Reliability Evaluation of Structural Safety Factor Using a Global Resistance Approach

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Abstract

This paper presents a discussion on different ways of assessing the safety of concrete structures applying Reliability Analyses. The usual design approach based on the Ultimate Limit States (ULS) is confronted with the Global Resistance Format, as defined in *fi*b Model Code 2010. The Global Format considers the several uncertainties present in the structural behaviour through a pre-defined limit state in which one or more loading variables are increased by a λ factor, until a collapse situation is attained. In this evaluation, the variables related to the actions and to the resistances are taken with their average value. The obtained values for the λ factor shall be compatible, in the safety point-of-view, with the β reliability factors corresponding to the required safety levels. A conventional building is analyzed, and the obtained reliability factors corresponding to the two approaches are presented. It is shown that the application of the Global Resistance Format can lead to more economical structures.

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Abstract

This paper presents a discussion on different ways of assessing the safety of concrete structures applying Reliability Analyses. The usual design approach based on the Ultimate Limit States (ULS) is confronted with the Global Resistance Format, as defined in *fi*b Model Code 2010. The Global Format considers the several uncertainties present in the structural behaviour through a pre-defined limit state in which one or more loading variables are increased by a λ factor, until a collapse situation is attained. In this evaluation, the variables related to the actions and to the resistances are taken with their average value. The obtained values for the λ factor shall be compatible, in the safety point-of-view, with the β reliability factors corresponding to the required safety levels. A conventional building is analyzed, and the obtained reliability factors corresponding to the two approaches are presented. It is shown that the application of the Global Resistance Format can lead to more economical structures.

Keywords: Reliability, Reinforced Concrete, Global Resistance.

1. Introduction

With the introduction of the concept of Global Resistance in the *fib* Model Code 2010 [1], in its item 4.6, from the conceptual aspects exposed, for instance, by Cervenka [2], new studies for evaluating the safety associated with the design following these new concepts are necessary.

The adequate approach for this evaluation is the Reliability Analysis, as exposed, for instance, by Melchers and Beck [3].

The analysis herein presented follow the research line developed by the authors, including new results and conclusions regarding previous papers already presented by them (for instance, [4]). The sequence of the paper follows the topics summarized in the sequel.

The central frame in a conventional building is selected for the presented analysis. A symmetrical conventional structure has been chosen, instead of a real one, in order to facilitate the analysis of results.

The building is subjected to a loading situation compatible with the one present in a real one, with the simultaneous application of dead loads and wind.

The design is done according with the Brazilian Standard for the Design of Concrete Structures [5], which follows the same design philosophy of the Eurocodes.

For the Global Resistance Analysis, it is necessary to revaluate the maximum resistant forces in the structural sections using, instead of the design values of the resistance, their average values. Then, a λ factor shall be found, that increases the forces up to the final collapse situation.

Reliability analyses are done using the computer program VAP [6]. Reliability indexes are found for a conventional sectional analysis and for the final collapse situation of the Global Analysis.

Obtained results are analysed and compared, leading to the conclusion that the Global Resistance Analysis can be applied without jeopardizing the structural safety.

2. Analysed structure

The structure analysed in this paper corresponds to a schematic building presenting thirteen floors. It is shown in Figure 1, the plan view of a typical floor of the building and a frontal view of its central plane frame.

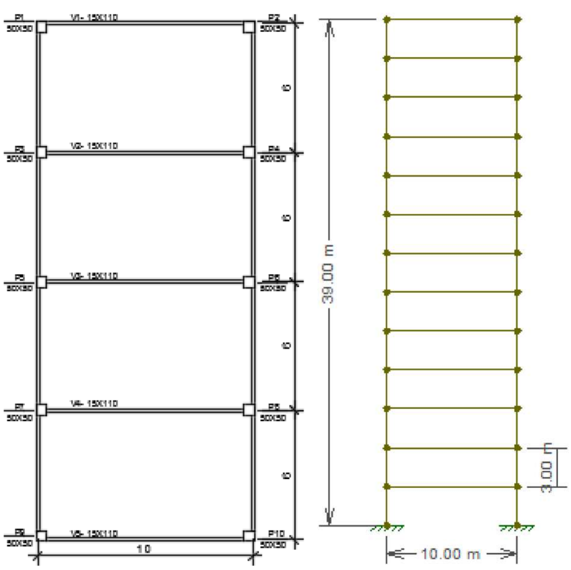

Figure 1. Plan view of a typical floor and the transversal plane frame of the analysed building

The two considered loadings correspond to dead loads and wind pressure. An area load of 8 kN/m^2 is assumed as the dead load and a resulting transversal horizontal area load of 1 kN/m^2 is considered for the wind load.

The resultant loads applied in the model are nodal loads of 18 kN (wind, in left side of the frame) e linear loads in the beams of 48 kN/m (dead load).

The considered dimensions and the necessary steel reinforcement are adjusted for resisting the forces obtained in the elastic analysis corresponding to the design in the Ultimate Limit State (ULS).

Characteristic values for the concrete and steel resistances are, respectively, f_{ck} = 30 MPa and f_{yk} = 500 MPa.

Columns have dimensions of 50 cm x 50 cm in plan and steel reinforcement corresponds to 72 cm^2, as schematically shown in Figure 2. This section is considered as constant throughout the height of the building.

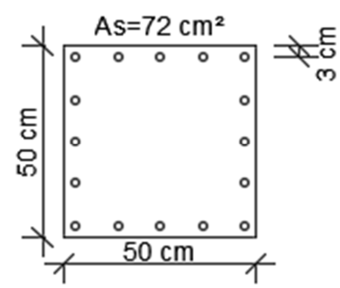

Figure 2. Sectional definition of the columns

The dimensions and reinforcement of the beams are adjusted in each floor, accordingly the acting forces. This is necessary in order that the posterior Global Analysis, which considers plastic hinges in each floor, be consistent, as is shown later on. The reinforcement of the beams is designed for the strictly necessary amount of steel.

For instance, in the first floor, the dimensions are of 15 cm x 110 cm and the reinforcement of 22.73 cm^2; in the eleventh floor, the dimensions are of 15 cm x 85 cm and the reinforcement of 16.7 cm^2.

3. Deterministic analysis – Ultimate Limit State

The more relevant results of the elastic structural analysis of the central frame are shown in Figure 3: maximum bending moment in the beam of the first floor and maximum moment and normal force in the first floor column base (results are characteristic values).

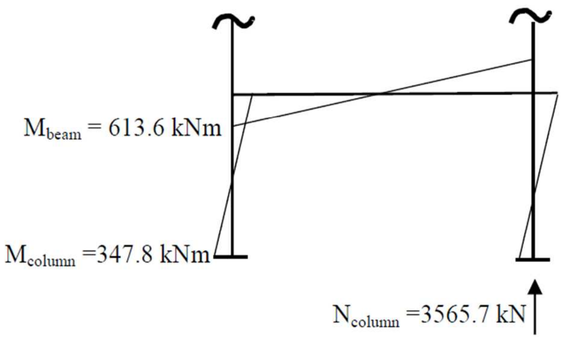

Figure 3. Relevant results of the elastic analysis

The reinforcement check of beams and columns for the maximum design forces is done using the spreadsheets developed by Santos [7]. The corresponding interaction curves are shown in Figures 4 (first floor beam) and 5 (column). The forces are increased by the partial factor γ_f = 1.4.

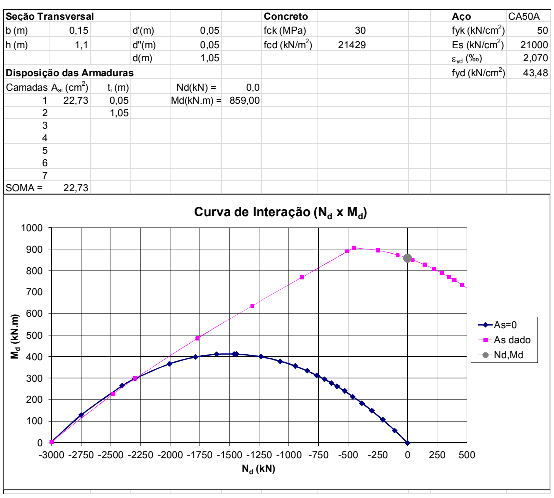

Figure 4. Interaction curve for the beam design

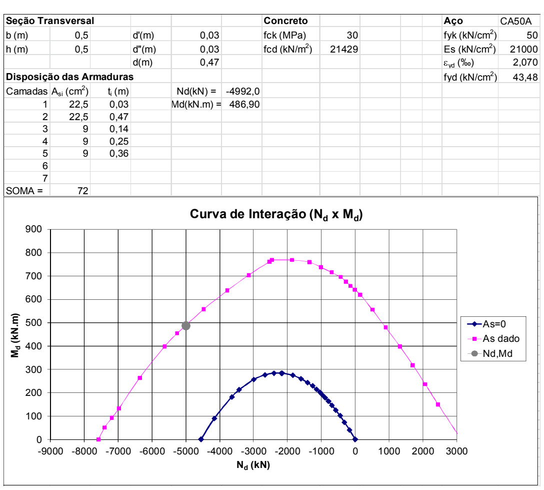

Figure 5. Interaction curve for the column design

4. Deterministic analysis – Ultimate Limit State

For the Global Analysis it is necessary initially to obtain the interaction curves corresponding to the resistance of beams and columns, considering the average values of the materials resistance, according to item 4.6.2.1 of *fib Model Code* 2010 [1]. Then, these curves for the previously analysed sections are drawn, for evaluating the forces resisted by then in this condition.

For obtaining the average values of the resistances, it is necessary to take into account that the definition of the characteristic values of the resistance of the materials considers the quantile of 5%.

Considering for the resistance of concrete and steel the coefficients of variation (COV = standard deviation/average value) respectively equal to 0.15 e a 0.05, the relationships between average values and characteristic values (*bias factors*) result to be 1.328 e 1.089. Then, the average values to be adopted are:

Concrete: f_{cm} = 1.328 . 30000 = 39840 kPa

Steel: f_{ym} = 1.089 . 500000 = 544500 kPa

For the first floor beam, using the interaction curve shown in Figure 6, drawn with the average values of the resistances, the obtained maximum resistant moment is 1151 kNm. This procedure is repeated for the beams of all the building floors.

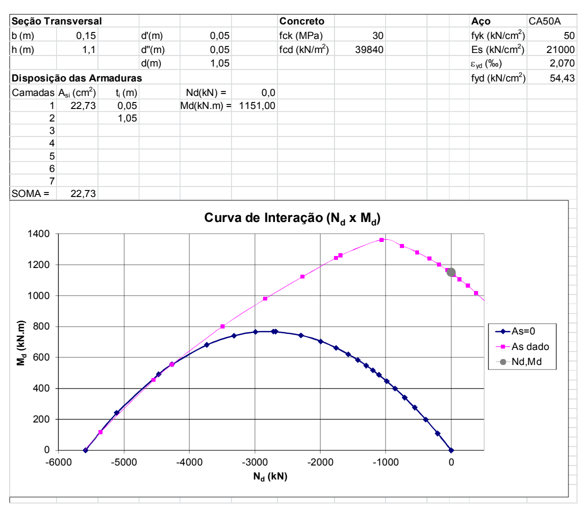

Figure 6. Interaction curve for the beams with average values for the resistances

Similarly, for the column, the interaction curve shown in Figure 7 is drawn, with the average values of the resistances. The maximum resistant moment is 1016 kNm, obtained after an interactive process, in which the final value of the maximum compressive normal force acting in the column is 5114 kN.

Seção Transversal					**Concreto**		**Aço**	CA50A
b (m)	0,5		d'(m)	0,03	fck (MPa)	30	fyk (kN/cm^2)	50
h (m)	0,5		d"(m)	0,03	fcd (kN/m^2)	39840	Es (kN/cm^2)	21000
			d(m)	0,47			ε_{yd} (‰)	2,070
Disposição das Armaduras							fyd (kN/cm^2)	54,43
Camadas	A_{si} (cm^2)	t_i (m)	Nd(kN) =	-5114,0				
1	22,5	0,03	Md(kN.m) =	1016,00				
2	22,5	0,47						
3	9	0,14						
4	9	0,25						
5	9	0,36						
6								
7								
SOMA =	72							

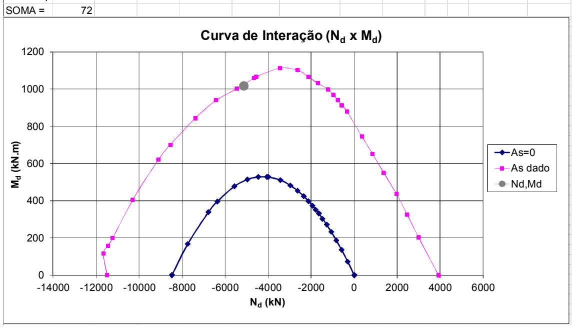

Figure 7. Interaction curve for the column with average values for the resistances

The formation of the plastic hinges in the beams is considered. Only after the formation of the last beam hinge, the hinges in the columns appears.

In this situation, the loading scheme that is imagined consists in to maintain the values of the dead loads in its average value and to increase progressively the horizontal wind loads multiplying them by a factor λ.

In the Global Resistance Analysis it is necessary initially to determine the average values of loads, according to item 4.6.2.1 of *fib Model Code* 2010.

For obtaining these average values, the relationships between average and characteristic values (*bias factors*) are taken as 1.05 for the dead loads and 1.187 for wind loads. This later value corresponds to a reference period of 50 years and to a coefficient of variation of 0.35.

In order to maintain the symmetry of the analysis and for facilitate the analysis of the formation of the plastic hinges, dead loads are considered as point loads acting in the nodes.

Therefore, the average values of the point loads to be applied in each node are:

Dead loads: D_m = 1.05 . 8 . 6 . 5 = 252 kN

Wind Loads: W_m = 1.187 . 9 = 10.68 kN

The analysis model is defined in order that as long as the horizontal force is being increased, plastic hinges progressively appears in the beams, as long as the maximum resistant moment is attained in them. The process stops when a final plastic hinge appears in the column.

Relevant final results obtained in the Global Resistance Analysis are shown in Figure 8.

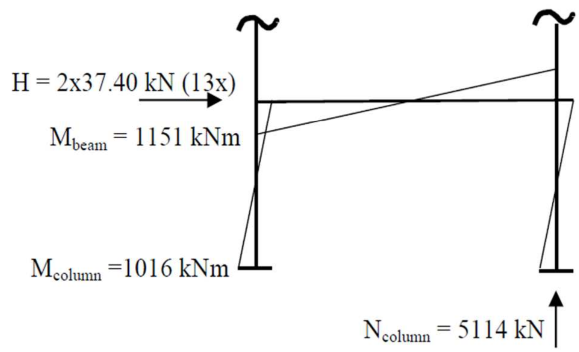

Figure 8. Relevant results of the global analysis

The global safety factor (λ) is obtained as:

$$\lambda = \frac{37.40}{10.68} = 3.50$$

It is to be observed that this relatively high value of the safety factor indicates that, considering the concepts of the Global Analysis, the design of the building can be optimized, allowing for a reduction in the cost of the structure.

5. Probabilistic analysis for the Ultimate Limit State - Beams

In the Ultimate Limit State, the probabilistic safety evaluation is performed for each section of the structure. Herein, the safety is evaluated in the critical sections in beams and columns.

For the beams, in pure bending, the sectional equilibrium is considered with an equivalent rectangular block of stresses in the concrete, following Brazilian standard, as shown in Figure 9.

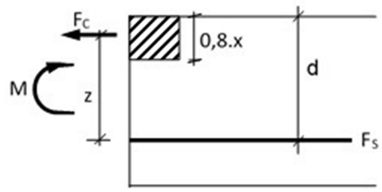

Figure 9. Equilibrium in the concrete section

Considering the equilibrium:

$$F_S = A_S . f_y \quad (1)$$

$$F_C = 0.85 . f_c . b . 0.8x \quad (2)$$

$$z = d - 0.4x \quad (3)$$

Then, with $F_S = F_C$:

$$M = A_S . f_y . z = A_S . f_y . (d - 0.4x) \quad (4)$$

$$M = A_S . f_y \left(d - \frac{0.5}{0.85} . \frac{A_S . f_y}{f_c . b} \right) \quad (5)$$

For the definition of the probabilistic variables, the values summarized in Table 1 are considered.

Table 1. Probabilistic characteristics for the reliability analyses

Variable	Distribution	Bias Factor	COV or σ
Dead load	Normal	1.05	0.05
Wind	Gumbel	1.19	0.35
Concrete resistance	Normal	1.328	0.15
Steel resistance	Normal	1.089	0.05
Steel area	Normal	1.00	0.015
Resistance modelling	Normal	1.00	0.05
Loading modelling	Normal	1.00	0.10
Sectional dimensions	Normal	1.00	4mm+0.006L ≤ 10mm

Variables related to uncertainties in the modelling of resistance and loads are accordingly considered in the reliability analysis, as shown in Table 1. For the sake of the simplicity they are not explicitly included the several reliability equations to be presented in the sequel.

The limit state function for bending moment failure is given by:

$$F_{lim} = A_S . f_y . \left(h - cob - 0,588 . A_S . \frac{f_y}{b . f_c} \right) - W \quad (6)$$

In this expression the variables still not defined are *b* (section width), *h* (section height), *cob* (distance between the reinforcement and the face of the section) and *W* (bending moment in the beam caused by wind action).

With basis in the values given in Table 1, the variables considered in the reliability analysis of the first floor beam are shown in Table 2.

Table 2. Probabilistic characteristics for the reliability analyses of the beam

Variable	Distribution	Average (μ)	Standard deviation (σ)
h (m)	Normal	1.10	0.01
b (m)	Normal	0.15	0.0049
cob (m)	Normal	0.05	0.005
A_s *(cm²)*	Normal	22.73	0.341
fc (kN/m²)	Normal	39840	5976
fy (kN/cm²)	Normal	54.45	2.7225
W (kNm)	Gumbel	728.3	254.91

The reliability analysis is done using the computer program VAP [6], applying the FORM method, with the input data shown in Figure 10.

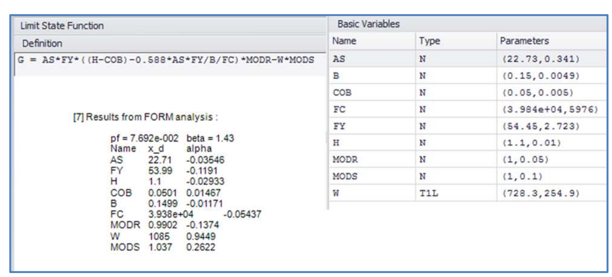

Figure 10. Probabilistic analysis of the beam

Main results of the analysis are:

Reliability index: $\beta = 1.43$
Probability of failure: $p_f = 7{,}692 \times 10^{-2}$

This very low value obtained for the reliability index β is compatible with the situation of a beam subjected only to the variable wind load, which presents a high coefficient of variation (*COV* = 0.35).

6. Probabilistic analysis for the Ultimate Limit State - Columns

For the columns, initially the variables, mechanical reinforcement ratio ω, reduced normal force η and reduced bending moment μ are defined:

$$\omega = \frac{A_S . f_y}{b.h.f_c} ;\ \eta = \frac{N}{b.h.f_c} ;\ \ \mu = \frac{M}{b.h^2 f_c} \qquad (7)$$

It is supposed that, around the design point, the variables ω, η and μ could by related through the linear relationship, given by Equation (8).

$$\omega = A + B\mu + C\eta \qquad (8)$$

This relationship is graphically shown in Figure 11, drawn on an adimensional chart for the design for eccentric compression presented in [7].

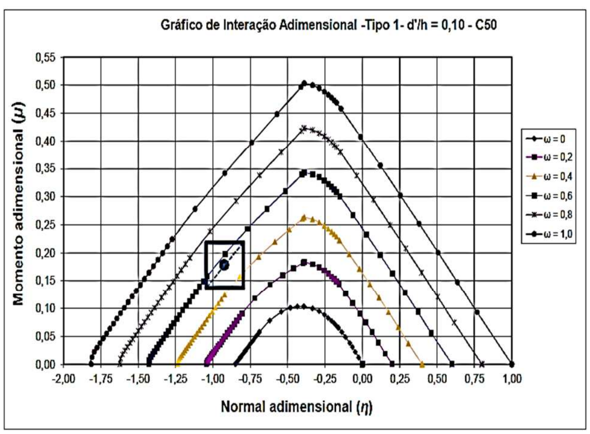

Figure 11. Linear relationship between adimensional variables

For the analysed column the following values are obtained:

A = -0.6129 ; B = 2.8802 ; C = 0.7268

(compression with positive value)

After some adequate substitutions, the limit state equation (9) is obtained:

$$F_{lim} = A_s \cdot f_y + 0.6129 \cdot b \cdot h \cdot f_c - \frac{2.8802 \cdot M}{h} - 0.7268 \cdot N \qquad (9)$$

With basis in the basic values given in Table 1, the probabilistic variables to be considered in the reliability analysis of the column are defined, as shown in Table 3.

Table 3. Probabilistic characteristics for the reliability analyses of the column

Variable	Distribution	Average (μ)	Standard deviation (σ)
b (m) = *h* (m)	Normal	0.50	0.007
M (kN.m)	Gumbel	412.9	144.5
N (kN)	Normal	3744.0	187.2
A_s (cm^2)	Normal	72.0	1.08
f_c (kN/m^2)	Normal	39840	5976
f_y (kN/cm^2)	Normal	54,45	2,72

The reliability analysis is done using the computer program VAP [6], applying the FORM method, with the input data shown in Figure 12.

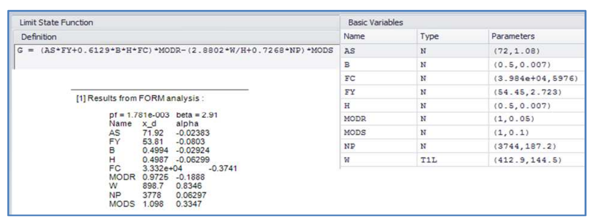

Figure 12. Probabilistic analysis of the column

Main results of the analysis are:

Reliability index: β = 2.91
Probability of failure: $p_f = 1.781 \times 10^{-3}$

This relatively low value obtained for the reliability index β can be explained, since the column is subjected to the simultaneous action of dead and variable loads, the later one with a high coefficient of variation and representing a relevant part of the total load (around 47%).

7. Probabilistic analysis for the Global Resistance Approach

For the reliability analysis in the collapse situation corresponding the Global Resistance Approach, it is initially necessary to relate, using equilibrium equations, the vertical and horizontal forces acting in this situation, the maximum bending moments resisted by the beams and normal forces and bending moments acting in the base section of the critical column.

In the analysis presented in the sequel, the reliability index β related to the factor λ equal to 3.5, corresponding to the conventional design in the Ultimate Limit State is determined.

The bending moment and normal force M_{column} N_{column} acting in the column base can be written as:

$$M_{column} = 273 \times F_H - \sum M_{beams} \tag{10}$$

$$N_{column} = \frac{(2 \times 273 \cdot F_H - 2 \times M_{column})}{10} + 13 \cdot F_V \tag{11}$$

In these equations, F_H and F_V are, respectively, the numerical values of the horizontal and vertical forces acting in each node of the frame.

After some adequate substitutions, the limit state function (12) is obtained:

$$F_{lim} = A_s \cdot f_y + 0.6129 \cdot b \cdot h \cdot f_c - \frac{2.8802 \cdot (273 \cdot F_H - \sum M_{beams})}{h} - 0.7268 \times \left(13 \cdot F_V + 0.2 \sum M_{beams}\right) \tag{12}$$

With basis in the basic values given in Table 1, the probabilistic variables to be considered in the reliability analysis of the column are defined as shown in Table 4. The considered values for the average value and standard deviation for the bending moments in the total thirteen beams are found applying the equations (13):

$$\mu = \sum \mu_{beams}; \sigma = \sqrt{\sum \sigma^2_{beams}} \tag{13}$$

Table 4. Probabilistic characteristics for the reliability analyses, Global Safety

Variable	Distribution	Average (μ)	Standard deviation (σ)
b (m) = h (m)	Normal	0.50	0.007
M_{beams} (kN.m)	Gumbel	9193	267.1
F_V (kN)	Normal	252.0	12.6
F_H (kN)	Normal	10.68	3.74
A_s (cm^2)	Normal	72.0	1.08
f_c (kN/m^2)	Normal	39840	5976
f_y (kN/cm^2)	Normal	54,45	2,72

The reliability analysis is done using the computer program VAP [6], applying the FORM method, with the input data shown in Figure 13.

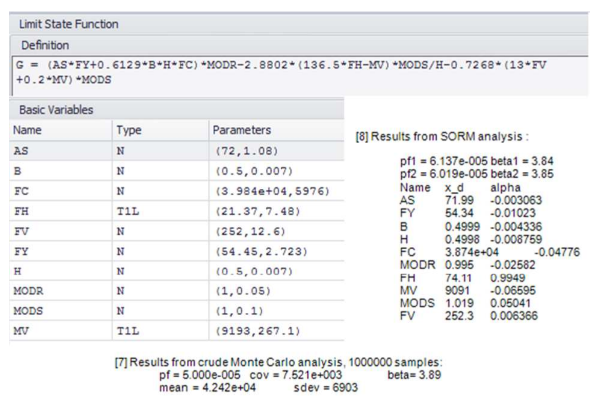

Figure 13. Probabilistic analysis, Global

Main results of the analysis are:

Reliability index: $\beta = 3.84$
Probability of failure: $p_f = 6.137 \times 10^{-5}$

The value obtained for the reliability index β, for the structure designed according the criteria of the Ultimate Limit State is superior to the usual limit $\beta = 3.8$ for the reference period of 50 years.

This means that, following the Global Resistance Approach, the design can be further optimized, leading to a more economical solution.

This can be seen graphically in Figure 14, where an evaluation of the relationship between the reliability index β with the parameter λ, global safety factor, evaluated using the average value of the involved variables, is shown.

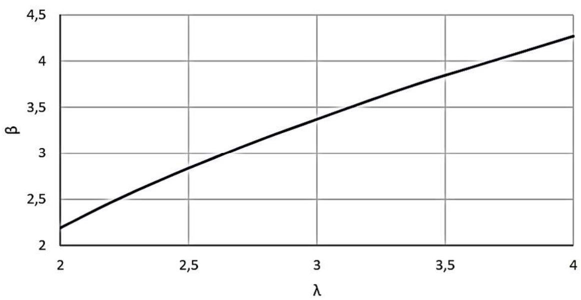

Figure 14. Relationship λ x β

8. Conclusions

From the results of the reliability analyses herein presented, it can be concluded that the application of the Global Resistance Approach can lead to more economical structures.

Table 5 summarizes the results obtained in the several analyses. The global safety factor λ corresponding to the usual design in Ultimate Limit States could be reduced without jeopardizing the global safety.

It can be noticed that usual reliability analysis in isolated sections can lead to distorted results, since the structural systems behave as a whole.

Table 5. Results of the several performed reliability analyses

Analysis	$\boldsymbol{\beta}$
Beam	1.43
Column	2.91
Global, λ = 3.50	3.85
Global, λ = 3.00	3.37

For the effective application of the herein exposed concepts more studies are necessary.

A normative definition for the maximum value for the parameter λ to be considered in the design is still necessary.

This global safety factor shall have higher values in situations in which fragile collapse can occur and lower in situations of ductile rupture.

9. References

[1] *fib* - International Federation for Structural Concrete. *Model Code for Concrete Structures 2010*; 2013.

[2] Cervenka, V. Reliability-based non-linear analysis according to *fib* Model Code 2010. *Structural Concrete*, 14, No. 1; 2013.

[3] Melchers, R. E.; Beck A. T. *Structural reliability analysis and prediction*. Third edition. Brisbane: John Wiley & Sons Ltd; 2018.

[4] Interlandi, C.; Martha, L. F.; Santos, S. H. C. Confiabilidade em Estruturas de Concreto Armado: Estudo Comparativo entre Enfoques de Estado Limite Último e de Segurança Global. *Proceedings of the X Congresso Brasileiro de Pontes e Estruturas*, Rio de Janeiro (in Portuguese); 2018.

[5] Associação Brasileira de Normas Técnicas. *ABNT NBR 6118. Projeto de Estruturas de Concreto – Procedimento* (in Portuguese); 2014.

[6] Petschacher Software und Projektentwicklungs GmbH - PSP Software. VAP – Variables Processor , Version 4.0; 2017.

[7] Santos, S. H. C. *Fundamentos de Concreto Armado II*, Escola Politécnica da Universidade Federal do Rio de Janeiro; 2018