On the Estimation of the Elastoplastic Work Needed to Initiate Crack Tearing

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ABSTRACT. The elastoplastic frontier of the plastic zones (pz) ahead of the crack fronts is obtained by incremental elastoplastic finite element three-dimensional calculations performed for cracked components with relatively high and low transversal constraints. Such detailed calculations consider all effects associated with the actual cracked component geometry and its loading conditions. From the numerical results, it is shown that, contrary to what is assumed in Fracture Mechanics estimates, the size of the plastic zones can vary significantly for a given Stress Intensity Factor. Since damage ahead of the crack front depends on the p_3 that always form ahead of crack fronts, or is at least much affected by it, this fact cannot be neglected. Indeed, it can have a major importance in practical applications, including in fatigue and in elastoplastic fracture estimates. This work proposes a methodology for evaluating the p_{z} volume based on a solid submodeling Finite Element (FE) analysis in which the smallest computation unit is the volume of influence of a plastified Gauss integration point, as opposed to considering an entire plastified FE as the smallest volume unit. Then, assuming that for tough metallic structural alloys their toughness can be obtained from the elastoplastic work needed to initiate crack tearing, the toughness is estimated from the stresses and strains inside the p_{3} . In the continuation of this work, such estimates will be compared with toughness values measured at the threshold of crack tearing in highly and lightly constrained specimens following ASTM E1820 testing procedure and an estimation procedure of J-resistance curves, respectively.

KEYWORDS. 3D finite elements; incremental elastoplastic calculations; 3D pz estimates; elastoplastic work; toughness estimates.

INTRODUCTION

The calculation of stress and strain fields around the crack tip in cracked structural components must be accurate because the crack fronts act as 3D stress intensifiers and are the primary cause for crack propagation, stable tearing and unstable fracture in them. Hence, this sort of analysis is important in engineering problems that involve fatigue and fracture analyses. A number of detailed 3D numerical studies to characterize the plastic zones (p_3) developed around crack fronts and to quantify their effects on the structural integrity of cracked mechanical components are available, see e.g. [1-3]. Such studies have shown that bilinear and trilinear elastoplastic material models provided almost identical results, when considering isotropic and kinematic hardening under monotonic loads [4]. In this work, a constitutive bilinear isotropic hardening model is used to numerically estimate the sizes and shapes of plastic zones in CT and SET specimens, as well as the elastoplastic (EP) stress/strain distributions inside them. A submodeling finite element (FE) technique [5] is adopted in the numerical calculations, using material properties taken from the literature [2]. First, the 3D EP submodeling FE technique was validated through direct comparison with recent numerical and experimental results [2, 3]. Then, the EP frontier of the p_{τ} ahead of the crack fronts and the stress/strain distributions inside them were obtained by careful incremental 3D EP submodeling FE analyses performed for some cases of cracked components with relatively high and low transversal constraints around the crack front. These constraints were varied changing the crack size and loading conditions. Geometry parameters are represented by crack length/specimen width (a/W) and specimen width/specimen thickness (W/B). Loading conditions are represented by nominal stress/yield strength (σ_{π}/S_{Y}) ratios for constant Stress Intensity Factors (SIF) K_I.

Study of the 3D plastic zone

One of the main objectives of this work is to show that, contrary to what is usually assumed in most Fracture Mechanics estimates, the size of the plastic zones can vary significantly for a given Stress Intensity Factor. Since both the material toughness and its resistance to fatigue crack growth depend on plastic work performed inside the pz, this fact can have a major importance in many engineering estimates. Therefore, several numerical FE analyses are performed, varying the parameters of the specimen models, but keeping a constant K_I SIF value. For each simulated geometry, a 3D EP global FE model was generated and meshed using more refined elements around the crack and increasing their size in regions away from the crack. Middle tension (MT), compact tension (CT) and Single Edge Tension (SET) specimens were used in this study. From the solution of the global model, the EP frontier of the pz in terms of the equivalent Mises strain was mapped in order to ensure that the total volume of the pz is entirely within the submodel with meshes of uniform element size for the final solution. The properties of the materials used in all simulations are presented in Tab. 1.

| Material | E [GPa] | ν[1] | S _Y [MPa] | H [MPa] | h[1] | H' [MPa] | H/E [1] |
|-----------|---------|------|----------------------|---------|-------|----------|---------|
| 2024-T3 | 73.1 | 0.33 | 345 | - | - | 984 | - |
| 2024-T351 | 73.5 | 0.33 | 425 | 685 | 0.073 | 220.5 | 0.003 |

Table 1: Materials and properties [2, 3].

For all the traditional specimens selected for this study, there are well-known expressions for K_I available in the literature: $K_I = P/(B\sqrt{W})f(a/W)_{Specimen}$, where $f(a/W)_{Specimen}$ is a geometry function that depends on the crack size (*a*) to specimen width (*W*) ratio. The equations 1-3 present these functions for the cracked components CT [6], MT [7] and SET [7].

$$f\left(\frac{a}{W}\right)_{CT} = \frac{\left(2 + \frac{a}{W}\right)}{\left(1 - \frac{a}{W}\right)^{3/2}} \left[0.886 + 4.64\left(\frac{a}{W}\right) - 13.32\left(\frac{a}{W}\right)^{2} + 14.72\left(\frac{a}{W}\right)^{3} - 5.6\left(\frac{a}{W}\right)^{4}\right]$$
(1)
$$f\left(\frac{a}{W}\right)_{MT} = \sqrt{\left(\frac{\pi a}{4W}\right) \sec\left(\frac{\pi a}{2W}\right)} \left[1 - 0.025\left(\frac{a}{W}\right)^{2} + 0.06\left(\frac{a}{W}\right)^{4}\right]$$
(2)

$$f\left(\frac{a}{W}\right)_{SET} = \frac{\sqrt{2\tan\left(\frac{\pi a}{2W}\right)}}{\cos\left(\frac{\pi a}{2W}\right)} \left[0.752 + 2.02\left(\frac{a}{W}\right) + 0.37\left(1 - \sin\frac{\pi a}{2W}\right)^3\right]$$
(3)

Numerical validation

To validate the FE numerical analysis, a comparison is made to two recent three-dimensional results taken from the literature [2, 3] for MT and for CT specimens. Geometry and loading of these specimens are summarized in Tab. 2. References [2, 3] used 3D SOLID185 elements with 8 nodes each, 8 Gaussian integration points per element and 3 degrees of freedom per node. In this work, the ANSYS software was used. For comparison, the same type of element of those references was adopted in the submodel that contains the pz. In the global model, since a less refined mesh is required, it uses larger 3D SOLID186 elements with 20 nodes each, 8 Gaussian integration points per element. A mesh convergence study (not shown here) was performed based on evaluations of the total volume (Vt) of the pz developed around the crack front for different mesh sizes. For a local analysis of stress and strain fields, it is recommended to apply a denser mesh at least around the crack front, as already suggested in many studies [1, 8, 9].

| Specimen | a/W[1] | <i>a</i> [mm] | W [mm] | W/B [1] | <i>P</i> [kN] | K₁ [MPa√m] | σ_n/S_Y [%] |
|----------|--------|---------------|--------|---------|---------------|------------|--------------------|
| МТ | 0.25 | 20.00 | 80.00 | 16.00 | 92.14 | 30.00 | 33.00 |
| | | | | 40.00 | 36.86 | | |
| | | | | 16.00 | 61.43 | 20.00 | 22.00 |
| | | | | 40.00 | 24.57 | | |
| СТ | 0.74 | 54.00 | 72.50 | 12.08 | 1.49 | 25.00 | 80.50 |
| | | | | 24.17 | 0.75 | | |
| | | | | 7.25 | 1.77 | 17.82 | 57.38 |

Table 2: Cases published in the literature [2, 3].

Figure 1 shows the shapes and volumes of only 1/4 of the pz on a MT specimen for $K_I = 30$ MPa \sqrt{m} for two thickness conditions. Reference [2] estimates the size of the *pz* based on the FEs that have Mises stress higher than S_Y in one of its integration points. The same criterion was adopted here and a similar value for V_t (19.96mm³) for the specimen MT with thickness B = 5mm was obtained. Figure 2 shows the numerical vertical displacements around the crack front on a CT specimen with 10mm thickness for $K_I = 17.82$ MPa \sqrt{m} at three different vertical levels, and compares these results with those obtained from reference [3].



Figure 1: 1/4 shape and volume of the *pz* developed around the crack front on MT component for $K_I = 30$ MPa \sqrt{m} : a) B = 5mm; e b) B = 2mm.

In spite of the good agreements with those references results, the criterion for selection of the smallest unit considered in the pz can be improved. The smallest unit of volume considered in the pz was the volume of the element (V_d) adopted in the modeling. In this case, even when only one Gauss integration point was plastified ($\mathcal{E}_{eq} \ge \mathcal{E}_Y$) at the elastoplastic frontier of the pz, the entire element was counted in the pz volume. This methodology adopted to calculate the volume of the pz can lead to different values when using linear elements and coarse meshes. This verification was carried out through numerical experiments performed using a simply supported beam uniformly loaded, which has an analytical solution [10]. Quadratic elements were in these FE simulations, and only the fractions of the volumes corresponding to their plastified Gauss integration points were counted as part of the pz around the crack tips. Thus, the smallest unit of volume treated in the pz models became $V_e/8$.

All numerical simulations were performed considering only 1/4 of the modeled specimens due to their symmetries, and using the following geometric and material parameters: L = 300 mm, L/B = 10, B/H = 1, E = 210 GPa, $S_Y = 300$ MPa, v = 0.3, $P_0 = 4BS_Y$, and $\rho = (P/P_0)(L/H)^2$, where P is the uniformly applied load, ρ defines the percentage of the cross section plastification and 2L, 2B and 2H are the beam length, thickness and height, respectively. Figure 3 shows the variations of the relative errors between the analytical (V_a) and numerical (V_n) plastic volumes as a function of the number of elements along B. Fig. 4 shows the analytical and numerical elastoplastic frontier of the plastic volumes considering a partial ($\rho = 0.8$) and a fully plastic cross section ($\rho = 1.05$). Comparisons between numerical analysis and analytical solutions indicate good correlations when the smallest unit of volume treated in the pz is $V_e/8$. The relative error between V_a and V_n is reduced from 35% to 6% using the improved criterion, as can be seen in Fig. 3.



Figure 2: Numerical vertical displacement on CT component for $K_I = 17.82$ MPa \sqrt{m} at the crack plane, 1.20mm and 2.90mm from the crack plane.



Figure 3: Errors between the analytical (V_a) and numerical (V_n) plastic volumes counting the entire volume of the element (V_e) and its fraction ($V_e/8$) in the *pz* volume for a partial cross section ($\rho = 0.8$).



Figure 4: Elastoplastic frontier for a partial ($\rho = 0.8$) and a fully plastic cross section ($\rho = 1.05$).

HIGHLY AND LIGHTLY CONSTRAINED SPECIMENS

To verify the proposed methodology of evaluating pz based on the volume of influence of each plastified Gauss point, such estimates should be compared with toughness values measured at the threshold of crack tearing in highly (CT) and lightly (SET) constrained specimens following ASTM E1820 testing procedure and an estimation procedure of J-resistance curves, respectively. For CT and SET specimens with $K_I = 30 \text{MPa}\sqrt{\text{m}}$, Fig. 5 shows the variation along B of the dimensionless ratio between the volume distribution of pz in slices of constant thickness and the volume of the element (V_s/V_e) , for 6 cases represented by a SET $(a/W = 0.2; \sigma_n/S_Y = 0.3)$ and a CT $(a/W = 0.4; \sigma_n/S_Y = 0.8)$, varying W/B (8, 12 and 16). Besides the cases presented in Fig. 5, there are 21 more cases as can be seen in Fig. 6, which shows that the volume of the *pz* ahead of the crack front represented by the dimensionless ratio between the total volume of pz and the volume of the element (V_t/V_e) can vary significantly for a given K_i . A remarkable difference in this ratio can be seen through a comparison between two cases: a SET specimen with geometry and loading ratios W/B = 8, a/W = 0.4, and $\sigma_n/S_Y = 0.3$ and a CT specimen with W/B = 8, a/W = 0.4, and $\sigma_n/S_Y = 0.8$. A surprisingly large factor of 9.6 was observed between them. Hence, based on damage arguments ahead of the crack front, it is possible to argue that J_{1C} measured toughness values may also dramatically change in those specimens.



Figure 5: p_{χ} volumes along the thickness.



Figure 6: Total pz volumes ahead of the crack front for several simulated cases.

CONCLUSIONS

This work presents 3D pz estimates using a 3D EP submodeling FE technique. The results obtained using this technique were validated through direct comparison with numerical results found in the literature that also contain experimental results for pz sizes and displacements measured on the surface of the specimens using Digital Image Correlation (DIC) techniques. In addition, the methodology for the calculation of pz volumes considering $V_e/8$ as the smallest unit of volume treated in the pz simulation was verified through numerical experiments validated by analytical solutions. Finally, it was observed from the results that for a given K_I the pz volumes are considerably different for several cases, which also implies different values of elastoplastic work in these volumes and, consequently, probably in their fracture toughnesses as well. In conclusion, estimation of fracture toughness from measured pz, both numerically and experimentally, is the major objective of this ongoing work, and it will be further discussed elsewhere.

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