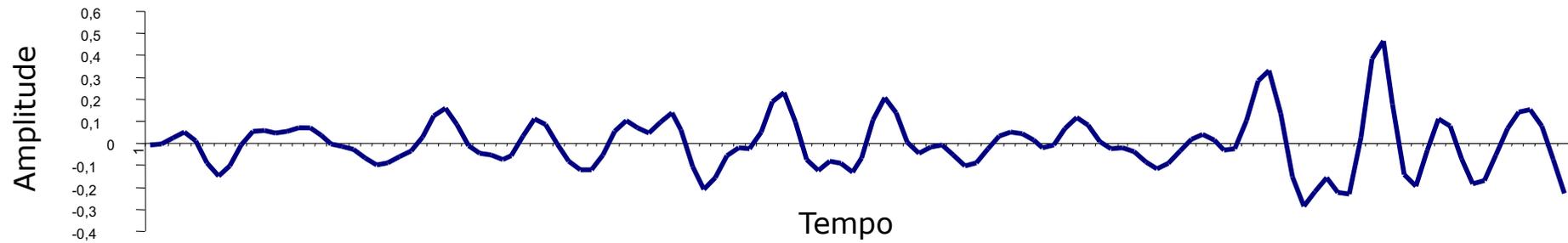
analógico

ponto-flutuante

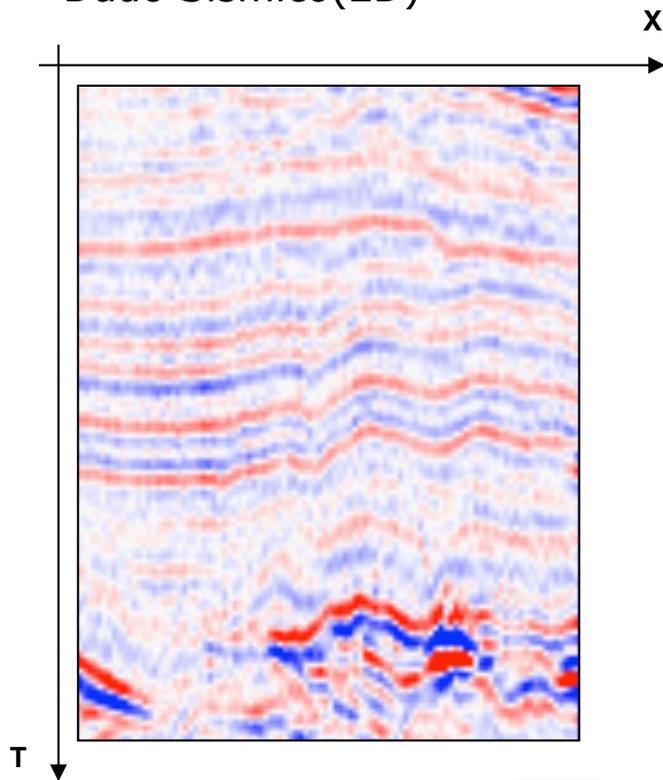
8-bit

Dado Sísmico

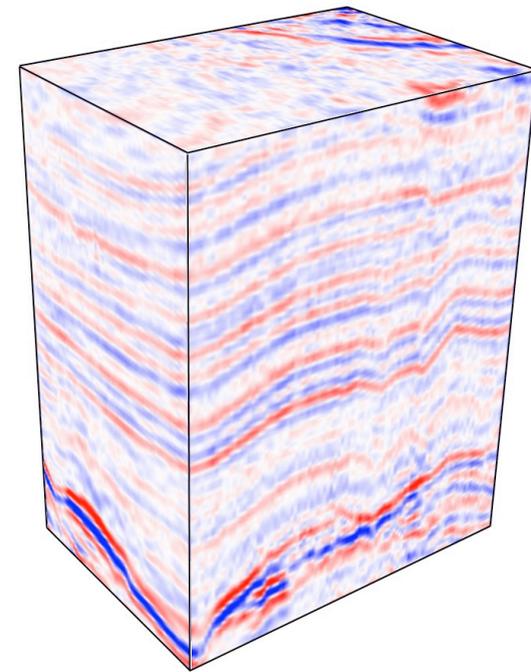
Traço Sísmico(1D)



Dado Sísmico(2D)



Dado Sísmico(3D)

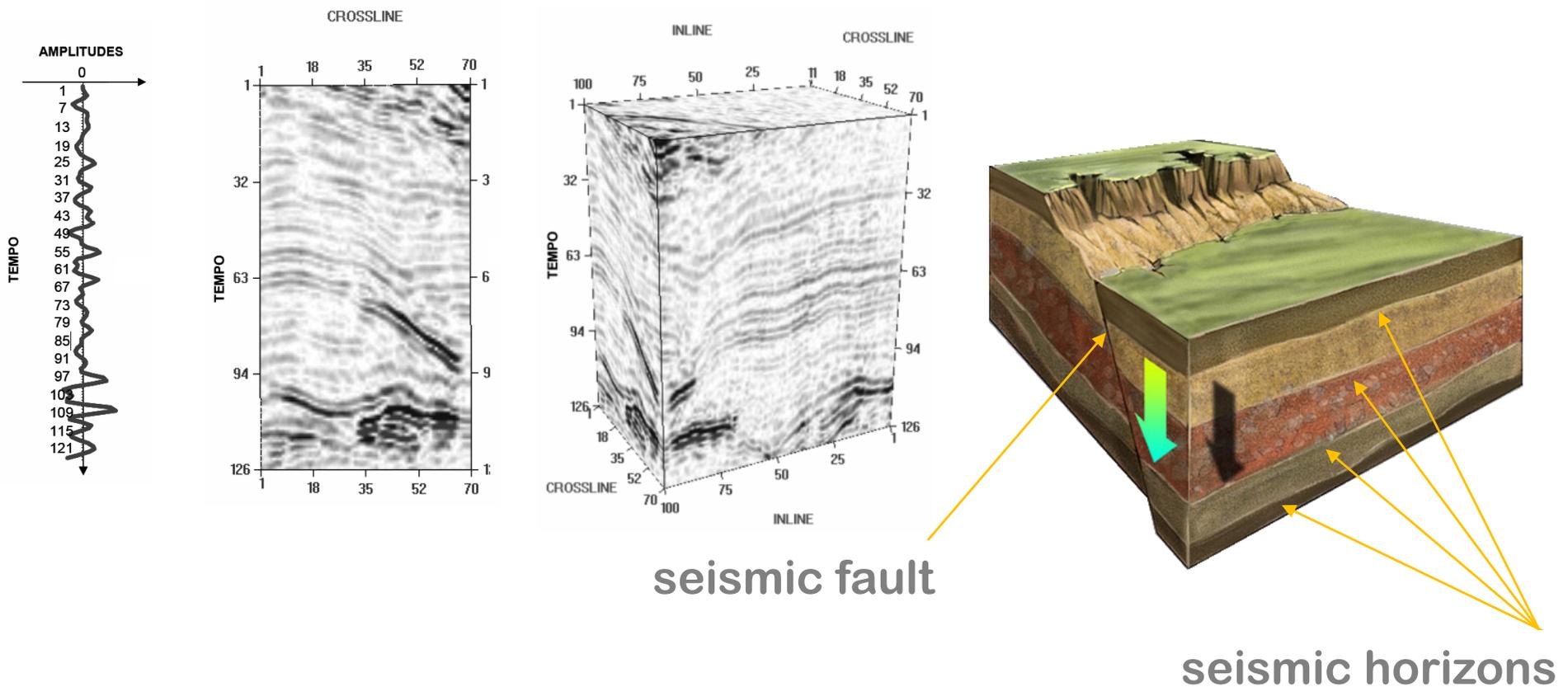


$$F(x, y, z)$$

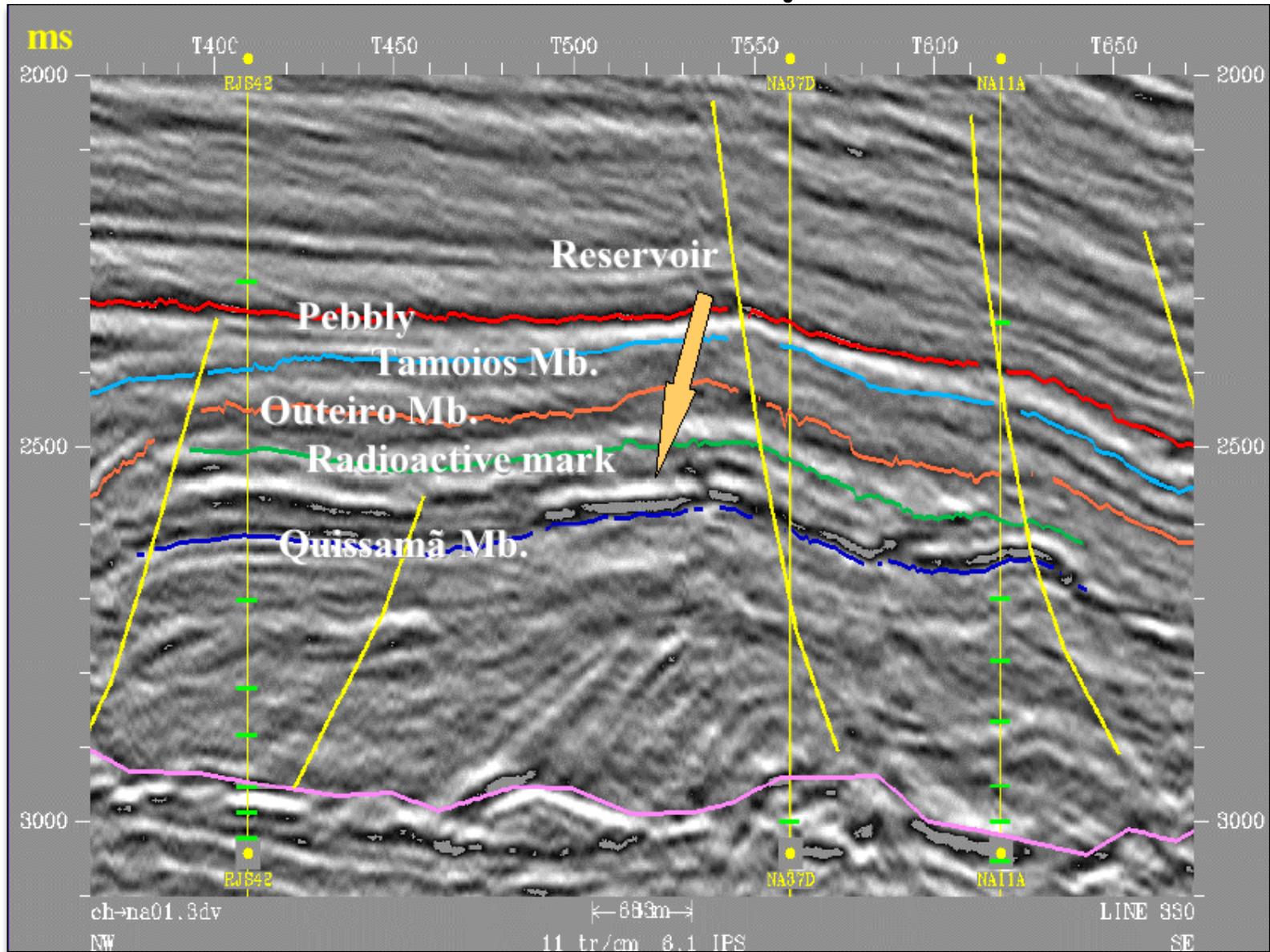


Seismic data

- Seismic traces: one temporal signal (1D)
- Seismic lines: 2D set of traces
- Seismic volumes: 3D set of traces
- 4D Seismic: seismic volumes in different times



Reservoir interpretation



(Johann, 2003)