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STRESS CONSTRAINED TOPOLOGY OPTIMIZATION VIA SEQUENTIAL SECOND ORDER CONE PROGRAMMING

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ABSTRACT

The main objective of structural design is to determine a structure that can carry the applied loads providing safety and without undergoing excessive displacements. In general, it is not clear, *a priori*, what is the most efficient shape that satisfies the above criteria requiring the smallest amount of material. In order to obtain this optimal structure the topology optimization method can be applied. It consists in finding the best material distribution that minimizes some performance measure (e.g.,the structural compliance). However, in some applications the optimal structure fails to meet the safety criteria due to stress concentration. To overcome this problem, stress constraints must be taken into account during the topology optimization process. In this work a sequential second order cone programming method is proposed to efficiently incorporate the stress constraints into the optimization problem. This method is well known in the field of limit analysis and it has shown to provide optimal solutions with very low computational cost. Numerical examples are presented here to demonstrate the efficiency and applicability of the proposed second order cone programming method.

Keywords: Stress Constraints, Topology Optimization, Second Order Cone Programming

1. INTRODUCTION

The main goal of structural engineering is design structures that are able to fulfill the safety criterion, which is not collapse under the action of the applied loads, and the serviceability condition, in other words avoid the appearance of great displacements. In addition, the structural design must fulfill these conditions spending less material as possible. Frequently, it is obvious, *a priori*, which is the most efficient design satisfying the above criteria requiring the smallest amount of material.

In order to aid the determination of such optimal design the techniques of structural optimization might be employed [1]. Among these techniques lies the topology optimization, in which the material distribution over a given domain is optimized. The most commonly used formulation for topology optimization seeks the material distribution that has at most a specified volume and renders the compliance its minimum [2, 3, 4, 5].

Although this formulation is well established and widely covered in the literature, the design provided, in some applications, fails to meet the safety criteria due to stress concentration [6]. Therefore, several attempts are being made to determine a formulation for stress constrained topology optimization [6, 7, 8, 9, 10, 11]. One of the main problems of this formulation is that in addition to the nonlinearity within the equilibrium equation, the stress constraints represent additional sources of nonlinearity to the optimization.

In this paper a sequential second-order cone programming method is proposed to efficiently incorporate the stress constraints into the optimization problem. This method is very well known in the field of limit analysis [12, 13, 14, 15, 16] and it has shown to provide optimal solutions with very low computational cost [17, 18, 19]. Numerical examples are presented here to demonstrate the efficiency and applicability of the proposed second order cone programming method.

2. SECOND-ORDER CONE PROGRAMMING

The second-order cone programming (SOCP) is a modern optimization technique which belongs to the extensive field of conic programming (CP). SOCP is capable of handling nonlinear convex problems including linear, quadratic and second-order cone constraints. Hence, linear programming (LP), convex quadratic programming (QP) and convex quadratically constrained quadratic programming (QCQP) are all particular cases of SOCP. Robust and efficient primal-dual interior points are available for SOCP, thus allowing for fast solutions of large scale-optimization problems. Furthermore, in [20] it is shown that a great variety of problems can be cast as SOCP problems, including sums of norms, problems with hyperbolic constraints and robust linear programming.

Conic programming is a subfield of convex optimization that studies a class of structured convex optimization problems called conic optimization problems. A conic optimization problem consists of minimizing a convex function over the intersection of an affine subspace and a convex cone. A general conic optimization problems may be stated as

$$\left\{ \begin{array}{ll} \min & \mathbf{c}^{\mathrm{T}}\mathbf{x} \\ \text{s.t.} & \begin{array}{l} \mathbf{A}\mathbf{x} = \mathbf{b} \\ \mathbf{x} \in \mathscr{K} \end{array} \end{array} \right. \tag{1}$$

in which, $\mathbf{x}$ are the design variables, $\mathbf{A}\mathbf{x} = \mathbf{b}$ is a set of linear constraints and $\mathscr{K}$ is the convex cone associated to the problem.

Some examples of conic programming are linear programming (LP), convex quadratic programming (Convex QP), second-order cone programming (SOCP) and the semidefinite programming (SDP). Figure 1 depicts the different subfields belonging to the conic programming.

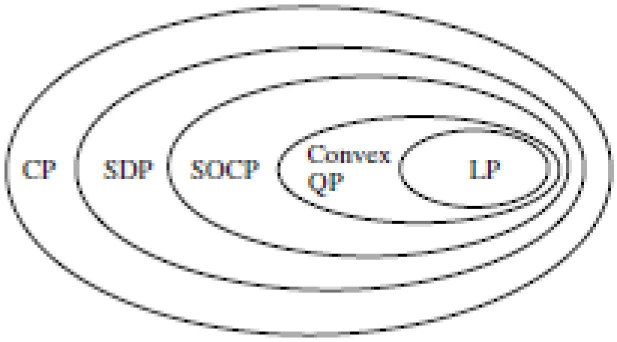

Figure 1. Conic programming subfields

Which one of these subfields is distinguished by the type of cone associated to the problem. In LP problems the associated cone is the so-called $\mathbb{R}_+$ cone, which is defined as:

$$\mathbb{R}_+ = \{x \in \mathbb{R} | x \geq 0\} \tag{2}$$

Thereby, a general LP problem may be written as:

$$\begin{cases} \min & \mathbf{c}^{\mathrm{T}}\mathbf{x} \\ \text{s.t.} & \begin{array}{l} \mathbf{A}\mathbf{x} = \mathbf{b} \\ \mathbf{x} \geq 0 \end{array} \end{cases} \quad \text{or} \quad \begin{cases} \min & \mathbf{c}^{\mathrm{T}}\mathbf{x} \\ \text{s.t.} & \begin{array}{l} \mathbf{A}\mathbf{x} = \mathbf{b} \\ \mathbf{x} \in \mathbb{R}^n_+ \end{array} \end{cases} \tag{3}$$

In SOCP problems the associated cone is called the second-order cone, also known as the ice-cream or the Lorentz cone, which is defined as

$$\mathscr{K}^q = \{\mathbf{x} \in \mathbb{R}^n | x_1 \geq \|x_{2:n}\|, x_1 \geq 0\} \tag{4}$$

Thus, a general SOCP problem may be written as

$$\begin{cases} \min & \mathbf{c}^{\mathrm{T}}\mathbf{x} \\ \text{s.t.} & \begin{array}{l} \mathbf{A}\mathbf{x} = \mathbf{b} \\ x_1 \geq \|x_{2:n}\| \\ x_1 \geq 0 \end{array} \end{cases} \quad \text{or} \quad \begin{cases} \min & \mathbf{c}^{\mathrm{T}}\mathbf{x} \\ \text{s.t.} & \begin{array}{l} \mathbf{A}\mathbf{x} = \mathbf{b} \\ \mathbf{x} \in \mathscr{K}^q \end{array} \end{cases} \tag{5}$$

Both the $\mathbb{R}_+$ and the

$$\mathscr{K}^q$$

cones belongs to a class of cones called self-scaled, see [17]. In [21] the extension of the primal-dual interior points algorithm for convex programming problems is presented. This formulation allows robust and highly efficient numerical implementations for solving SOCP. According to [20], worst-case theoretical analysis shows that the number of iterations required to solve a SOCP problem grows at most as the square root of the number of design variables, while numerical experiments indicate that the typical number of iterations ranges between 5 and 50, almost independent of the problem size. This feature allows for the solution of large scale problems with minor computation expenses. A step by step numerical implementation of the primal-dual interior-point algorithm for conic quadratic optimization is introduced in [17].

Given its numerical efficiency and stability, SOCP has been successfully applied to large scale limit analysis problems [14, 22]. SOCP may be applied to limit analysis since several yield criteria can be casted into a second-order cone constraint [12, 13, 23]. In this paper this efficient representation of the material yield criteria is coupled with the stress constrained topology optimization. Therefore, this cast of material yield criteria into second-order cone constraints is herein reviewed.

3. MATERIAL YIELD CRITERIA AS SECOND-ORDER CONES

All the most common yield criteria used in practice are amenable to be cast into semidefinite conic form, while in some particular cases the criteria may be cast into second-order cone constraints. It is presented in [23] the conic representation of several yield criteria found in the literature. Particularly, the Mohr-Coulomb, Rankine and Tresca criteria are representable as positive semidefinite cones, while the von Mises and Drucker Prager criteria may be rewritten as second-order conic constraints. For clarification purposes, the casting of the von Mises yield criterion into a second order cone is herein demonstrated.

The von Mises criterion states that yielding begins when the octahedral shearing stress reaches a critical value σ_{y}, the material yield stress. For a plane stress state the criterion is given as

$$\tau_{oct} = \sqrt{\sigma_{xx}^2 + \sigma_{yy}^2 - \sigma_{xx}\sigma_{yy} + 3\tau_{xy}^2} \leq \sigma_{\mathrm{y}} \tag{6}$$

in which, τ_{oct} is the octahedral shearing stress, σ_{xx}, σ_{yy} and τ_{xy} are the Cauchy stress tensor components.

Eq. 6 may be rewritten in matrix form as

$$\tau_{oct} = \sqrt{\boldsymbol{\sigma}^{\mathrm{T}}\mathbf{M}\boldsymbol{\sigma}} \leq k \tag{7}$$

in which,

$$\mathbf{M} := \begin{bmatrix} 1 & -0.5 & 0 \\ -0.5 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}; \quad \boldsymbol{\sigma} := \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix} \tag{8}$$

Since matrix **M** is positive definite a Cholesky factorization is proven to exists, thus Eq. 7 is restated as

$$\tau_{oct} = \sqrt{\boldsymbol{\sigma}^{\mathrm{T}}\mathbf{M}\boldsymbol{\sigma}} = \sqrt{\boldsymbol{\sigma}^{\mathrm{T}}\mathbf{L}^{\mathrm{T}}\mathbf{L}\boldsymbol{\sigma}} = \sqrt{\mathbf{y}^{\mathrm{T}}\mathbf{y}} \leq \sigma_{\mathrm{y}} \Leftrightarrow \|\mathbf{y}\| \leq \sigma_{\mathrm{y}} \tag{9}$$

in which,

$$\mathbf{y} = \mathbf{L}\boldsymbol{\sigma} \tag{10}$$

Defining **x** as

$$\mathbf{x} = \begin{bmatrix} \sigma_{\mathrm{y}} & y_1 & y_2 & y_3 \end{bmatrix} \tag{11}$$

it is straightforward to see that **x** in the four dimensional cone, as defined in Eq. 4, implies that the yield criterion is verified. Explicitly, the von Mises criterion for plane stress is restated as

$$\mathscr{C}^{VM} = \left\{ \mathbf{x} \in \mathbb{R}^4 | x_1 \geq \|x_{2:4}\|, x_1 \geq 0 \right\} \tag{12}$$

in which, the second-order cone $\mathscr{C}^{VM}$ is known as the von Mises cone.

4. STRESS CONSTRAINED TOPOLOGY OPTIMIZATION

Density-based topology optimization with stress constraints focuses in determining the best material distribution that minimizes the weight of a structure while satisfying given yield criteria. In order to enforce the stress constraints, the most common approach employed in the literature is the so-called direct method. This approach relies on limiting the stresses on given points of each finite element, possibly the integration points. However, density-based topology optimization with stress constraints usually relies on a fine discretization of the domain, thus leading to a great number of design variables as well as a large number of stress constraints. In order to solve such a large-scale optimization problem, in [24] the authors propose the use of the Augmented Lagrangian method, while in [7] a mathematical programming method developed by the authors themselves is proposed.

Other author propose methods that seek to reduce the number of constraints by using the so-called aggregating functions, such as the P-norm [6], and the KS-function [25], thus making the problem amenable to be solved using general non-linear algorithms. However, these ad hoc methods can lead to poor control of the local behavior of the stresses, thus impairing the solution of the problem. In the present work a procedure based on the solution of a sequence of SOCP subproblems, in which the stress constraints are rewritten as second-order cones, is proposed. It is noteworthy that, since SOCP is able to solve large scale problem efficiently, this procedure is particularly suitable when the local method is employed. In the following the step-by-step of the procedure is introduced.

The topology optimization problem addressed in this paper is stated as find the density field which renders the minimum volume such that the yield criterion is met for the elastic stresses in all stress control points, e.g. gauss points. In other words,

$$\begin{cases} \min\limits_{\rho} & \sum\limits_{i=1}^{n} \rho_i V_i \\ \text{with} & \mathbf{K}(\rho)\mathbf{u} = \mathbf{f} \\ \text{s.t.} & \begin{array}{l} f_{\mathrm{y}}\left(\sigma_j, \sigma_{\mathrm{y}}\right) \leq 0 \ \forall j = 1...m \\ 0 < \rho_i \leq 1 \ \forall i = 1...n \end{array} \end{cases} \tag{13}$$

in which, ρ_i and V_i are, respectively, the density and the volume of the ith element, $\mathbf{K}(\rho)$ is the global stiffness matrix, $\mathbf{u}$ is the displacement used to calculate the stresses, $\mathbf{f}$ is the load vector, $f_{\mathrm{y}}\left(\sigma_j, \sigma_{\mathrm{y}}\right)$ is an implicit function representing a given yield criterion surface, σ_j is a vector containing the stresses of the jth stress control point and σ_{y} is the material yield stress.

For the particular case of the von Mises yield criterion the problem may be written as

$$\begin{cases} \min\limits_{\rho} & \sum\limits_{i=1}^{n} \rho_i V_i \\ \text{with} & \mathbf{K}(\rho)\mathbf{u} = \mathbf{f} \\ \text{s.t.} & \begin{array}{l} \sqrt{\sigma_j{}^{\tau}\mathbf{M}\sigma_j} - \sigma_{\mathrm{y}} \leq 0 \ \forall j = 1...n \\ 0 < \rho_i \leq 1 \ \forall i = 1...n \end{array} \end{cases} \tag{14}$$

Taking the first order Taylor's series expansion of stresses with respect to the design densities:

$$\sigma_i(\rho_0 + \Delta\rho) \approx \sigma_i(\rho_0) + \nabla\sigma_i(\rho_0)\Delta\rho \tag{15}$$

in which, ρ_0 is the material distribution of the current iteration.

Substituting the approximation of equation 15 into the stress constraints of equation 14 gives

$$\sqrt{(\sigma_i(\rho_0) + \nabla\sigma_i(\rho_0)\Delta\rho)^{\mathrm{T}}\mathbf{L}^{\mathrm{T}}\mathbf{L}(\sigma_i(\rho_0) + \nabla\sigma_i(\rho_0)\Delta\rho)} - \sigma_{\mathrm{y}} \leq 0 \tag{16}$$

Thereby, employing the aforementioned casting of the von Mises stresses into second-order conic constraints, the solution to the problem in Eq. 13 may be found by subsequently solving the following SOCP subproblem

$$\begin{cases} \min\limits_{\Delta\rho, \mathbf{y}, k} & \sum\limits_{i=1}^{n} (\Delta\rho_i + \rho_{0i}) V_i \\ with & \mathbf{K}\mathbf{u} = \mathbf{f} \\ s.t. & \begin{array}{l} \mathbf{y}_j = \mathbf{L}\left(\sigma_j(\rho_0) + \nabla\sigma_j(\rho_0)\Delta\rho\right) \ \forall j = 1...n \\ \left\|\mathbf{y}_j\right\| \leq \sigma_{\mathrm{y}} \ \forall j = 1...n \\ -\rho_{0i} < \Delta\rho_i \leq 1 - \rho_{0i} \ \forall i = 1...n \end{array} \end{cases} \tag{17}$$

From a practical point of view, each subproblem seeks to approximate the real problem through a quadratic conic model which in turn must be solved in order to determine a step of the optimization. In general, this class of sequential optimization algorithms can be implemented using either the linear search or the trust-region method [26]. The line search method requires a great number of evaluations of both the objective function and the constraints of the problem. Thus, since each evaluation of the topology optimization problem requires a complete structural analysis, this method leads to an expensive computational alternative. On the other hand, the trust-region requires only one step of the structural analysis per iteration, thus alleviating much of the computational effort involved in each optimization step.

The trust-region method can be naturally coupled to subproblem of Eq. 17 by simply introducing the following second-order cone constraint

$$\|\Delta\rho\| \leq r \tag{18}$$

in which, r is the trust-region radius.

The constraint (1.18) restrains the subproblem solution to the ball of radius r. Hence, a key ingredient of the method is the choice of the radius for the trust-region per subproblem. In general, this choice is based on the agreement between the real problem and the quadratic conic model adopted. In the present work, the model is based on the approximation of the von Mises stresses per iteration of the real problem, thus the agreement is given by the following rate:

$$\varepsilon := \left\| \frac{\sqrt{\sigma_j(\rho_0+\Delta\rho)^{\tau}\mathbf{M}\,\sigma_j\,(\rho_0+\Delta\rho)} - \sqrt{\sigma_j(\rho_0)^{\tau}\mathbf{M}\,\sigma_j\,(\rho_0)}}{\|\mathbf{y}_j\| - \sqrt{\sigma_j(\rho_0)^{\tau}\mathbf{M}\,\sigma_j\,(\rho_0)}} \right\|_{\infty} \tag{19}$$

Based on this agreement rate a simple algorithm [26] may be employed to adjust the trust-region radius and also to accept or reject the step in each iteration of the optimization procedure.

5. RESULTS

In order to attest the correctness of the proposed approach some numerical examples were run and the results are presented hereafter. The sequential approach was implemented in MATLAB and the subproblems were solved using MOSEK.

5.1 The MBB-beam

The MBB-beam is a benchmark example in topology optimization. It consists of a simply supported beam with a mid-span applied load. It is shown in Figure 2 the model, considering symmetry, used in the analysis. The adopted material has YoungâĂŹs modulus 71 GPa, PoissonâĂŹs ratio 0.33 and yield limit 350MPa; the applied load is 1,500 kN; the plate thickness is 1 mm; and the geometry of the plate is given by L = 100 mm.

The obtained results are shown in Figure 3 for a mesh with 300 elements. In Figure 4 it is shown the corresponding penalized stress field. In order to verify that the proposed optimization scheme is mesh independent the problem is analyzed with a more refined mesh. The optimum topology for a finer mesh of 1200 elements is presented in Figure 5. Furthermore, the penalized stress field of this finer mesh is illustrated in Figure 6.

It may be observed that the results present a qualitative resemblance of topology. Therefore, the optimization methodology presented âĂŞ sequential second-order cone programming approach âĂŞ has proved itself to be a stable numerical procedure for the stress constrained topology optimization.

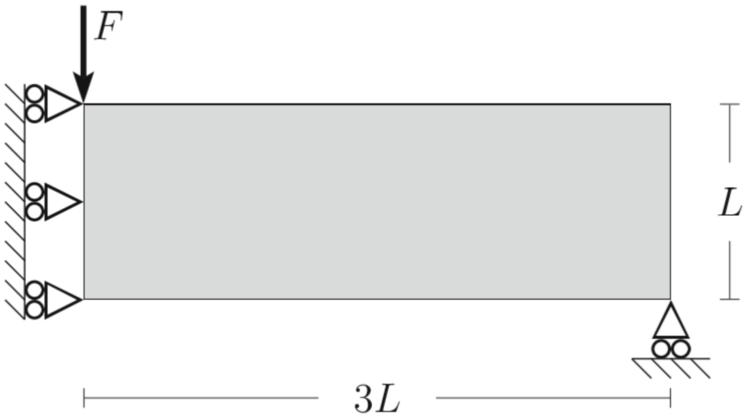

Figure 2. Geometry of the MBB problem [6]

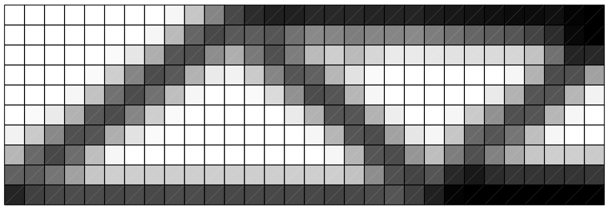

Figure 3. Optimum topology for a 300 elements mesh

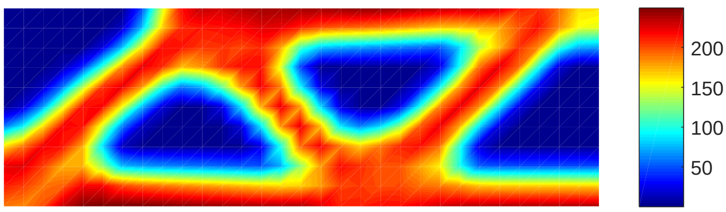

Figure 4. Penalized stress field for a 300 elements mesh

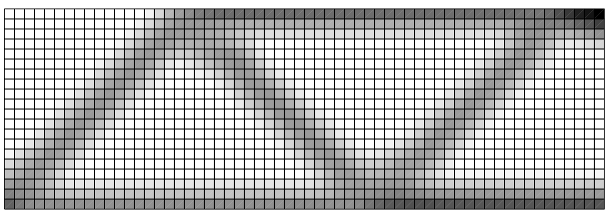

Figure 5. Optimum topology for a 1200 elements mesh

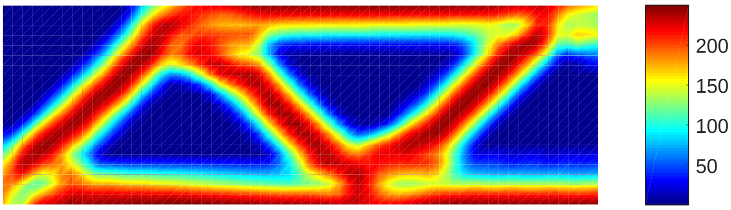

Figure 6. Penalized stress field for a 1200 elements mesh

6. CONCLUSIONS

The second-order cone programming approach has successfully allowed an aggregation free formulation for stress constrained topology optimization. Therefore, this formulation provides more accurate results, since the stress constraints are met everywhere in the structural domain.

However providing quality results, the computational efficient were below expected for practical problems. It failed the expectation that the optimization would be as fast as it is in limit analysis problems. Nevertheless, it is hoped that this results can be bettered using a warm-start strategy based on the solution of the previous subproblemâĂŹs solution. In addition, it was observed that the matrix concerning the variables in limit analysis is sparser than the one in the elastic formulation, which might have hold back the solver performance.

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