The impact of uncertainties model considered in reliability analysis for the design of reinforced concrete deep beams using strut-and-tie model.

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Summary

The design of engineering applications requires the assurance of an appropriate reliability level. Uncertainties are observed in mechanical and geometrical proprieties as well as in external loads. Consequently, a rational structural design requires the consideration of properties and loads uncertainties. Recently, this rational approach has been applied successfully in many areas of structural engineering. An important issue concerning design and detailing of reinforcement concrete members is the investigation of the so-called strut-and-tie models (STM). In these models, a reinforcement member is reduced to a truss-like structure, i.e. a set of compressive struts and tensile ties, in order to find a feasible statically admissible transfer mechanism of the applied load to the supporting system or ground foundation. This approach has often been used when some kind of discontinuity is present in the concrete element or structure. In addition, one may take into account the limited capacity of concrete to sustain plastic deformation providing distributed reinforcement to ensure a more ductile behavior and requiring that concrete members, represented by struts and nodes in the model, do not collapse before yielding of steel ties. Therefore, an efficiently formulation to verify the security and ductility behavior of strut-and-tie models is necessary. The present work uses the Crude Monte Carlo simulation method and FORM/SORM approximation methods for attesting a ductile behavior of a strut-and-tie model. These numerical stochastic methods can be applied to estimate the failure probability. With this estimation, a reliability index can be evaluated and compared with a target reliability index. The probability of each failure mode is obtained and design modifications are proposed to assure the ductility behavior of the strut-and-tie model. The stochastic distribution of concrete and reinforcement properties and of self weight and live loads follow the JCSS model and ACI code recommendations. This design approach follows ACI 318-05 guidelines that recommend the adoption of a reliability index and a target ductility behavior in design procedures of reinforced concrete structures.

Keywords

Reliability analysis, concrete structures, strut-and-tie models, uncertainties model, epistemic uncertainties.

Theme

buildings - construction - conventional loads / fire - concrete

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1. Introduction

Deep beams are structural elements that have small span/depth ratios. They have useful applications in many structures, such as tall buildings, foundations, offshore structures, and several others. This kind of element has the structural behavior commanded by shear failure mechanism. The shear failure mechanism of reinforced concrete structures members is not yet fully understood and more progress needs to be done to achieve an adequate rational design level. In this context, one interesting design tool is the truss analogy method. In this method, the internal force transfer in the continuum is replaced by a truss mechanism that represents a fully cracked reinforced concrete component. Strut-and-tie models are a set of truss-like mechanisms first proposed by Ritter and Morsch in the early 1900s. The original truss analogy concept assumes that concrete after cracking is not capable of resisting tension and postulates that cracked reinforced concrete beam acts as a truss with chords and a web composed of concrete struts and transverse ties. Based on plasticity theory this method was modified by others researches, such as Thurlimann , Muller and Marti. Further work by Schlaich et al (1987) extended the beam truss model with the use of uniformly inclined diagonals, thereby enabling application to all parts of the structures in the form of generalized strut-and-tie systems. Nowadays, strut-and-tie models are included in several codes and guidelines, such as ACI 318-05, EUROCO-2004, CEB-1992, and others.

Some provisions in ACI 318-05 have applied strut-and-tie concepts in the design of deep beams. Using these concepts, the designer is capable of predicting the load capacity of many types of deep beams. However, experimental results demonstrate that the actual behavior of a reinforced concrete component and its prediction using ACI code are quite different. Collins and Kuchma (1999) showed experimentally that ACI code overestimated the shear capacity of reinforced concrete deep beams having large sections and small reinforced ratios. Therefore, it is necessary to investigate possible ways to surpass these difficulties.

The main objective of this paper is to present a reliability criterion for analysis and design of deep beams using the strut-and-tie model. This approach permits that all kind of uncertainties present in problems of structural analysis of deep beams could be incorporated into the design process. Moreover, the proprosed reliability analysis assesses the failure probability associated with each failure mechanism of a deep beam. Using this approach, the designer estimates the probabability of occurance of a ductile mode or brittle mode and proposes design modifications to avoid an undesirable failure mode. To demonstrate the capability of this approach, the design of a deep beam tested experimentally is presented and analyzed. Model uncertainties values are obtained from the experimental results of 214 reinforced concrete deep beams that were tested to failure and reported in the literature by others researches and presented by Park and Kuchma (2007).

2. Reliability analysis

The analysis of a reinforced concrete structure involves several uncertainties related to concrete and steel properties, structural dimensions and position of steel reinforcement, loads and boundary conditions. Furthermore, uncertainties associated to the employed epistemic models are little used. For a realistic analysis, it is necessary to look for expected values and variances of the structural response, considering random input parameters. Several methods for probabilistic structural analysis have been studied in the last years. Generally, the methods employed are the Monte Carlo simulation method, First Order Reliability Method - FORM and Second Order Reliability Method. The Monte Carlo method is the most simple and evident way to accomplish a probabilistic analysis, and for that reason is widely used. In this method, material properties, loads and other random variables are introduced by digital simulation, without any significant modification of the algorithm used in the deterministic analysis. Moreover, Monte Carlo method is statistically consistent and may be employed to test other techniques. However, Monte Carlo method may be computationally very expensive in problems with several degrees of freedom and when many simulations are necessary to obtain the statistical descriptors of the structural response. In this case the FORM/SORM methods have often been applied. In the design criteria of reinforced concrete structures based on ultimate limit states, safety is reached by means of partial safety factors. These factors are introduced with different values to increase or to reduce the magnitude of the random variables involved in the analysis. Usually, loads, materials strengths and structural dimensions are the basic

random variables considered in the design. Partial safety factors are introduced to increase loads and to reduce steel and concrete strengths. For concrete and steel, the partial factors cover the deviations of the nominal dimensions and the difference between the strength obtained from test specimens and the strength in the actual structure. The use of partial safety factors, although convenient, is not sufficient to determine the safety level obtained in the design. In fact, safety depends on the structural response due to the actions and this involves interdependence among all random variables. A consistent evaluation of the safety level requires the determination of the structural failure probability. This probability can be estimated if the probability distribution of a certain random variable representing a given safety margin for the structure is known. Unfortunately, it is not always possible or practical to obtain this probability distribution. An alternative to obtain the safety level consists in the evaluation of the reliability index. This index, takes into account all random variables involved and the way the structure responds to the actions. The reliability index is associated to a failure probability, although this relationship is not explicit. Several works have been developed to evaluate the reliability of reinforced concrete structures. However, the reliability analysis applied in strut-and-tie models is a new research field to be developed. In general, there are several methods to solve the probability of failure for systems that represents the structural behavior. A general problem in reliability analysis can be shown graphically by Fig 1.



Figure 1: Design domain in reliability analysis.

Simulation methods and approximate methods like First Order Reliability Method (FORM) and Second Order Reliability Method (SORM) are used in this paper to estimate a probability of failure of a reinforced concrete deep beam. Five modes of failure (bearing plates for support and points of load, horizontal and diagonal struts and tie) are considered into a system that represents the structural behavior. The FERUM software developed in University of Berkeley (Der Kiureghian el at [7]) and another developed by the authors are used for this proposal.

2.1 Crude Monte Carlo simulation

This method of simulation is widely used when accurate estimative of probability of failure is required. A necessary condition for MCS used is that probability distribution of each random variable involved in the problem need to be known. After that, a random generation of value based on statistical parameters (mean and standard deviation) and correspondent probability distributions is made and the probability of failure is estimated. In crude Monte Carlo simulation P_f is estimated by:

$$P_f = \int_{\Omega_F} f_X(x) dx = \int_{R^n} I(x) f_X(x) dx = E[I(x)]$$
(1)

Where E[I(x)] is mathematical expectance of random variable I(x). The estimator I(x) can be defined by:

$$I(X) = \begin{cases} 1 \to x \in \Omega_{\rm f} \\ 0 \to x \notin \Omega_{\rm f} \end{cases}$$

Since the number of simulation is large N, the empirical average of I(X) values can be understood like a failure probability estimator. Then:

$$\bar{P}_f = \frac{\sum_{j=1}^{N} \{ I(G(U) \le 0) \}}{N}$$

A graphic visualization about Monte Carlo simulation is shown in Fig 2. The integral result is done around the mean value whether N will be sufficiently large.



Figure 2: Monte Carlo simulation method.

2.2 FORM/SORM Methods

The First Order Reliability Method (FORM) is a gradient-based search algorithm to locate the nearest point in the parameter space that yields a failure. In this so-called design point, a linear approximation of the Limit State Function (LSF) is used The First Order Reliability Method (FORM) is a gradient-based search algorithm. The First Order Reliability Method (FORM) is a gradient-based search algorithm to locate the nearest point in the parameter space that yields a failure. In this so-called design point, a linear approximation of the Limit State Function (LSF) is used as an approximate boundary between the safe and failure domain. Using this boundary, an estimate of the failure probability can be computed. In this paper, the HLRF method (Hansofer and Lind [8]) has been applied, a widely used algorithm that is generally fast and reliable. Each iteration step consists of a numerical gradient evaluation to find the direction of steepest descent towards the design point, followed by an optimization loop to set an optimal step in that direction. A disadvantage of the FORM algorithm is that no measure of accuracy is provided for the estimated failure probability. When a problem has multiple failure criteria, a FORM analysis must be performed for each criterion.

The Second Order Reliability Method (SORM) the limit-state surface is approximated by a parabolic surface with its principal curvatures fitted to the principal curvatures of the limit-state surface at the design point. Two algorithms are available for determining the principal curvatures and the corresponding principal directions of the limit-state surface at the design point. One algorithm determines the principal curvatures (eigenvalues) and the principal directions (eigenvectors) by solving an eigenvalue problem involving the Hessian (second-derivative matrix) of the limit-state surface. The Hessian is computed by finite difference calculations in the standard normal space. Another algorithm computes the principal curvatures in the order of decreasing magnitude by iterative calculations in the course of finding the design point by the improved HLRF algorithm (Der Kiureghian et al [6]). This approach is advantageous for reliability problems with large number of random variables, since calculations can be stopped when the magnitude of the last curvature found is sufficiently small. Once the principal curvatures are determined, the asymptotic formula by Breitung [5] or the exact formula by Tvedt [37] is used to compute the probability content of the fitted parabolic as the SORM approximation of the failure probability for each component. The design point for FORM/SORM approximation methods are shown in Fig3.



Figure 3: Design point in FORM/SORM approximation methods

3. Uncertainties

The presence of uncertainty in engineering, therefore, is clearly unavoidable. The available data are often incomplete or insufficient and invariably contain variability. Moreover, engineering planning and design must rely on predictions or estimation based on idealized models with unknown degrees of imperfections relative to reality, and thus involves additional uncertainty. In practice, two types of uncertainty may be identifying: aleatory uncertainty and epistemic uncertainty. For the deep beam problem used in this paper both types are considered.

3.1 The aleatory uncertainty or uncertainty associated with randomness

Many phenomena or process of concern to engineers contain randomness. It means that expected outcomes are unpredictable (to some degree). Such phenomena are characterized by field or experimental data that contains significant variability that represents the natural randomness of an underlying phenomenon; i.e., the observed measurements are different from one experiment to another, even if conducted or measured under apparently identical conditions. There is a range of measured or

observed values of the experimental results; moreover, within this range certain values may occur more frequently than others. The variability inherent in such data or information is statistical in nature, and the realization of a specific value or range of values involves probability. In general, this kind of approach is considering trough basic variables that are more relevant for the problem. Then, specifics limit state equations are developed to represent each mode of failure of the structure. The calculation model for each limit state considered should contain a specified set of basic variables, i.e. physical quantities which characterize actions and environmental influences, material and geometrical quantities. The basic variables are assumed to carry the entire input information to the calculation model. Each basic variable is defined by a number of parameters such means, standard deviation, type of probabilistic distributions, parameters determining the correlation structures, etc. Considering the deep beam problem, four basic variables are taken: two of them represent the material randomness (concrete and steel) and two others for actions randomness (permanent and live load). In Table 1, the statistical parameters for associated basic variables are taken from JCSS 2001.

Random variable	Distribution	Mean	δ
Permanent load	Normal	Pgm	0,03
Live load	Gumbel	Pqm	0,30
Compression concrete strength	Lognormal	fcm	0,17
Yield strength	Lognormal	fym	0,05

Table1 - Statistical prope	erties for the model
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The compression concrete strength and yield strength values used in this paper are the average of values obtained in the laboratory results. The load values are obtained through ACI 318-05 for the less value of failure in the model. A value of 0.5 in the relation between permanent and total load are used to simulate a real design situation.

3.2 The epistemic uncertainty or uncertainty associated with imperfect knowledge

In engineering, there are idealized models of the real world in our analysis and the predictions for the purpose of making decisions or planning and developing criteria for design of an engineering system. These idealized models, which may be mathematical or simulation models or even laboratory models, are imperfect representations of the real world. Consequently, the results of analysis, estimations or predictions obtained on the basis of such models are inaccurate, with some unknown degree of error, and thus also contain uncertainty. Such uncertainties are, therefore, knowledge based and are of the epistemic type. Quite often, this epistemic uncertainty may be more significant than the aleatory uncertainty. In performing a prediction or estimation with a idealized model, the objective is invariably to obtain a specific quantity of interest; this may be the mean-value or median value of a variable. Therefore, in considering the epistemic uncertainty it is reasonable in practice, to limit our consideration to the accuracy in calculating or estimating the central value, such mean-value or median value. A calculation model is a physically based or empirical relation between relevant variables, which are in general random variables:

$$T = G(X_1, X_2, \dots, X_i)$$

Where *T* is the model output, *G* is the model function and X_i the basic variables of the problem. In fact, the model function *G* often will be inexact. Then, the difference between the model prediction the real outcome of the experiment can be written down as:

$$T = G'(X_1, \ldots, X_i, \theta_1, \ldots, \theta_j)$$

The parameters namely θ_i are referred to as parameters which contain the model uncertainties and random variables.

•				
Random variable	Distribution	Mean	δ	
				ņ
Uncertainty of resistance	Normal	1,76 *	0,18 *	
Uncertainty of load effect	Normal	1	0,05	

Table2	- Epistemic	uncertaities	of the	model
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* Values obtained by model versus experimental results

Their statistical properties are derived from experiments or observations. The mean of these parameters should be determined in such a way that, on average, the calculation model correctly predicts the test results. In Table 2, the parameters for epistemic uncertainties consideration are shown. The proposed method used the values of average and coefficient of variation obtained by Park and Kuchma (2007) to calculate the capacity of 214 reinforced concrete deep beams that were tested to failure and reported in the literature by others investigations using ACI 318-05 orientations.

4. Example application

To illustrate the proposed approach for the assessment of the safety and ductility level on the strut-and-tie model, an example of a reinforced concrete deep beam is presented and analyzed considering the orientations presents in ACI 318-05. The compression concrete strength values were increased to show their influence in the behavior of model.

4.1. Deep beam structure

The external geometry of the deep beam used for this example is shown in Fig. 4. A simply supported deep beam to two-point top loading is used. For hydrostatic nodes, the struts e tie dimensions are controlled by the size of the bearing plates and for existent angle for the ratio a/d_{ef} , only. Unfortunately, since the width of strut and tie are not known initially, some iterative process is required after initial values are adopted.

In this work none hydrostatic node is not required, then the width of the horizontal strut and tie are modified and its influence evaluated. From the force equilibrium at the top node, the compressive forces F_d in the diagonal strut, F_h in the horizontal strut and T in the horizontal tie can be determined by the following equations:

$$F_{d} = \frac{P}{\sin\theta_{s}} \qquad \qquad F_{h} = \frac{P}{\tan\theta_{s}} \qquad \qquad T = \frac{P}{\tan\theta_{s}} \qquad (2)$$

The inclined angle θ_s of the diagonal strut is given by:

$$tan\theta_s = \frac{d_{ef}}{a}$$

Where, a is the shear span and d_{ef} is the lever arm of main reinforcement to the center of concrete stress block. The term As is the cross-sectional area of the diagonal strut and is computed by:

$$A_{D_strut} = b_w(w_t \cos \theta_s + l_b \sin \theta_s)$$



Figure 4: Strut-and-tie model for simply supported deep beams

Where, b_w is the beam width, w_t is the depth of the bottom node, taken differently by one case to another and lb is the width of the support plates. The lever arm of main reinforcement d_{ef} is:

$$d_{ef} = h_v - \frac{w_t}{2} - \frac{w_h}{2}$$

The effective depth of the top horizontal concrete strut was taken as:

$$w_h = kd$$

Where, d is the effective depth of deep beam and k was derived from the classical bending theory for a single reinforced beam section. For more details see Park and Kuchma (2007). The effective depth of the diagonal concrete strut was taken as:

$$w_{da} = \frac{a}{2}sin(\theta_s) + kdcos(\theta_s)$$
(3)

Where, a/2 should not be less than the length of the loading plate, kd is the depth of the compression zone at the section. The inclined angle of the diagonal strut can be evaluated by:

$$tan\theta_s = \frac{h_v - \frac{w_t}{2} - \frac{w_h}{2}}{a}$$

The effective depth of the diagonal strut is adopted for the less value between Eq. 3 and the similar expression used for the bottom node.

Table	3	shows	the	values	used	in	the	example	considered	in	this	paper.	These	values	are	obtained	in
pape	r of	Park a	nd k	Kuchma	(2007	7).											

Table 3 - Model proprieties										
Pn										
KN										
73,42										

Beam 0A0-44 - Table 1 - Park and Kuchma (2007) SI Units

6. Methodology

The approach used in the paper has two steps. In first step laboratorial conditions are simulated in way to compare results with researches previously done. Failure modes are checked and the numerical stochastic model calibrated. It means that only permanent load is considered with any correction because it represents a mean value. It is possible once there are controlled loading conditions inside the laboratory. In the same way, concrete and steel proprieties values are used like mean values once represent mean values for laboratorial tests. In the second step, real design conditions are applied. It means that total load is half represented by permanent load (normal distribution) and half by live load (Gumbel distribution). Also the characteristic values adopted by resistance and load variables have to be correct and the correspondent means values obtained for each correspondent distribution. In this step JCSS code orientations are adopted. All the steps are done considering the epistemic uncertainties in the model like presented in Table 2.

6. Discussion and conclusions

A value of 20.5MPa was used in the original deep beam. A brittle mode of failure, namely the concrete crushing, was obtained in the test. A similar result was obtained by the numerical model too. As shown in Fig.5, using laboratory conditions, high values of brittle failure are obtained for all values of compressive strength of concrete.



Figure 5: Reliability analysis using laboratory conditions

In spite of the brittle behavior the values of reliability index is satisfactory for values above 26MPa. For simplicity, only important modes of failure were shown, i.e. the failure of mode 4 associated a concrete crushing and failure mode 5 associated a yielding of steel. Even considering uncertainties into the model the ACI 318-05 formulation was not capable to lead the structure for ductile modes of failure.

In the second step, the design conditions were applied into the numerical model. Since, the design conditions are harder then laboratory conditions an increment into the values of compressive strength of concrete was necessary to guaranty the same level of safety. However, the brittle behavior continues present for all values of compressive strength of concrete. The modes 4 and 5 had a similar behavior obtained previously, with high values for crushing concrete and low values for yielding of steel.



Figure 6: Reliability analysis using design conditions

A modified design condition was proposal to surpass the brittle behavior. A modification in the width of the beam was done in association a decrease in the steel area present in the model. A value of 12.2 cm was used for deep width with a reduction of 34% in the area of steel present in the tie. The Fig. 7 shows the results for the modified design condition.



Figure 7: Reliability analysis using modified design conditions

Despite of the level of safety were reached above 26MPa, the desire ductile behavior could be observed only after values above 35MPa. This is an important observation. Using this approach is possible controlling the ductile level of the model by some modification in the variables of the problem. Also, a safety level for the model can be estimated.

A numerical approach using the reliability analysis of a concrete deep beam including the uncertainties of model is done. The ACI 318-05 orientations recommended for strut-and-tie models in the design concrete structures were used and compared with a previously tested model. Some modifications into the variables of the problem were necessary to improve a better behavior for the model. A safety level and a significant control of the failure mode of the structure were approved to be done. It can be concluded that reliability analysis using an experimental calibration is a satisfactory approach to be used in design and analysis of reinforced concretes using strut-and-tie methods.

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