

Crack Path Predictions on Modified C(T) Specimens under Variable Amplitude Loading

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ABSTRACT. *A hybrid global-local methodology to predict fatigue crack propagation in 2D structures is extended to model crack retardation effects induced by variable-amplitude (VA) loading histories. First, finite element (FE) models are used at each propagation step to calculate the generally curved fatigue crack path. However, the FE approach alone is not computationally efficient to predict crack growth rate, because it would require time-consuming remeshing of the entire structure after each event in VA loading. Therefore, the crack path and their mixed-mode stress intensity factors are FE calculated under constant-amplitude (CA) loading using fixed crack increments, requiring only relatively few remeshing steps. An analytical expression is then fitted to the calculated K_I values, which is used in a local-approach fatigue design program to predict crack propagation lives under VA loading, considering load interaction effects such as crack retardation or arrest after overloads. This methodology is experimentally validated by fatigue crack growth tests on compact tension C(T) specimens, modified with holes positioned to attract or to deflect the fatigue cracks.*

INTRODUCTION

Fatigue life prediction of cracked two-dimensional (2D) structural components requires the calculation of the generally curved crack path, the associated stress intensity factors (SIF) K_I and K_{II} , and the crack propagation rate at each load step [1]. A finite element (FE) global discretization of the component, using specialized crack tip elements to predict the crack path and to calculate its associated SIF, is a standard practice. However, this global calculation method is not computationally efficient under variable amplitude (VA) loading to predict fatigue lives, because it requires time-consuming remeshing procedures and FE recalculations after each loading event.

On the other hand, the so-called local approach, based on the direct integration of the crack propagation equation, can be efficiently used to calculate the crack increment at each load cycle, considering crack retardation effects if necessary. However, this approach requires the stress intensity expression K_I for the crack, which is not available for most real components.

Since the advantages of these two approaches are complementary, the problem can be divided into two steps. First, the curved fatigue crack path and its SIF are calculated under constant amplitude (CA) loading in a specialized FE program, using small crack increments and automatic remeshing. Numerical methods are used to calculate the crack propagation path, based on the computation of the crack incremental direction, and the associated SIF $K_I(\mathbf{a})$ and $K_{II}(\mathbf{a})$, where \mathbf{a} is the length along the crack path. The $K_I(\mathbf{a})$ values are then used as an input to a fatigue program based on the local approach, where the actual VA loading is efficiently treated by the integration of the crack propagation equation, considering overload-induced retardation effects if required [1].

This hybrid methodology has been experimentally validated through crack growth experiments under CA loading on modified compact tension C(T) specimens, in which holes were machined to curve the crack path [2]. In this work, the methodology is extended to VA loading, considering load interaction effects, by first testing standard C(T) specimens under VA loading to calibrate several crack retardation models, to then use the calibrated parameters to predict the fatigue lives of the modified C(T) specimens under similar loading conditions.

ANALYTICAL BACKGROUND

To compute the SIF along a (generally curved) crack path under mixed mode I - mode II loading, at least three methods can be chosen: (i) the displacement correlation technique [3], (ii) the potential energy release rate computed by means of a modified crack-closure integral technique [4-5], and (iii) the J-integral computed by means of the equivalent domain integral (EDI) together with a mode decomposition scheme [6-7]. Bittencourt et al. [8] showed that for sufficiently refined FE meshes all three methods predict essentially the same results.

In addition, to calculate the crack incremental growth direction in the linear-elastic regime in 2D FE analysis, three criteria can be used: (i) the Maximum Circumferential Stress (S_{qmax}), (ii) the Maximum Potential Energy Release Rate (G_{qmax}), and (iii) the Minimum Strain Energy Density (U_{qmin}) [1, 2].

Two complementary pieces of software, named **Quebra2D** and **ViDa** [1, 2, 9], have been developed to implement the two steps of this hybrid methodology. A brief description of both programs is presented below.

Quebra2D is an interactive graphical program for simulating two-dimensional fracture processes based on a FE self-adaptive mesh-generation strategy [2, 10]. This program includes all methods described above to compute the crack increment direction and the associated stress-intensity factors along the crack path. Moreover, its adaptive FE analyses are coupled with modern and very efficient automatic remeshing schemes, which substantially decrease the computational effort.

The automatic calculation procedure in **Quebra2D** is performed in 4 steps: (i) the FE model of the cracked structure is solved to obtain K_I and K_{II} and to calculate the corresponding crack propagation direction; (ii) the crack is increased in the growth direction by a (small) required step; (iii) the model is remeshed to account for the new

crack size; and (iv) the process is iterated until rupture or until a specified crack size is reached. As a result, a list of K_I and K_{II} values is generated at short but discrete intervals along the predicted crack paths.

The second program, named **ViDa**, is a general-purpose fatigue design program developed to predict both initiation and propagation fatigue lives under VA loading by all classical design methods, including the SN, the IIW (for welded structures) and the eN for crack initiation, and the da/dN for crack propagation. It includes several load interaction models to predict overload and underload-induced crack retardation and acceleration. The program includes comprehensive database with mechanical properties of more than 13000 materials, hundreds of editable K_I and K_{II} SIF expressions and da/dN curves to be used in the calculations. In particular, **ViDa** accepts any crack growth equation and any SIF expression, making it an ideal companion to **Quebra2D**, which can be used to generate the required DK expression if not available in its database.

EXPERIMENTAL RESULTS

The FCG experiments were performed on cold-rolled SAE 1020 steel with yield strength **285MPa**, ultimate strength **491MPa**, Young modulus **205GPa**, and reduction in area **54%**, measured according to the ASTM E 8M-99 standard, with the analyzed weight percent composition: C 0.19, Mn 0.46, Si 0.14, Cu 0.11, Ni 0.052, Cr 0.045, Mo 0.007, Nb 0.002, Ti 0.002, Fe balance. The tests were performed at two $R = K_{min}/K_{max}$ ratios, $R = 0.1$ and $R = 0.7$, in a 250kN computer-controlled servo-hydraulic testing machine. The crack length was measured following ASTM E 647-99 procedures. The measured growth rates on 16 standard compact tension C(T) test specimens were fitted by a modified McEvily da/dN equation (in m/cycle), as shown in Figure 1, where the propagation threshold under $R = 0$ is $DK_0 = 11.5 \text{ MPa}\sqrt{\text{m}}$, and the fracture toughness is $K_C = 280 \text{ MPa}\sqrt{\text{m}}$.

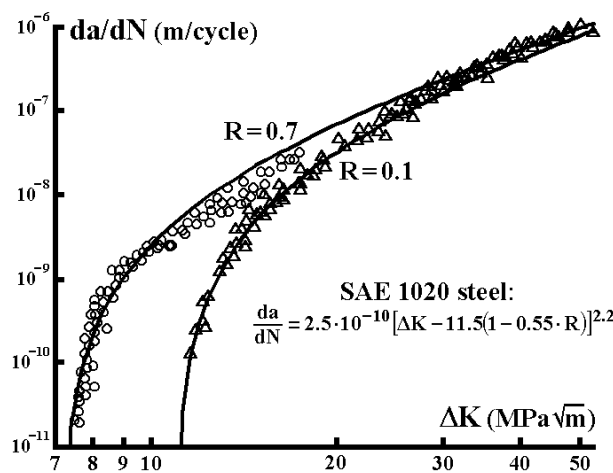


Figure 1. Modified McEvily da/dN equation fitted to the SAE 1020 steel data.

Three modified C(T) specimens were designed and tested, with width $w = 29.5\text{mm}$ and thickness $t = 8\text{mm}$. Each one had a 7mm-diameter hole positioned at a slightly different horizontal distance **A** and vertical distance **B** from the notch root, as shown in Figure 2(a). Two very different crack growth behaviors had been predicted by the FE modeling of the C(T) specimens, depending on the hole position. The predictions indicated that the fatigue crack was always attracted by the hole, but it could either curve its path and grow toward the hole (“sink in the hole” behavior) or just be deflected by the hole and continue to propagate after missing it (“miss the hole” behavior).

Using the **Quebra2D** program, the transition point between the “sink in the hole” and the “miss the hole” crack growth behaviors was identified. The three modified C(T) specimens were designed so that specimens named CT1(CA) and CT1(VA) had the hole just half a millimeter above the transition point, and a specimen named CT2(CA) had the hole half a millimeter below it. The chosen specimen geometries were machined, measured, and FE remodeled, to account for small deviations in the machining process (Figure 3). In this way, it could be assured that the numerical models used in the predictions reproduced the real geometry of the tested specimens.

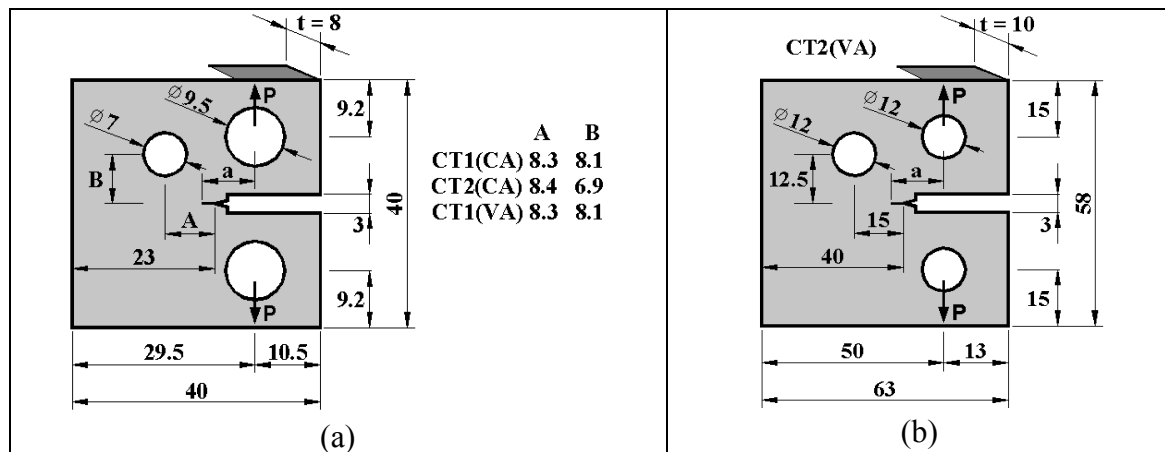


Figure 2. Measured dimensions of the hole-modified C(T) specimens (mm).

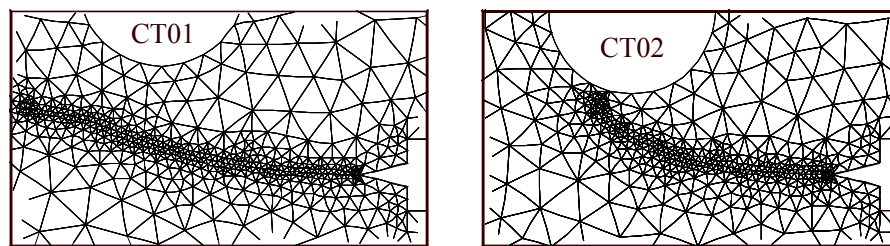


Figure 3. Automatically generated FE mesh of the CT1(CA) and CT2(CA) specimens.

Specimens CT1(CA) and CT2(CA) were tested under CA loading, under a quasi-constant stress-intensity range $DK_I \gg 20\text{MPa}\sqrt{\text{m}}$ and load ratio $R = 0.1$.

Two specimens were tested under VA loading: one standard C(T) specimen, and the holed CT1(VA). The VA load histories applied to the specimens are shown in Fig. 4.

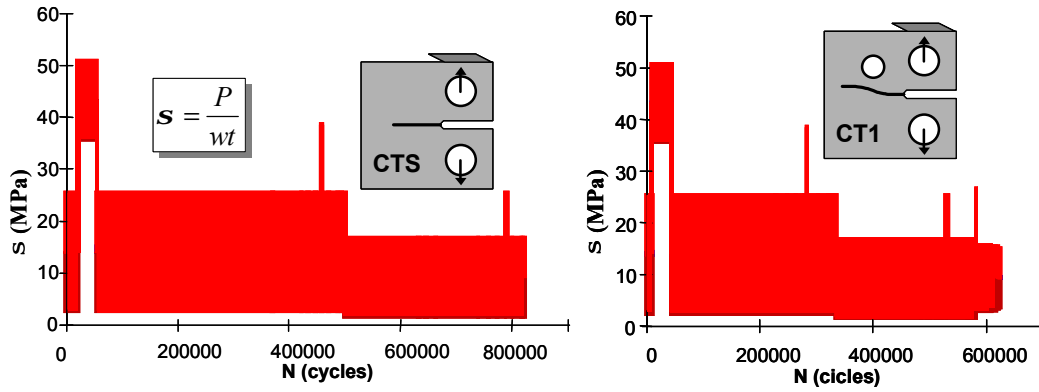


Figure 4. Applied load history (in kN) for standard C(T) and modified CT1(VA).

Figure 5 shows the predicted and measured crack paths for the three modified specimens (in mm) under CA or VA loading, presenting a very good match. This suggests that the curved crack paths predicted under CA loading give good estimates of the measured paths under VA loading. Therefore, assuming that the Linear Elastic Fracture Mechanics (LEFM) conditions apply, the discussed two-step methodology can be generalized to the VA loading case.

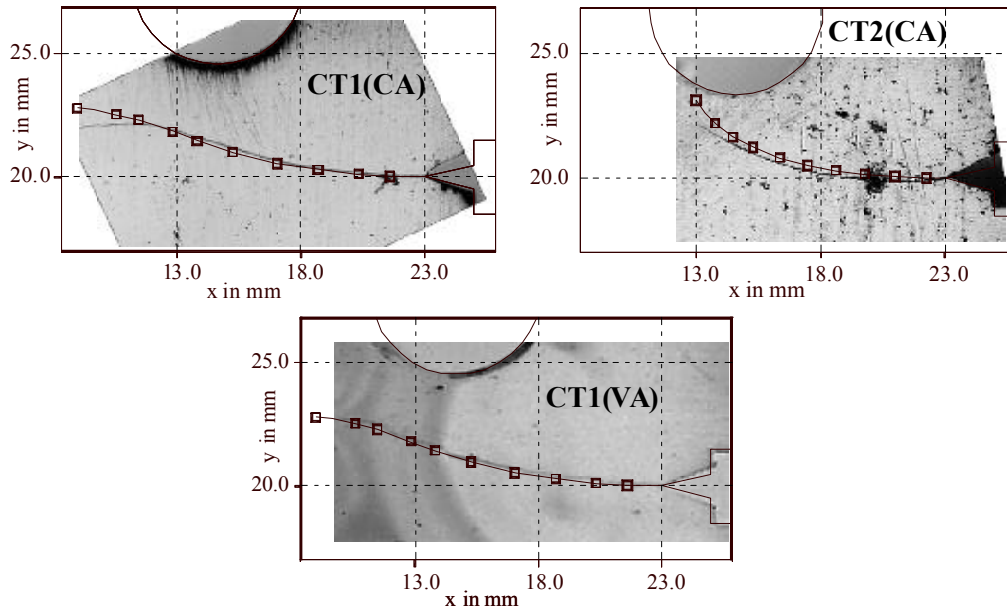


Figure 5. Predicted and measured crack paths for the modified C(T) specimens (mm).

The SIF values calculated under CA loading along the crack path using the **Quebra2D** program were exported to the **ViDa** software to predict fatigue life, considering load interaction effects. Figure 6 shows predicted and measured crack sizes for the modified C(T) specimens under CA loading.

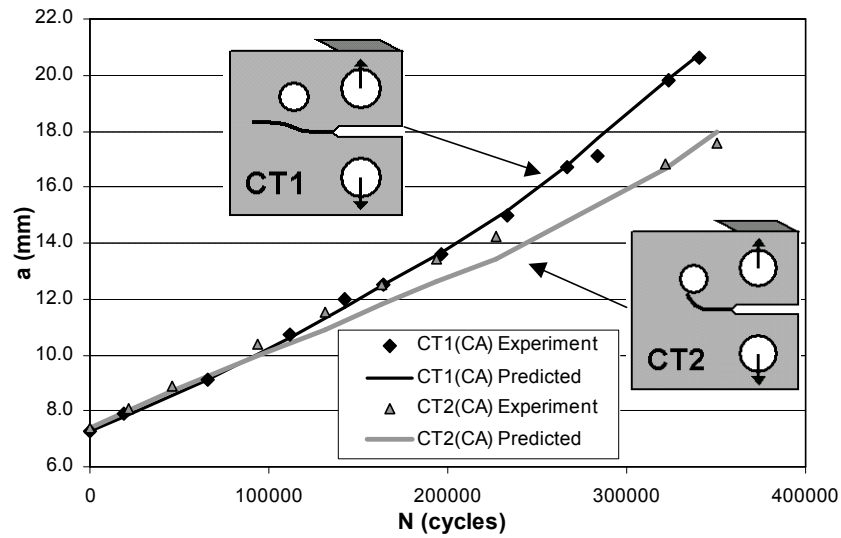


Figure 6. Predicted and measured FCG for modified C(T) specimens under CA loading.

Several crack retardation models were calibrated based on the standard C(T) data under VA loading, including the Constant Closure model [11] (where the crack opening load K_{op} was calibrated as 26% of the maximum overload SIF, $K_{ol,max}$), a modified Wheeler model [12-13] (where the model's exponent was estimated as 0.51), and Newman's closure model [14] (generalized for the VA loading case, where the stress-state constraint factor was fitted as $\alpha = 1.07$, suggesting dominant plane-stress FCG conditions). The fitted load interaction parameters were then used to predict in the **ViDa** program the crack growth behavior under VA loading of the hole-modified CT1(VA) specimen, see Fig. 7. The significant retardation effects of the CT1(VA) specimen were very well predicted using these three load interaction models.

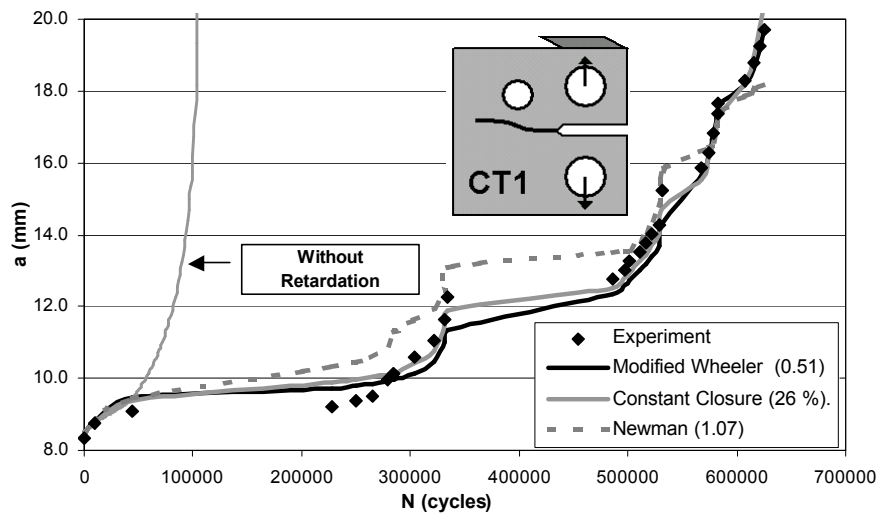


Figure 7. Crack growth predictions (based on straight-crack calibrations) on a modified C(T) specimen under VA loading.

Finally, a larger modified C(T) specimen named CT2(VA) has been designed and tested under VA loading, with sizes shown in Figure 2(b). The applied VA load history is shown in Fig. 8. As seen in Fig. 9(a), in the beginning there is a good match between the predicted and measured crack paths. However, after an overload at about 750,000 load cycles in the history, there is a significant deviation in the crack path. After carefully examining the specimen surface, it was found that the crack tip had unexpectedly bifurcated due to the overload, see Fig. 9(b). Even though such crack bifurcations can be easily modeled using the **Quebra2D** program [15], it is very difficult to predict whether and when they are induced. In addition, this overload generated a very large plastic zone ahead of the bifurcated crack tip, with dimensions comparable to the length of the residual ligament between the crack and the hole, invalidating LEFM assumptions. Therefore, elastic-plastic FE calculations considering bifurcation effects would be required to predict the crack path of this specimen.

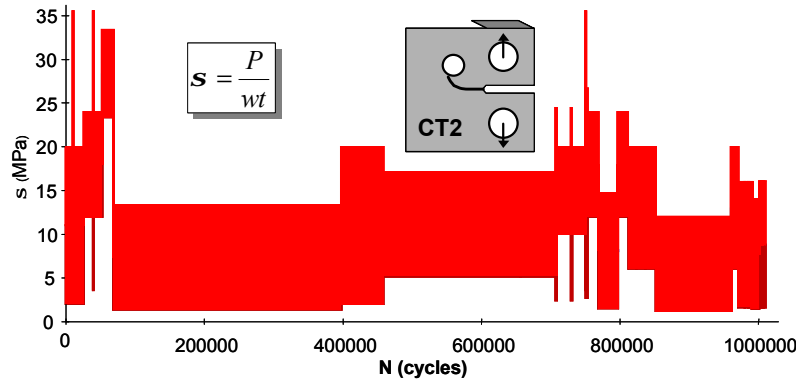


Figure 8. Applied load history for standard modified CT2(VA) specimens.

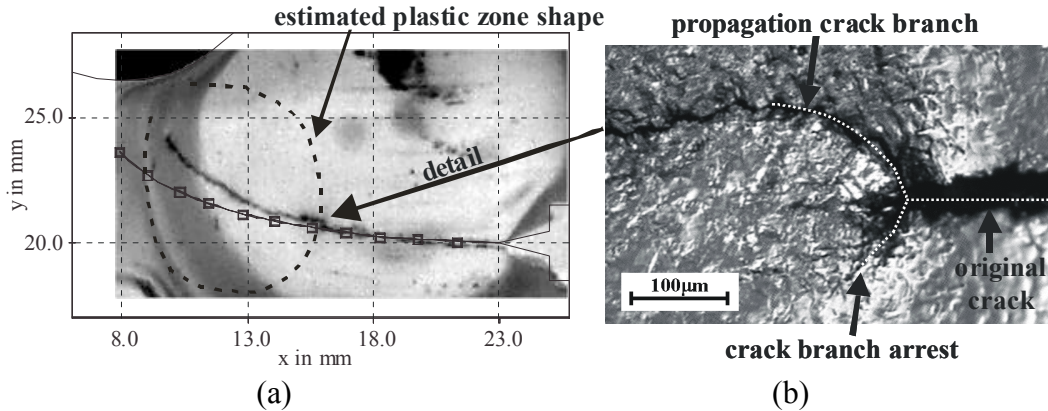


Figure 9. Predicted and measured crack paths for the CT2(VA) specimens.

CONCLUSIONS

In this paper, a methodology to predict fatigue crack propagation in generic 2D structures was extended to VA loading histories, modeling crack retardation effects. Experimental results were performed on hole-modified compact tension specimens to validate

the approach. It was found that overloads did not deviate significantly the crack path predicted under constant amplitude loading, provided that they did not induce crack bifurcation or plastic zones with sizes comparable to the residual ligament.

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