# Reliability analysis of concrete structures applied to strut-and-tie model

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ABSTRACT: This paper shows a methodology for reliability analysis of reinforced concrete structures applying the strut-and-tie method. A Monte Carlo simulation is used to calculate the reliability index and probability of failure modes, considering both safety and ductile behavior of the strut-and-tie model. Three different formulations presented in the literature to compute the effective compression resistance of the concrete are considered: ACI-318, CEB-Fib and Schlaich & Schaefer. Both reliability index value and ductility behavior are considered in this comparison. An example of design of a deep beam using reliability analysis for validation of safety and ductility aspects is presented.

# 1 INTRODUCTION

The design of engineering applications requires the assurance of an appropriate reliability level. Uncertainties are present in mechanical and geometrical proprieties as well as in external loads distribution and values. Consequently, a rational approach in structural design requires that uncertainties should be taken into account. Recently, this approach has been applied successfully in the area of structural engineering (Paliga, 2008; Graziano, 2005). An important issue concerning design and detailing of reinforced concrete members is the investigation of the so-called strut-and-tie models (STM). According to this approach, a reinforced concrete member is modeled as a truss-like model, i.e., a set of compressive struts and tensile ties, in order to find a feasible statically admissible transfer mechanism of the applied load to other members or to the structure foundation. This approach has often been used when some kind of discontinuity is present in the concrete element or structure. When applying this methodology, one may take into account the limited capacity of concrete to sustain tensile plastic deformation placing reinforcement steel bars in the tensile regions. The result is a composite material that ensures a ductile behavior, provided that the concrete members in the model, representing by struts and nodes, do not collapse before yielding of steel ties. Therefore, an efficient formulation to verify the

safety and ductility behavior of strut-and-tie models is desired.

This work used a Monte Carlo simulation method to perform a reliability analysis of reinforced concrete structures applying the strut-and-tie method. Monte Carlo simulations allow the estimation of the failure probability for any type of random problem. It has two main advantages: a possibility to deal with practically any mechanical and physical model, and an easy implementation without any modification of the mechanical model. With the calculated failure probability, the reliability index can be obtained and compared with a target reliability index proposed by the Joint Committee on Structural Safety (JCSS) model. The JCSS Probabilistic model code gives guidance on the modeling of random variables in structural engineering. In this work, the failure probability of some failure mechanisms is obtained, and modifications will be proposed to assure a ductility behavior of the strut-and-tie model. Nondeterministic distribution of all modeling parameters, such as values of concrete and reinforcement properties, self weight and live load, were obtained from the JCSS code. For struts and nodes calculations, three different formulations presented in the literature to compute the effective compression resistance of the concrete will be considered: ACI-318, CEB-Fib and Schlaich & Schaefer. Both reliability index value and ductility behavior are considered in this comparison.



Figure 1. Deep beam analyzed by Souza (20066).

#### 2 PROBLEM DESCRIPTION

#### 2.1 Analyzed structure

An application example of a deep beeam previously analyzed by Souza (2006) using the sttrut-and-tie method is adopted. This work used the same model to perform a reliability analysis thought a Monte Carlo simulation. The model characteristicss are shown in the figure 1. The adopted strut-and-tiee model can be observed in the same figure.

#### 2.2 Mode failures



Figure 2. Strut-and-tie model applied in reeinforced concrete deep beam.

Table 1. Statistical properties for the model.

Random variable	Distribution	Mean	Standard deviation	Cov
Permanent load	Normal	Pg(kN)	0.03 Pg	0.03
Variable load	Gamma	Pq(kN)	1.5 Pq	1.50
Compression concrete strength	Lognormal	fckm (kN/cm2)	0.17 fckm	0.17
Yield strength	Lognormal	fykm (kN/cm2)	0.05 fykm	0.05



Figure 3. Model reliability index in function of permanent load variation.

#### 2.3 Random variables

The statistical properties of the model random variables are shown in table 1.

## 3 RESULTS

A series of tests is performed to obtain the reliability index of the proposed moodel. The estimation of the effective concrete resistannce in compression considers the different formulattions mentioned previously. A variation between thee percentage of permanent load  $q_p$  and total load  $q_T$  is also considered to measure its influence on the reeliability index and ductility characteristics of the moddel. The ductility behavior is intrinsically related too the mobilization of the equivalent truss structuree within the concrete. The reliability index  $\beta$  represents the safety level of the model and the probabilitiies obtained from each failure mode are used to deetermine the model ductile capacity. In a ductile behhavior, if a structure failure has already occurred, thhe probability of crushing concrete struts should be low (less than 5%) and the probability of reinforcemeent yielding should be high (above 95%). As mentionned, the random analysis is performed by a Monte Carlo simulation, and different values for the reliability index are obtained using different formulations andd considering different percentage of permanent andd total loads. The Figure 3 shows clearly that for loww values of permanent load the safety criterion requireed by JCSS, represented by a target index, is not met. This is observed using all the studied formulations. Just for permanent load up about 70% of total



Figure 4. Model brittle failure probability in function of permanent load variation.



Figure 5. Model reliability index in function of steel area reinforcement  $(q_p/q_T = 50\%)$ .

loadd, the values of minimum safety requirements can bbe met.

The influence of the relaative intensity between permanent and total loads is also observed in the evaluation of model ductility. As shown in Figure 3, using the CEB-Fib formulations, high values of brittle failure probability are obtained for low values of permanent to total load relation. The ACI-318 formulation presents lower sensitivity to the parameter qp/qt than the CEB-Fib formulation. The Schlaich & Schaefer formulation is the one that obtains the lower values of brittle failure probability. Figure 4 refers to the relation between brittle failure (mode 5) probability, represented by the crushing of strut concrete member, and total failure probability.

To increase the values of reliability index  $\beta$  obtained previously, as shown in Figure 1, this work proposed the modification of some parameter values to ensure the minimum safety level recommended by JCSS. As the model is, in general, ductile, increasing  $f'_c$  does not result in significant increase of  $\beta$ . Therefore, a gradual increase of reinforcement steel area is proposed, and the effects on the reliability indices are observed, as shown in Figure 5. The parameter  $f'_c$  is fixed at 25 MPa and a value of 50% to the relation  $q_p/q_T$  is adopted.

The Schlaich & Schaefer formulation is able to produce reliability indices greater than the reference value, unlike the formulations proposed by ACI and CEB-Fib. Such formulations also lead to brittle models when steel area reinforcement increases, as shown in Figure 6.

Figure 7 refers to the relation between ductile failure (mode 6) probability, represented by steel yielding, and total failure probability. The formulation proposed



Figure 6. Model brittle failure probability in function of steel area reinforcement ( $q_p/q_T = 50\%$ ).



Figure 7. Model ductile failure probability in function of steel area reinforcement ( $q_p/q_T = 50\%$ ).



Figure 8. Model reliability index in function of steel area reinforcement with different  $f'_c$  values  $(q_p/q_T = 50\%)$ .

by Schlaich & Schaefer indicates a ductile behavior. The other formulations, however, lead to a gradual reduction of  $P_{Ductil}$  (%), indicating structure brittle behavior.

It can be observed that CEB-Fib formulation is more conservative in ensuring ductility behavior because it needs a greater value to present a similar satisfactory behavior. The formulations of the ACI-318 & Schlaich and Schaefer have similar behavior; however the ACI-318 is more conservative. One may note that the various formulations proposed for evaluation of the concrete effective compressive strength in strut and tie model can lead to different behaviors in terms of both safety and ductility. Figure 8 shows reliability indices curves to the formulations studied considering both safety and ductility requirements.

### 4 CONCLUSIONS

The formulations presented to concrete effective compression resistance in the strut and tie model lead to satisfactory levels of reliability for the studied structure only for high values of permanent and total load relation. As the structure is, in general, ductile, high values of  $f'_c$  does not lead to significant improvements in structural safety. This gain in reliability is achieved by increasing the area of steel. However this procedure leads to brittle structures when using CEB-Fib and ACI-318 formulations. With Schlaich & Schaefer formulation, it is possible to obtain, in the case treated at this work, a safe and ductile structure.

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