A Finite Element Approach for Geological Section Reconstruction

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Abstract

This work presents a new approach for the restoration of geological cross-sections based on physical modeling and numerical simulation. Its main purpose is to introduce Continuum Mechanics concepts into the geological restoration process in order to consider physical properties of the materials during the simulation of the movement of a rock block along a fault. The adopted strategy uses a dynamic relaxation algorithm to solve the equation system that arises from the numerical simulation based on the Finite Element Method, together with some specific boundary conditions to represent the movement of the rock block over the fault.

1 Introduction

The geological cross-section reconstruction or restoration process is one of the existing techniques for obtaining an accurate and consistent interpreted geological section from seismic or well data. Its main purpose is to validate the structural interpretation of a cross section and its geometry, and it may define a valid or an admissible section. The importance of such technique in the petroleum industry is to reduce the risks of exploration by validating the structural model.

In general, cross-section restoration is an interactive technique that depends largely on human interpretation. In this process, the user (usually a geologist or geophysicist) decides, step by step, on the validity of each operation that composes the whole process. Therefore, this technique is typically a trial-and-error process.

Geological section restoration techniques can be subdivided into classic and non-classic techniques (Tearpock & Bischke, 1991). Classic restorations are empiric and based on geological premises to simulate deformations on the Earth's crust. They use geometric principles as modeling tools for the geological premises (Goguel, 1962; Dahlstron, 1969). On the other hand, non-classic techniques, even though still basically geometric, consider measured deformations to simulate the phenomena involved in the formation of geological structures (Suppe et al., 1983, 1985; Tearpock & Bischke, 1991). In addition, they use numerical methods for automation purposes. However, in both approaches, these techniques are basically empiric and purely geometric.

This work proposes an algorithm for the deformation of a geological block whose main objective is to incorporate, as a first step, physical modeling to the cross-section restoration problem. The basic idea is to introduce Continuum Mechanics concepts into the geological restoration process in order to consider physical properties of the material during the simulation of the movement of a block along a fault. This is a first step because a simplified

linear-elastic material behavior is considered and because there are other typical operations commonly used in geological restoration, such as section decompaction, that have not been considered.

2 The Restoration Process

The geological restoration is a process to geometrically validate an interpretation of a geological cross section by a geologist or geophysicist (Dahlstron, 1969). It consists of an attempt to rebuild the original geometry of the layers of a section before the deformations suffered along time. As very little knowledge is available on the deformation mechanisms occurred in the past, the restoration is based on geological premises that can be simulated using geometrical principles (Ferraz, 1993).

In the classic restoration approach, these premises are usually based on quite simple theories that reflect the high degree of uncertainty relative to the tectonic processes that took place in the formation of the geological structures. One of these premises is the "volume conservation law", which establishes that the geological features are restored to a pre-unstrained state without volume loss of the geological material, so that the disposition of strata and the length and thickness of each layer maintain a coherent picture (Dahlstron, 1969).

Geological cross-section restoration is an iterative process, represented by several steps. Using a structural interpretation based on well or seismic data, a geological section is obtained that undergoes several stages of restoration until a valid section is reached. If a restoration is not possible, it means that a new interpretation should be performed. Figure 1 summarizes the iterative restoration process, from field data to a possible validation of a geological section.
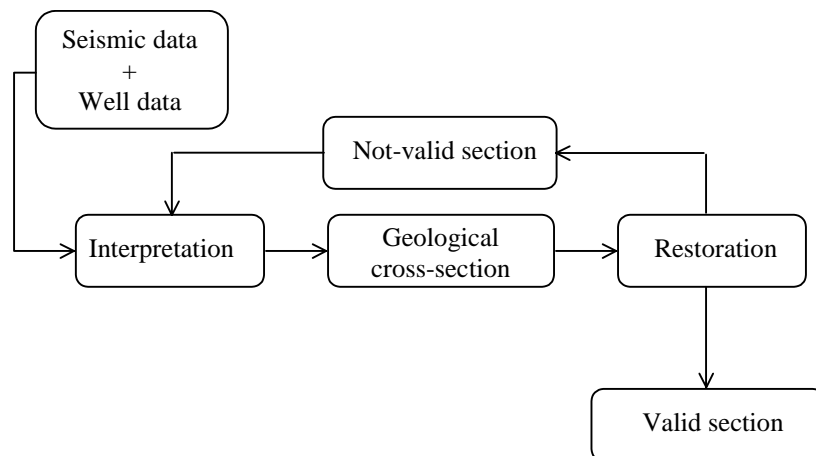

Figure 1 – Geological section restoration diagram.

3 The New Approach

The non-classic restoration techniques generally use numerical methods that take into account strain measures for a better simulation of the geological mechanisms. An approach based on the discretization of a continuous deformation field into finite-size strains was originally developed by Etchecopar (1974), who used least-square best-fitting of discrete translations, rotations and internal slips to study the deformation of a crystalline aggregate. Since that original work, there have been several approaches using similar techniques.

Finite element techniques for the restoration problem were first developed for regions of ductile deformation (Schwerdtner, 1977; Cobbold, 1977, 1979). The basic idea was to use least-square minimization of spurious gaps and overlaps. The purpose of this method was to unstrain finite regions in which the strain is known and then used least-square fitting of discrete translations and rotations to reconstruct the initial state.

In 1993, Rouby and collaborators (Rouby et al., 1993) described a new least-square method for the restoration of faulted sedimentary horizons in regions of dominantly extensional tectonics. Such technique was later adapted for compressional tectonics regions (Borgeous et al., 1997).

It may be observed that the evolution of geological restoration techniques tends to abandon the uncertainties of the traditional methodologies and moves in a direction that takes into account the physical phenomena involved in the formation of the geological structures. The first step was the introduction of deformation measures as results and sometimes as input data for the restoration process. The consideration of deformation measures in the process is certainly a gain, since it allows a more accurate restoration. The determination of deformation magnitudes and directions in geological blocks helps predicting the intensity, orientation, and time span of faults (Erickson et. al., 2000).

Several methods found in the literature use discretization techniques and take into account strain measures, but no method was found that considers the physical properties of the rock blocks in the restoration process. In other words, none of the existing restoration techniques simulates the problem as a Continuum Mechanics deformation process. The referred non-classic methodologies, despite considering deformation as part of the process, still represent basically geometric tools for the restoration.

To treat the problem in a Continuum Mechanics approach, it is necessary to add other parameters to the process, such as constitutive relations of the deformed material. This implies the definition of physical properties such as Young's module and Poisson's ratio (Timoshenko and Goodier, 1970)

This work proposes a new approach for the restoration process that takes into account these mechanical material behavior properties. In other words, one of the purposes of this work is to study deformations in the restoration process using Continuum Mechanics principles, in a

non-empiric way, to obtain the geometry of a geological block deformed by a movement over a fault.

The approach used here for simulating the sliding of rock blocks on faults employs the Dynamic Relaxation Method (Underwood, 1983) coupled with the Finite Element Method (Zienkiewicz, 2000). The Dynamic Relaxation (DR) Method is an explicit iterative method for the static solution of structural mechanics problems. It is based on the fact that the static solution is the steady-state part of the transient response for a temporal load. The DR method is especially attractive for problems with highly nonlinear geometric and material behavior (Underwood, 1983). The fact of being explicit in time makes it computationally interesting because all quantities may be treated as vectors. The Finite Element (FE) Method is applied to obtain such static solution in each step (time interval). Therefore, the geological section's block that is being transformed is discretized in a FE mesh. In this work, a mesh of constant-strain triangular elements is adopted.

Figure 2 presents, in a schematic way, the DR algorithm. In this figure, **u** is the nodal displacement vector, **F** is the nodal force vector, $\boldsymbol{\varepsilon}$ is the FE strain tensor, **K** is the FE stiffness matrix, and **m** is the node (fictitious) mass.

Figure 2 – Schematic diagram of the DR algorithm.

The DR algorithm evaluates in each step the unbalanced forces, that is, the equilibrium between external and internal forces is verified for each FE mesh node of the geological block that is being deformed. The convergence of the algorithm is associated with the minimization of the unbalanced forces. Therefore, in each step, the algorithm employs the equation that governs the movement law (second law of Newton) and the constitutive equations. The enforcement of the movement law is done sequentially, which facilitates the introduction of

the mixed boundary conditions in terms of prescribed displacements and/or velocities and applied forces.

In each step, the unbalanced forces provoke movements in the mesh nodes. The resulting displacements are obtained by means of successive numeric integrations in time of accelerations and velocities. All nodes are moved according to the computed displacements and to their boundary restriction conditions. This causes deformations in each finite element, which results in finite element stresses defined by the material's constitutive relationship. From these stresses, the internal forces in mesh nodes are computed and, after properly discounted for the applied external forces, originate the new unbalanced forces, restarting the iteration (Figueiredo, 1991).

4 The Strategy

The idea of the proposed move-on-fault transformation algorithm is to define restrictions to the displacements of the FE mesh nodes in contact with the fault. The algorithm creates a local coordinate system for these nodes, defined according to the fault segment's inclination in contact with the restricted node, as shown in Figure 3. Node displacement is free along the direction of the fault (direction 1) and constrained in the normal direction (direction 2). As the rock block moves along the fault, the node local system changes according to the fault's inclination, as illustrated in Figure 4.
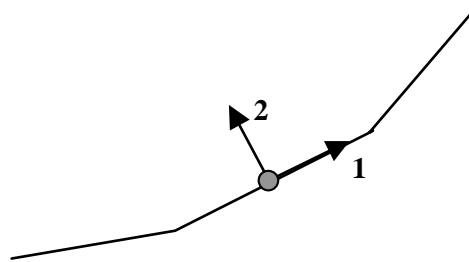
Figure 3 – Local coordinate system.
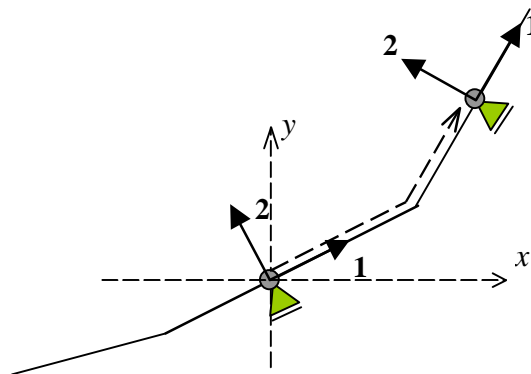
Figure 4 – Variable boundary conditions along the fault.

The constrained nodes always move along the geometry of the polygonal line that defines the fault. Therefore, in the case of listric faults, the constrained normal direction might vary along the block's movement, because a node can move from one fault segment to another. This means that, at the end of each step, the algorithm must update the local coordinate restriction system and adjust the position of the node accordingly, as shown in Figure 5.
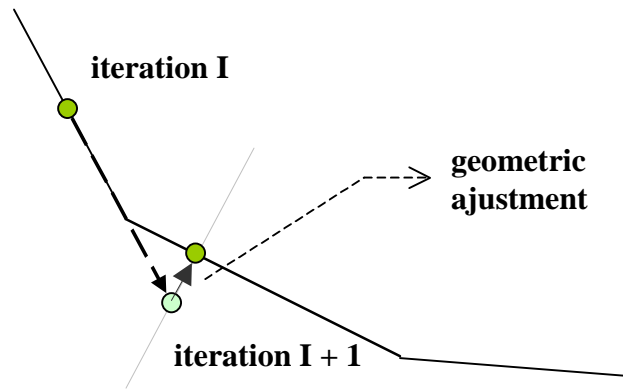

Figure 5 – Geometric adjustment of a node moving along a fault.

The transformation induced by the proposed move-on-fault algorithm is applied to a geological block by means of a prescribed displacement field. In the first DR iteration step, the prescribed displacements will result in the initial unbalanced nodal forces. The final block configuration will correspond to an equilibrium stage of nodal forces, as described in the previous section.

Two different approaches exist to apply the prescribed displacements to a geological block, as shown in Figure 6. The prescribed displacement field may be applied to the upper horizon board of the block (Figure 6a) or to the top node in contact with the fault (Figure 6b). The boundary of the block is divided into three board types. In one board, the boundary conditions correspond to the prescribed displacements. The second board represents the contact of the block with the fault. In this board, the mesh nodes are constrained to move only along the fault. In the remaining board, the nodes are free to move (no displacement boundary condition is applied).
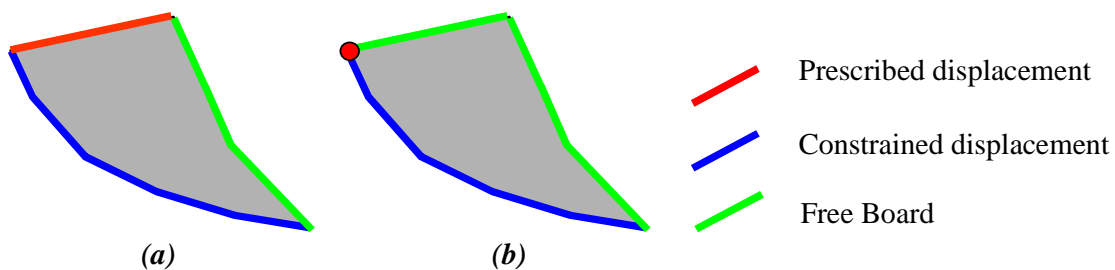

Figure 6 – Displacement boundary conditions of a block.

The prescribed displacement field is specified by a "destination" geometry of the prescribed displacement board. Figure 7 illustrates this for a prescribed displacement field applied to the upper horizon of the block.
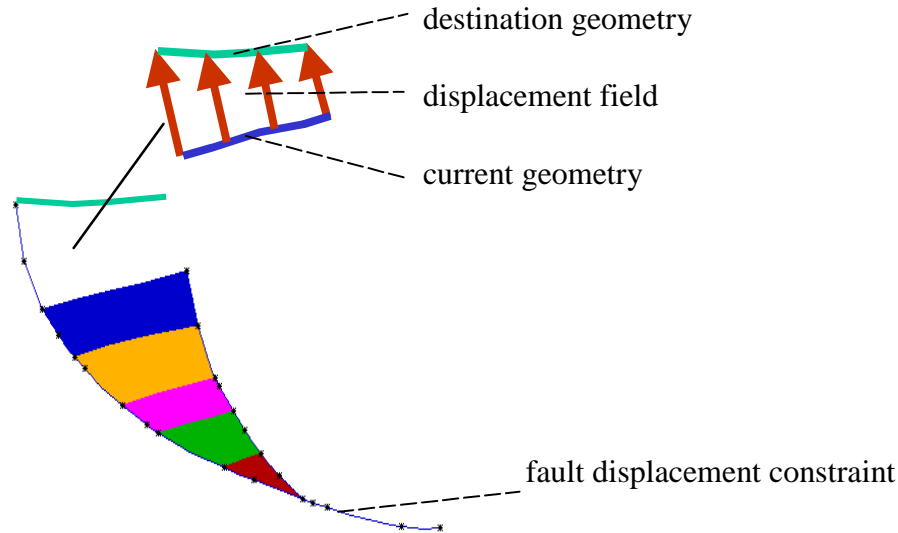

destination geometry

displacement field

current geometry

fault displacement constraint

Figure 7 – Prescribed displacement field.

4 Examples

The proposed algorithm has been implemented in an existing system for geological section reconstruction. This system, called Recon (Ferraz, 1993; Martha, 1994; Santi 2002), already contained a set of geometric transformations that simulate the geological premises in the context of a classic restoration process.

To implement the new move-on-fault transformation algorithm in Recon, it was necessary to integrate it to a FE analysis program that implements the DR algorithm (Figueiredo, 1991). In addition, a mesh generation procedure (Miranda et al., 1999), a module to manage the communication with the analysis program, and a module for post-processing strain results were incorporated to Recon.

To illustrate the new capabilities of Recon, Figures 8 to 12 show one step of the restoration process of a geological section. Figure 8 shows the initial configuration of a rock block to be transformed. Figure 9 illustrates the configuration of this block after being transformed by the DR move-on-fault algorithm. In this example, the prescribed displacement field was applied only to the top node in contact with the fault.
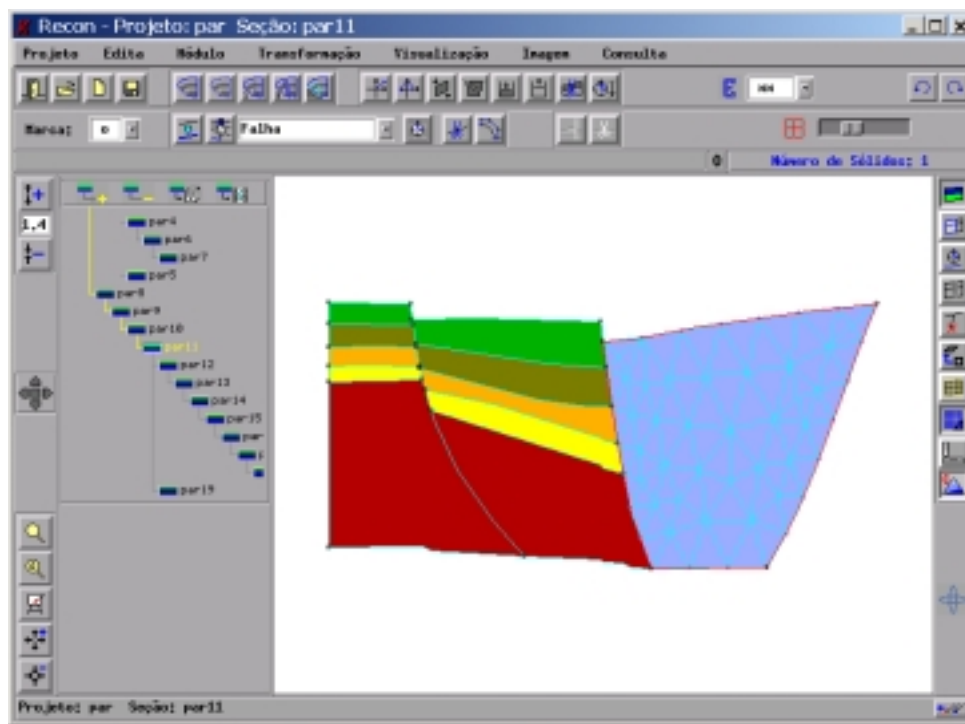
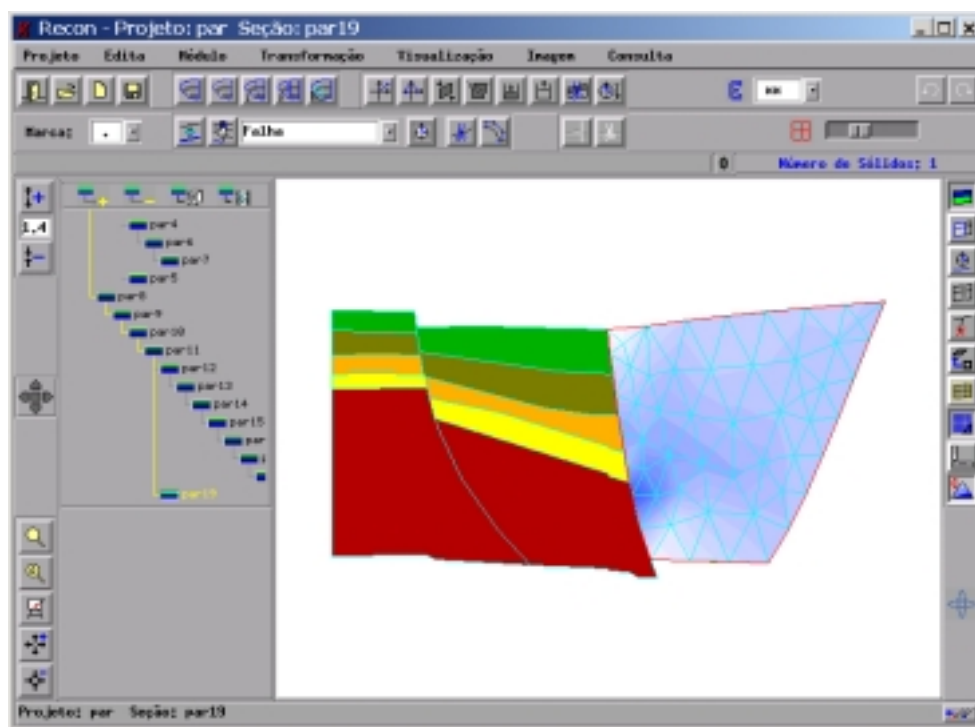**Figure 8 – Current configuration of a geological block.**

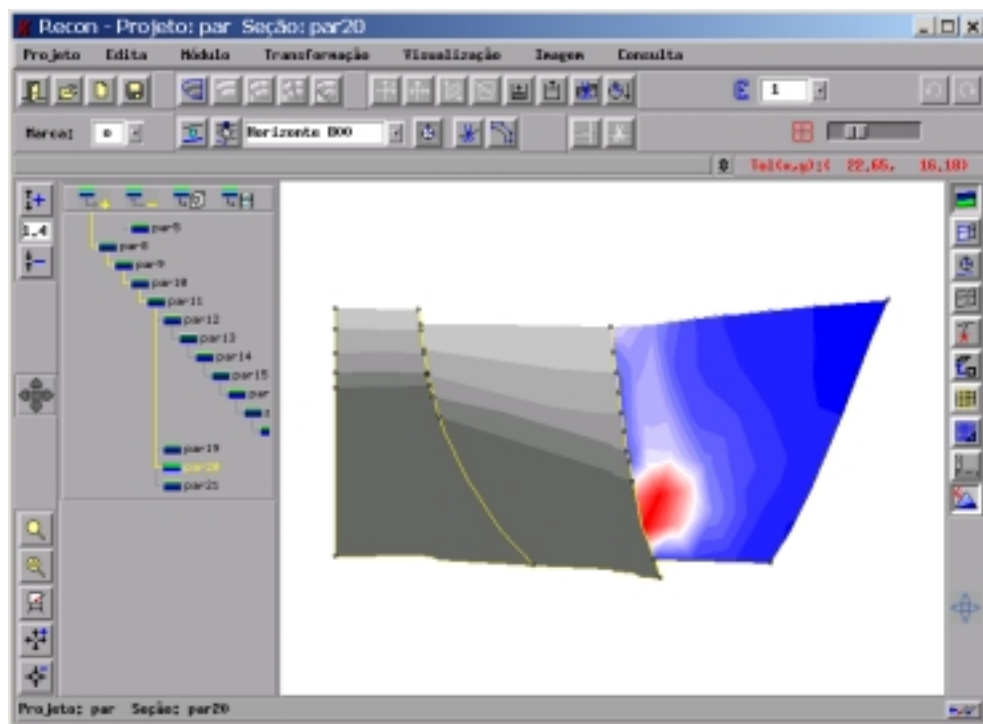**Figure 9 – Block transformed by the DR move-on-fault algorithm.**

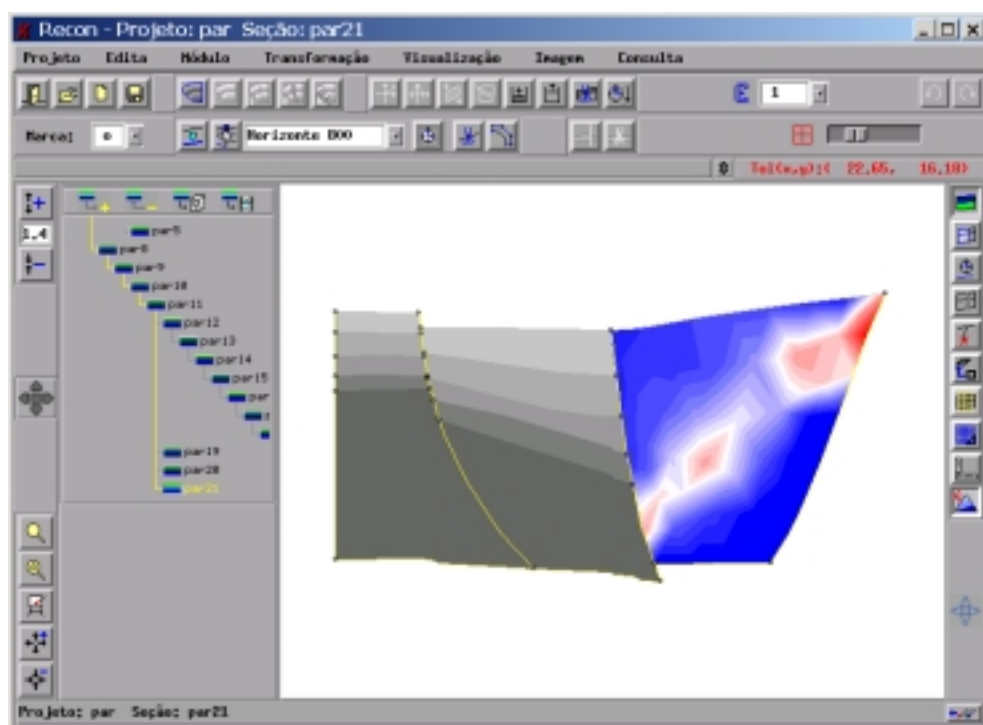**Figure 10 – Maximum principal strain results for the DR move-on-fault approach.**

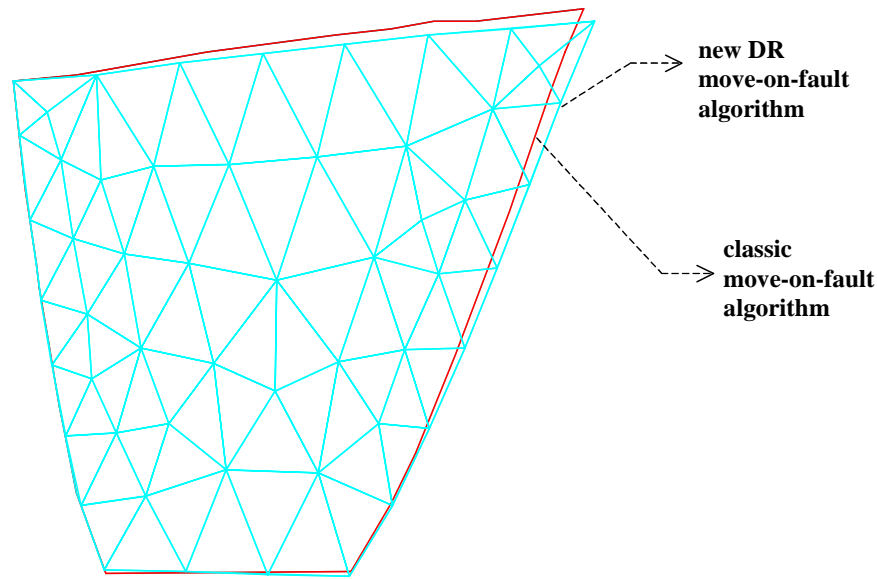**Figure 11 – Maximum principal strain results for the classic move-on-fault approach.**

Figure 12 – Geometric difference between classic and proposed move-on-fault approach.

Figure 10 shows a contour of maximum principal strain results for the new move-on-fault algorithm. Figure 11 presents the same contour for a classic move-on-fault transformation, which is based on a rigid displacement field and a fixed-angle shear deformation. Comparing these two images, one may observe that the proposed move-on-fault algorithm, which is based on Continuum Mechanics principles, provided more reasonable results. The strain contour of the new approach in Figure 10 presents a strain concentration in the region close to a kink in the fault's geometry. On the other hand, in the classic move-on-fault approach, the effect of the fault geometry's kink spreads over the entire block.

Figure 12 compares the deformed geometry of the block defined by the new and the classic move-on-fault algorithms. It may be observed that the geometric transformation of the new move-on-fault algorithm is more rigid. However, it is possible to change the values of the physical properties of the geological material.

Another example is shown in Figures 13 and 14. One can note in these images that the maximum principal strain contour presented for the new approach is more coherent than the strain contour obtained by the classic algorithm. In the new approach (Figure 13), it is possible to identify the area of the rock block that moved rigidly and the deformed regions that are in contact with the fault, while in the classic approach (Figure 14) the results apparently do not suggest any observation of this type.
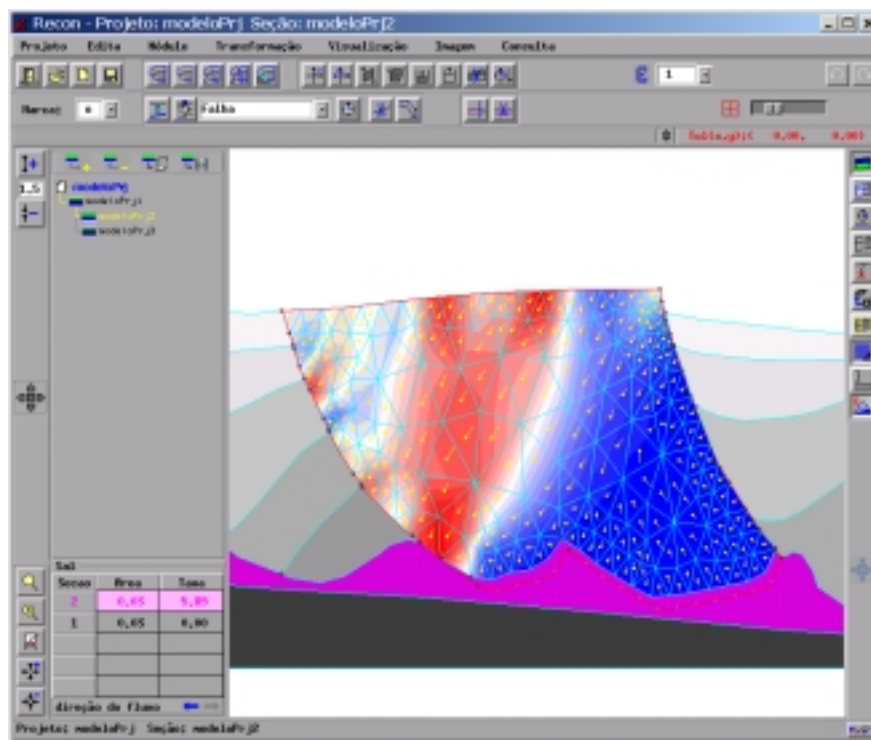
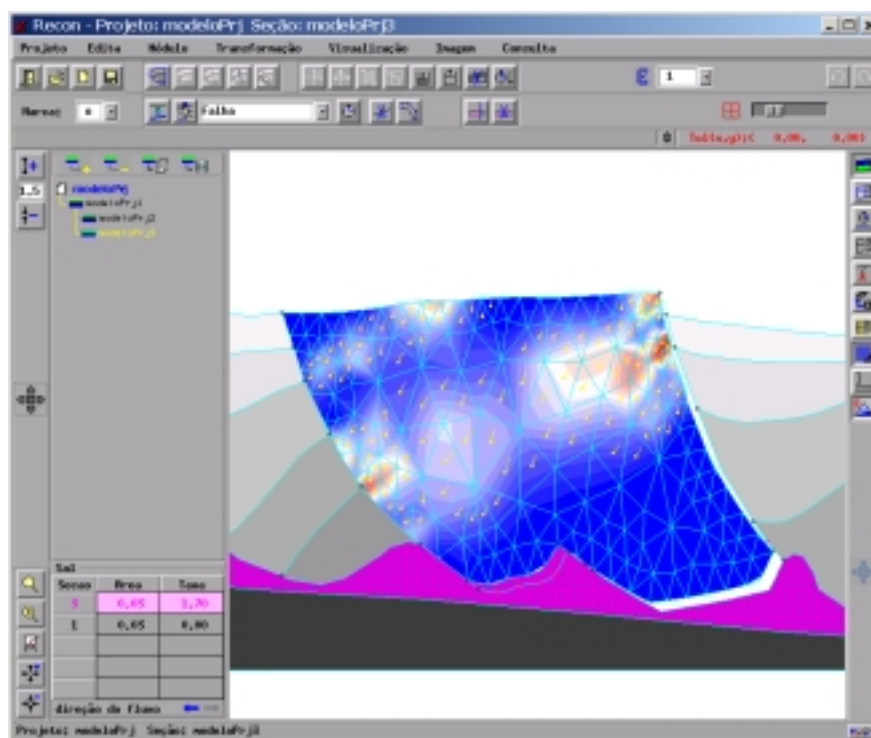**Figure 13 – Maximum principal strain results for the DR move-on-fault approach.**

Figure 14 – Maximum principal strain results for the classic move-on-fault approach.

5 Conclusion

This work has proposed a numerical technique for the restoration of geological cross sections. The idea was to introduce physically based principles in the geological transformation algorithms. The new technique consists of a move-on-fault transformation that is based on Continuum Mechanics principles. The authors believe that, in the restoration process, the application of a numerical method that considers the constituent relationships of the geological material in its formulation determines a new approach for the problem when compared to classic, empirically based, transformation algorithms.

The proposed procedure is considered a first step in the direction of a physically based geological restoration. Only a linear-elastic behavior of the material was taken into account. A more realistic elastic-plastic behavior may be considered in the future. The iterative Dynamic Relaxation algorithm is well suited to consider non-linear material behavior.

One important consequence of the use of a numerical technique is a greater degree of automation in the restoration process. This is particularly important in three-dimensional geological restoration. One of future objectives of this work is to implement the proposed move-on-fault transformation algorithm in a 3D modeling system, such as gOcad.

Acknowledgements

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