

3D Geological Restoration using a Finite Element Approach

Márcio Rodrigues de Santi, João Luiz Elias Campos and Luiz Fernando Martha
Tecgraf/PUC-Rio – Brazil

Abstract

This work presents a 3D finite element approach for the restoration of geological models. This is a natural extension of a previous work presented by de Santi et al. (2002), which addressed the 2D problem. As in the 2D problem, the use of physical modeling and numerical simulation improves the restoration results by taking into account physical properties of the rock materials and by automating the geological restoration process. A finite element gOcad plug-in (Martha, Campos & Cavalcante Neto, 2001) is used to create a finite element model: a tetrahedral mesh suitable for finite element simulation, material properties, and boundary conditions representing some displacement restrictions and loading that must be applied to simulate the events needed during the restoration process. The analysis program uses a dynamic-relaxation solution algorithm to solve the finite element equation system that represents the geological model. Some modifications were made in the original analysis program to add 3D capabilities: tetrahedral finite elements and a special boundary condition to simulate normal displacement restriction that represents the movement of a rock block over a fault. An example is presented to illustrate the proposed approach in the geological restoration process.

1 Introduction

The geological restoration process is one of the existing techniques for obtaining an accurate and consistent interpreted geological section from seismic or well data. Its main purpose is to validate the structural interpretation of a geological formation and its geometry, and it may define a valid or an admissible model. The importance of such technique in the petroleum industry is to reduce the risks of exploration by validating the structural model.

Traditionally, the reconstruction process is performed in two dimensions using geological cross-sections. In general, the cross-section restoration is an interactive technique that depends largely on human interpretation. In this process, the user (usually a geologist or geophysicist) decides, step by step, on the validity of each operation that composes the whole process. Therefore, this technique is typically a trial-and-error process.

Geological section restoration techniques can be subdivided into classic and non-classic techniques (Tearpock & Bischke, 1991). Classic restorations are empiric and based on geological premises to simulate deformations on the Earth's crust. They use geometric principles as modeling tools for the geological premises (Goguel, 1962; Dahlstron, 1969). On the other hand, non-classic techniques, even though still basically geometric, consider measured deformations to simulate the phenomena involved in the formation of geological structures (Suppe et al., 1983, 1985; Tearpock & Bischke, 1991). In addition, they use numerical methods for automation purposes. However, in both approaches, these techniques are basically empiric and purely geometric.

Recently, the authors proposed a 2D algorithm (Santi, Campos & Martha, 2002) for the deformation of a geological block whose main objective was to incorporate physical modeling to the cross-section restoration problem. The basic idea was to introduce Continuum Mechanics concepts into the geological restoration process in order to consider physical properties of the material during the simulation of the movement of a block along a fault. This may be considered a simplified first step because a linear-elastic material behavior is considered and because there are other typical operations commonly used in geological restoration, such as section decompression, that have not been considered.

Recently, three-dimensional geological restoration algorithms and tools have been developed, which consider both the classic geometrical approach (e.g., Midland Valley–3Dmove) and the numerical procedure (e.g., Muron & Mallet, 2003). A straightforward extension of the 2D classic process, which is based on empirical geometrical transformations, in three dimensions is complex, since this approach requires the definition of some markup features in 3D space. The numerical simulation approach is simpler than the geometrical transformation approach because only source and destination geometric features need to be specified.

This work presents a 3D finite element approach for the restoration of geological models. This is a natural extension of the previous 2D work presented by the authors (2002). As in the 2D problem, the use of physical modeling and numerical simulation improves the restoration results by taking into account physical properties of the rock materials and by automating the geological restoration process. The analysis program uses a dynamic-relaxation solution algorithm to solve the finite element equation system that represents the geological model. Some modifications were made in the original analysis program to add 3D capabilities: tetrahedral finite elements and a special boundary condition to simulate normal displacement restriction that represents the movement of a rock block over a fault.

In this work, a finite element gOcad plug-in (Martha, Campos & Cavalcante Neto, 2001) is used to create a finite element model: a tetrahedral mesh suitable for finite element simulation, material properties, and boundary conditions representing some displacement restrictions and loading that must be applied to simulate the events needed during the restoration process. The gFEM plug-in implements an algorithm for generating unstructured tetrahedral finite element meshes in arbitrarily shaped three-dimensional regions. This meshing algorithm (Cavalcante Neto et al., 2001) incorporates aspects of well-known meshing procedures, but includes some original steps. It uses an advancing front technique that generates good shape quality elements, along with an octree to develop local guidelines for the size of generated elements. The advancing front technique is based on a standard procedure found in the literature with two additional steps to ensure valid volume mesh generation for virtually any domain. The first additional step is related to the generation of elements only considering the topology of the current front and the second additional step is a backtracking procedure with face deletion, to ensure that a mesh can be generated even when problems happen during the advance of the front. To improve mesh quality (as far as element shape is concerned), an a posteriori local mesh improvement procedure is used. A description of the object classes introduced in gOcad data structure for the implementation of the finite element meshing and result visualization capabilities is available in a previous article (Martha, Campos & Cavalcante Neto, 2001).

2 The Restoration Process

The geological restoration is a process to geometrically validate an interpretation of a geological cross section by a geologist or geophysicist (Dahlstron, 1969). It consists of an attempt to rebuild the original geometry of the layers of a section before the deformations suffered along time. As very little knowledge is available on the deformation mechanisms occurred in the past, the restoration is based on geological premises that can be simulated using geometrical principles (Ferraz, 1993).

In the classic restoration approach, these premises are usually based on quite simple theories that reflect the high degree of uncertainty relative to the tectonic processes that took place in the formation of the geological structures. One of these premises is the “volume conservation law”, which establishes that the geological features are restored to a pre-unstrained state without volume loss of the geological material, so that the disposition of strata and the length and thickness of each layer maintain a coherent picture (Dahlstron, 1969).

Geological cross-section restoration is an iterative process, represented by several steps. Using a structural interpretation based on well or seismic data, a geological section is obtained that undergoes several stages of restoration until a valid section is reached. If a restoration is not possible, it means that a new interpretation should be performed. Figure 1 summarizes the iterative restoration process, from field data to a possible validation of a geological section.

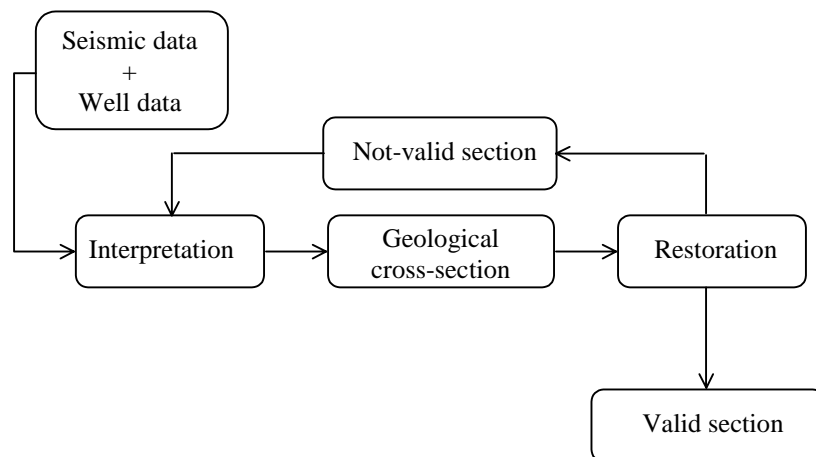


Figure 1 – Geological section restoration diagram.

3 The Finite Element Approach

The non-classic restoration techniques generally use numerical methods that take into account strain measures for a better simulation of the geological mechanisms. An approach based on the discretization of a continuous deformation field into finite-size strains was originally developed by Etchecopar (1974), who used least-square best-fitting of discrete translations, rotations and internal slips to study the deformation of a crystalline aggregate. Since that original work, there have been several approaches using similar techniques.

Finite element techniques for the restoration problem were first developed for regions of ductile deformation (Schwerdtner, 1977; Cobbold, 1977, 1979). The basic idea was to use least-square minimization of spurious gaps and overlaps. The purpose of this method was to unstrain finite regions in which the strain is known and then used least-square fitting of discrete translations and rotations to reconstruct the initial state.

In 1993, Rouby and collaborators (Rouby et al., 1993) described a new least-square method for the restoration of faulted sedimentary horizons in regions of dominantly extensional tectonics. Such technique was later adapted for compressional tectonics regions (Borgeous et al., 1997).

It may be observed that the evolution of geological restoration techniques tends to abandon the uncertainties of the traditional methodologies and moves in a direction that takes into account the physical phenomena involved in the formation of the geological structures. The first step was the introduction of deformation measures as results and sometimes as input data for the restoration process. The consideration of deformation measures in the process is certainly a gain, since it allows a more accurate restoration. The determination of deformation magnitudes and directions in geological blocks helps predicting the intensity, orientation, and time span of faults (Erickson et. al., 2000).

Several methods found in the literature use discretization techniques and take into account strain measures, but no method was found that considers the physical properties of the rock blocks in the restoration process. In other words, none of the existing restoration techniques simulates the problem as a Continuum Mechanics deformation process. The referred non-classic methodologies, despite considering deformation as part of the process, still represent basically geometric tools for the restoration.

To treat the problem in a Continuum Mechanics approach, it is necessary to add other parameters to the process, such as constitutive relations of the deformed material. This implies the definition of physical properties such as Young's module and Poisson's ratio (Timoshenko and Goodier, 1970)

This work extends a new approach for the restoration process that takes into account these mechanical material behavior properties. This approach was initially implemented in two dimensions (Santi, Campos & Martha, 2002), and is extended in this work for 3D. One of the purposes of this approach is to study deformations in the restoration process using Continuum Mechanics principles, in a non-empiric way, to obtain the geometry of a geological block deformed by a movement over a fault.

The procedure used here for simulating the sliding of rock blocks on faults employs the Dynamic Relaxation Method (Underwood, 1983) coupled with the Finite Element Method (Zienkiewicz, 2000). The Dynamic Relaxation (DR) Method is an explicit iterative method for the static solution of structural mechanics problems. It is based on the fact that the static solution is the steady-state part of the transient response for a temporal load. The DR method is especially attractive for problems with highly nonlinear geometric and material behavior (Underwood, 1983). The fact of being explicit in time makes it computationally interesting because all quantities may be treated as vectors. The Finite Element (FE) Method is applied to obtain such static solution in each step (time interval). Therefore, the geological block that is being transformed is discretized in a FE mesh. In this work, a mesh of constant-strain tetrahedral elements is adopted.

Figure 2 presents, in a schematic way, the DR algorithm. In this figure, \mathbf{u} is the nodal displacement vector, \mathbf{F} is the nodal force vector, $\boldsymbol{\varepsilon}$ is the FE strain tensor, \mathbf{K} is the FE stiffness matrix, and \mathbf{m} is the node (fictitious) mass.

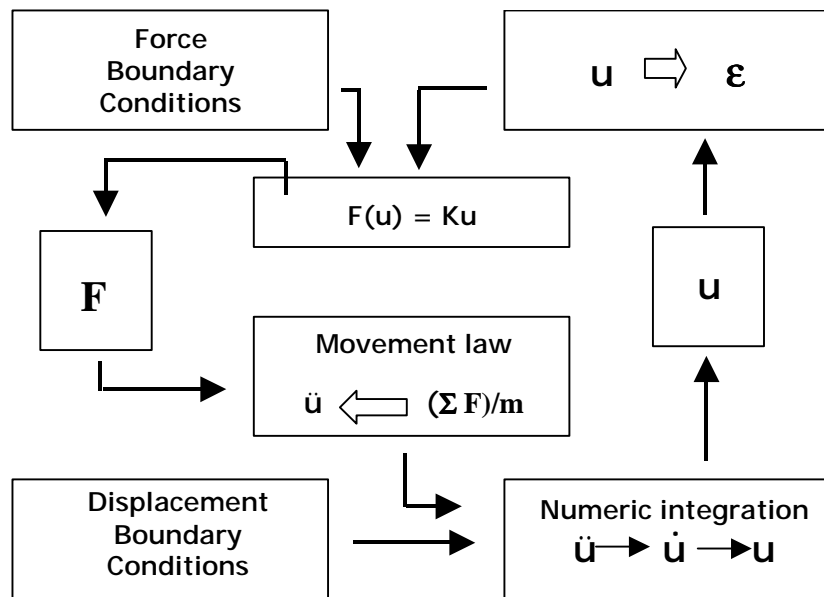


Figure 2 – Schematic diagram of the DR algorithm.

The DR algorithm evaluates in each step the unbalanced forces, that is, the equilibrium between external and internal forces is verified for each FE mesh node of the geological block that is being deformed. The convergence of the algorithm is associated with the minimization of the unbalanced forces. Therefore, in each step, the algorithm employs the equation that governs the movement law (second law of Newton) and the constitutive equations. The enforcement of the movement law is done sequentially, which facilitates the introduction of the mixed boundary conditions in terms of prescribed displacements and/or velocities and applied forces.

In each step, the unbalanced forces provoke movements in the mesh nodes. The resulting displacements are obtained by means of successive numeric integrations in time of accelerations and velocities. All nodes are moved according to the computed displacements and to their boundary restriction conditions. This causes deformations in each finite element, which results in finite element stresses defined by the material's constitutive relationship. From these stresses, the internal forces in mesh nodes are computed and, after properly discounted for the applied external forces, originate the new unbalanced forces, restarting the iteration (Figueiredo, 1991).

4 The Strategy

The idea of the proposed move-on-fault transformation algorithm is to define restrictions to the displacements of the FE mesh nodes in contact with the fault. The algorithm creates a local coordinate system for these nodes, defined according to a triangle along the fault surface in contact with the restricted node, as shown in Figure 3. Node displacement is free along the fault surface (directions u and v) and constrained in the normal direction (direction w). As the rock block moves along the fault, the node local system changes according to the contact triangle on the fault surface, as illustrated in Figure 4.

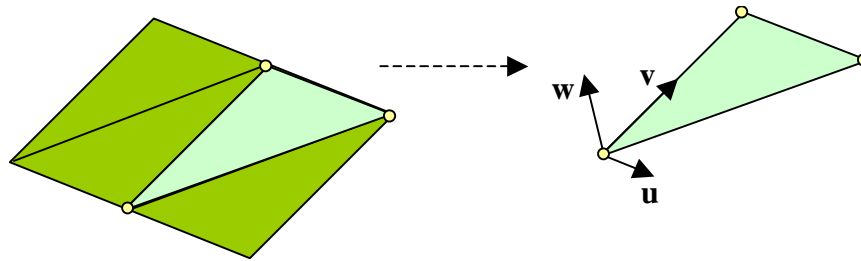


Figure 3 – Local coordinate system.

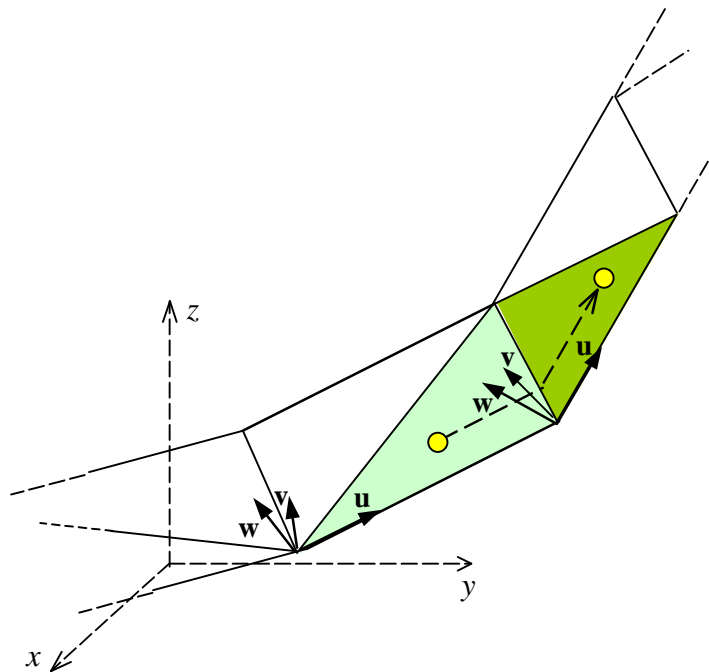


Figure 4 – Variable boundary conditions along the fault.

The constrained nodes always move along the geometry of the triangulated surface that defines the fault. Therefore, in the case of listric faults, the constrained normal direction might vary along the block's movement, because a node can move from one fault triangle to another. This means that, at the end of each step, the algorithm must update the local coordinate restriction system and adjust the position of the node accordingly, as shown in Figure 5.

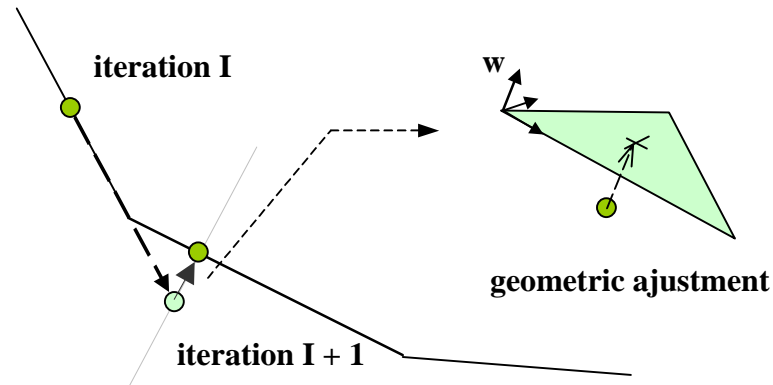


Figure 5 – Geometric adjustment of a node moving along a surface fault.

The transformation induced by the proposed move-on-fault algorithm is applied to a geological block by means of a prescribed displacement field. In the first DR iteration step, the prescribed displacements will result in the initial unbalanced nodal forces. The final block configuration will correspond to an equilibrium stage of nodal forces, as described in the previous section.

Two different approaches exist to apply the prescribed displacements to a geological block, as shown in Figure 6 (for simplicity a two-dimensional example is depicted). The prescribed displacement field may be applied to the upper horizon board of the block (Figure 6-a) or to a curve on the top of the block (Figure 6-b). The boundary of the block is divided into three board types. In one board, the boundary conditions correspond to the prescribed displacements. The second board represents the contact of the block with the fault. In this board, the mesh nodes are constrained to move only along the fault. In the remaining board, the nodes are free to move (no displacement boundary condition is applied).

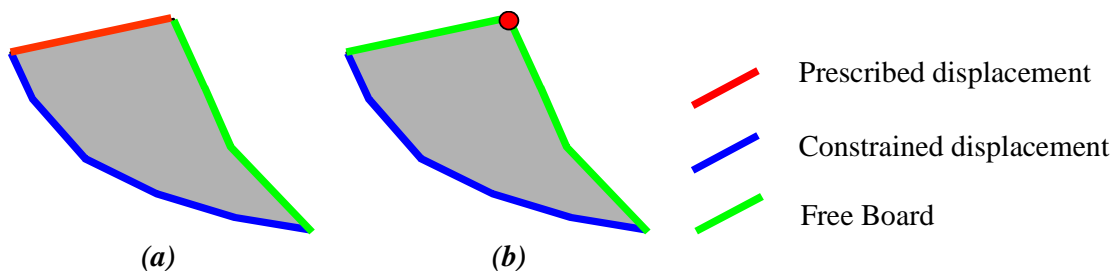


Figure 6 – Displacement boundary conditions of a block (2D example).

The prescribed displacement field is specified by a “destination” geometry of the prescribed displacement board. Figure 7 illustrates this for a prescribed displacement field applied to the upper horizon of the block.

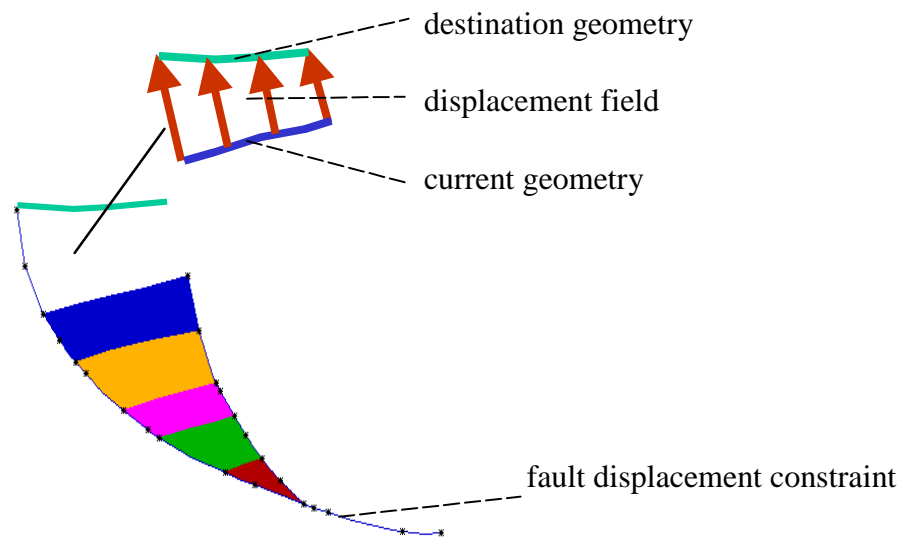


Figure 7 – Prescribed displacement field (2D example).

4 Example

One example illustrates the capabilities of the proposed 3D geological restoration process. Figures 8 shows the initial geometry of a block to be restored. The model was created in gOcad.

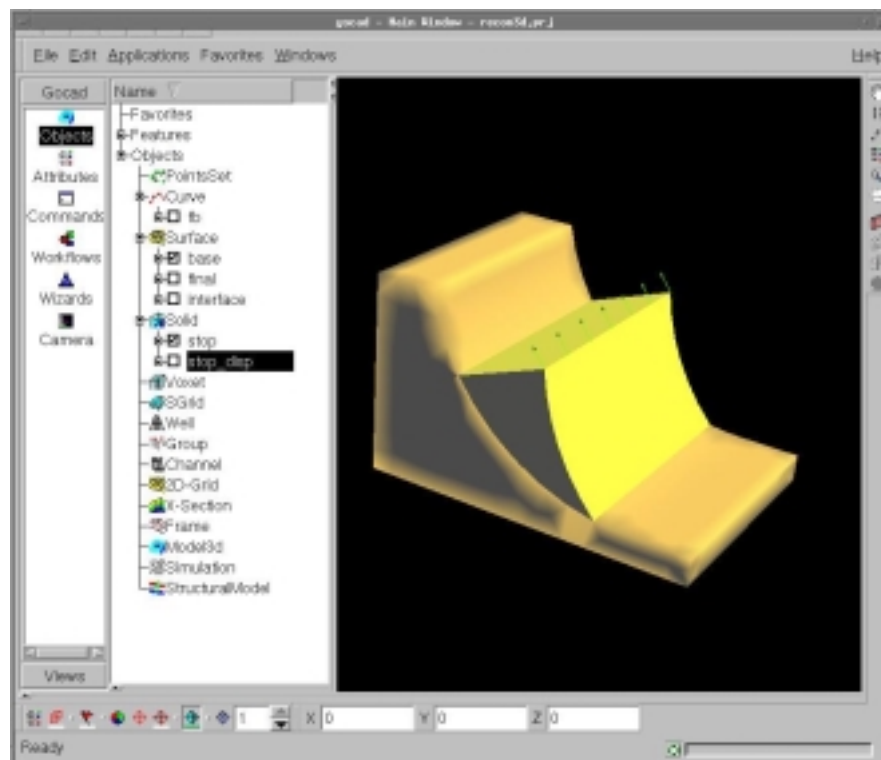


Figure 8 – Initial geometry of block to be restored.

Figure 9 shows the finite element mesh that was generated by the gFEM gOcad plug-in.

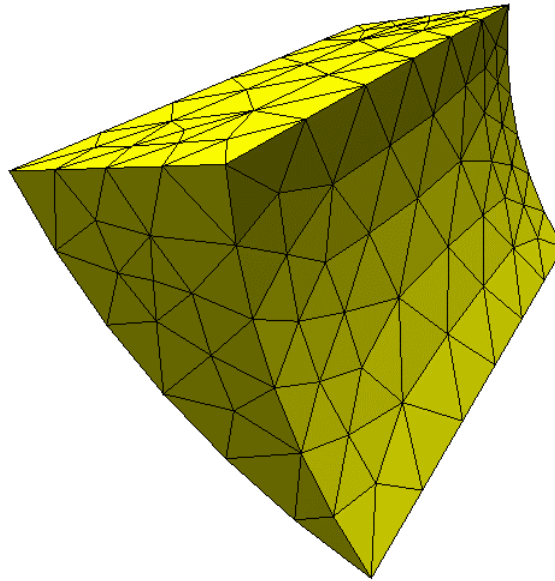


Figure 9 – Finite element mesh of block in the initial geometry.

Three steps of the DR iterative process are shown in Figures 10 to 12. The figures show contoured images of a principal strain component. The mesh at the initial configuration is also shown in the figures. Figure 10 illustrates the first step, in which only the nodes with prescribed displacements have moved. An intermediate step is depicted in Figure 11. The final step, after convergency of the iterative process, is shown in Figure 12.

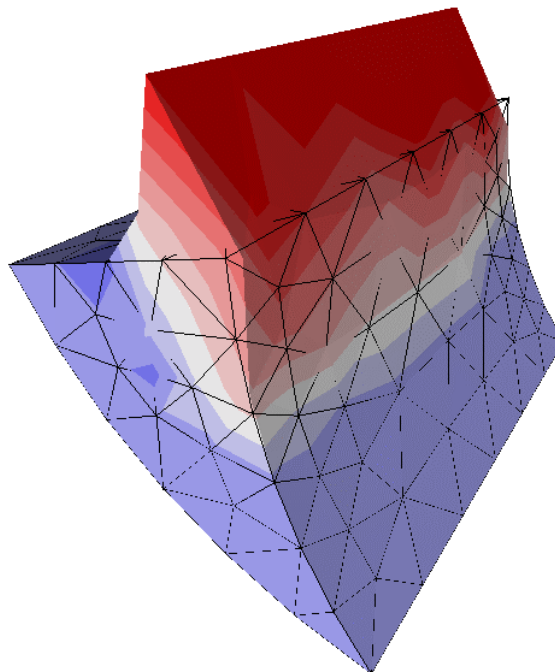


Figure 10 – Principal strain results for the first step of DR move-on-fault approach.

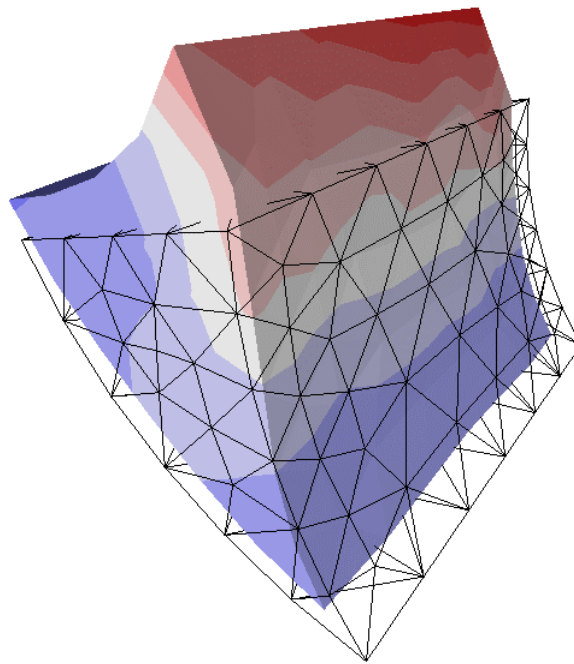


Figure 11 – Principal strain results for an intermediate step of DR move-on-fault approach.

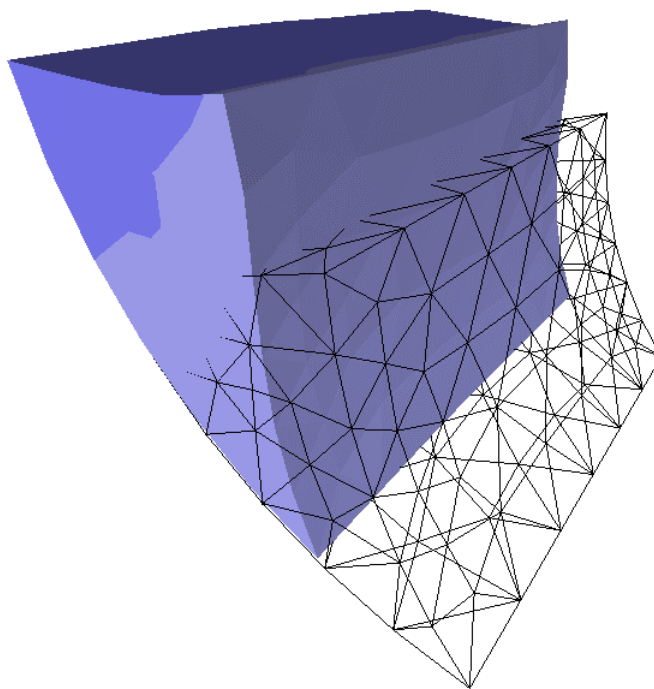


Figure 12 – Principal strain results for the final step of DR move-on-fault approach.

Finally, the restored geometry of the block is incorporated to the gOcad data structure, as shown in Figure 13.

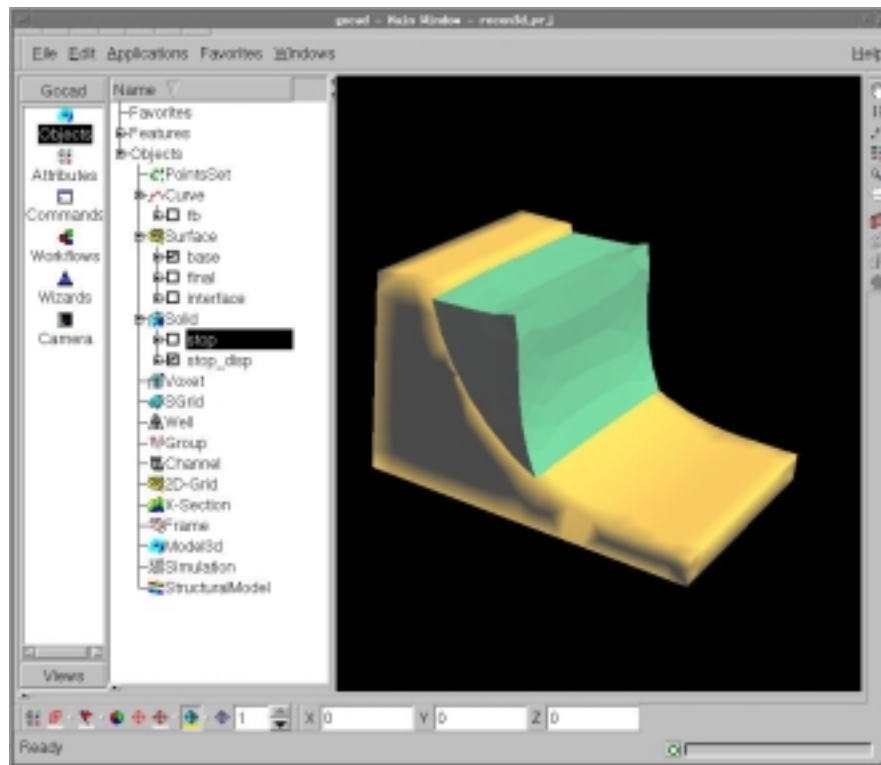


Figure 13 – Block transformed by the DR move-on-fault algorithm.

5 Conclusion

This work has extended a 2D numerical technique for the restoration of geological cross sections. A three-dimensional move-on-fault transformation algorithm, which is based on Continuum Mechanics principles, was incorporated to gOcad. This algorithm introduces physically based principles in the geological transformation algorithms. The authors believe that, in the restoration process, the application of a numerical method that considers the constituent relationships of the geological material in its formulation determines a new approach for the problem when compared to classic, empirically based, transformation algorithms.

The proposed procedure is considered a first step in the direction of a physically based geological restoration. Only a linear-elastic behavior of the material was taken into account. A more realistic elastic-plastic behavior may be considered in the future. The iterative Dynamic Relaxation algorithm is well suited to consider non-linear material behavior.

One important consequence of the use of a numerical technique is a greater degree of automation in the restoration process. This is particularly important in three-dimensional geological restoration.

Several additional extensions are still necessary in the current implementation. For example, multiple fault constraints need to be considered. This paper is just an initial description of the proposed 3D geological restoration approach, which has not been tested in real geological models yet.

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