Reliability-Based Design Optimization of Space Trusses

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Motivation

This paper deals with uncertainties and optimization in the design of space trusses. It considers the geometric nonlinear behaviour of the structure.

- **Uncertainties** are inevitable due to the random nature of the material properties and loads.

- **Optimization** is necessary to obtain an economical design.

RBDO

DDO

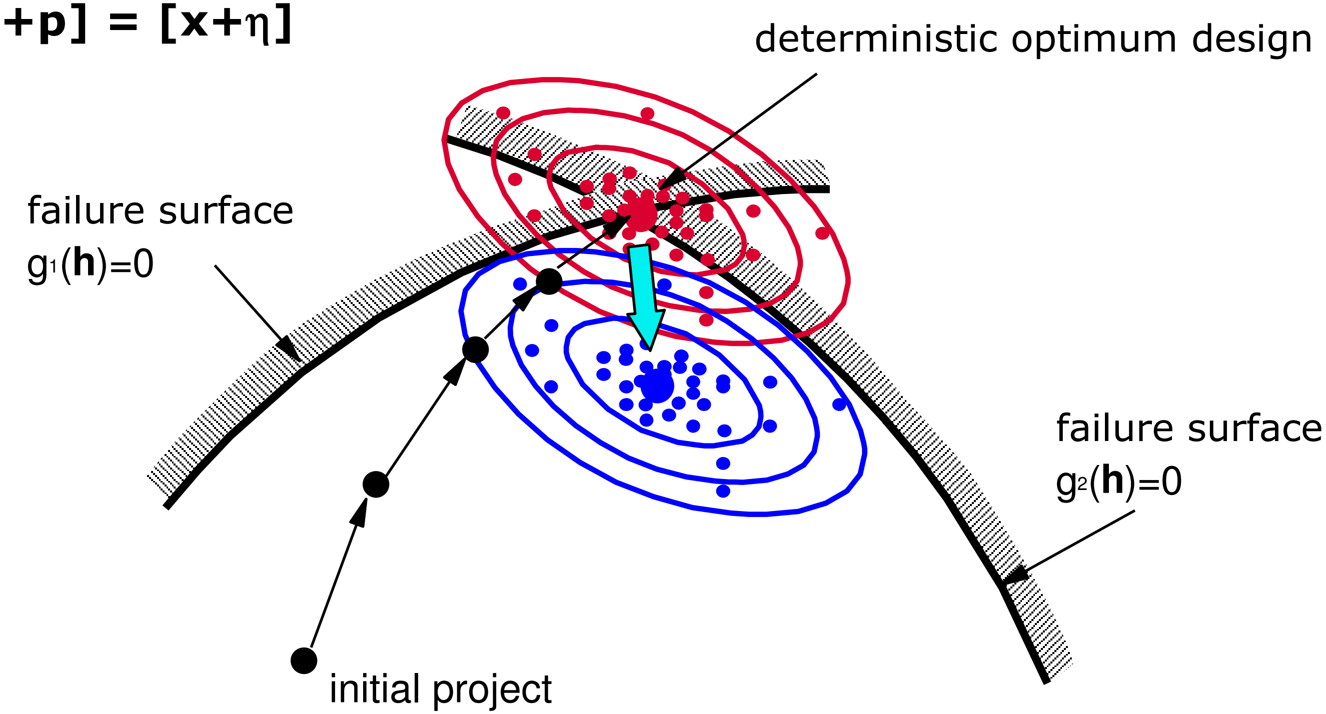
- min f(**h**,**p**)
- g_i (**h**,**p**) > 0

RBDO

- min f(**h**,**p**)
- P_t - P(g_i (**x**,η) < 0) >0

h - design variables (areas and/or coordinates) in the optimization problem
p - parameters in the optimization problem
x - random variables in the reliability analysis
η - deterministic parameters in the reliability analysis

[h+p] = [x+η]



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Reliability Analysis

Methods for the structural reliability analysis allows for the determination of the probability of failure (pf) of a given structure.

$$pf = \int_{g(\mathbf{x},\boldsymbol{\eta})\leq 0} f_{\mathbf{x}}(\mathbf{x})d\mathbf{x}$$

f (x) - multi-variable probability density function

$\mathbf{x}$ - random variables

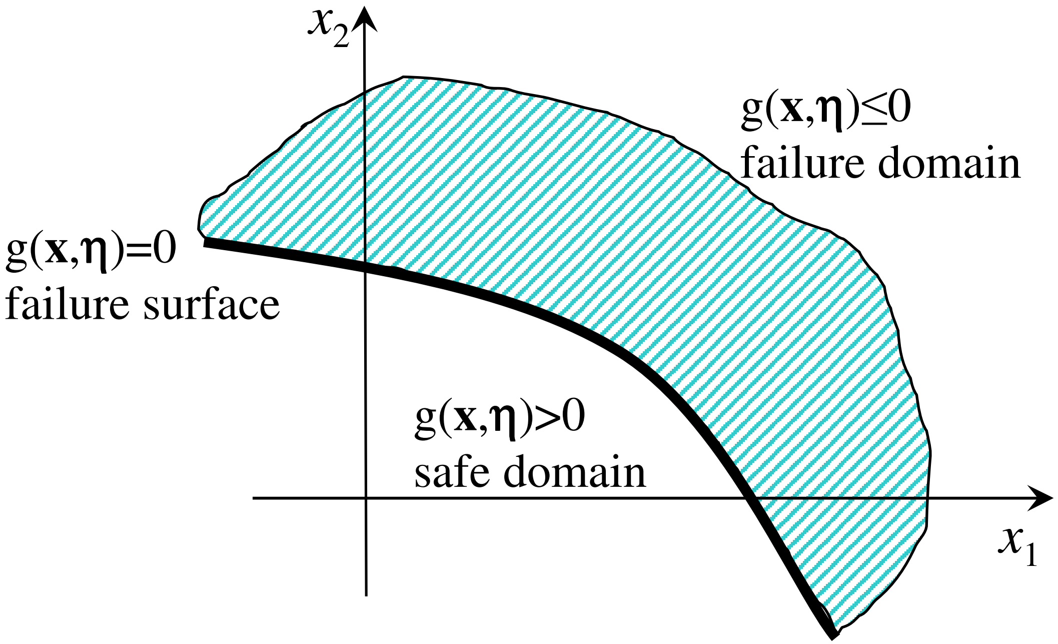
η - determiníistic parameters

$g(\mathbf{x},\eta) \leq 0$ - failure domain

$\beta=\Phi^{-1}[1\text{-}pf]$ - reliability index

Reliability Analysis (cont.)

Performance Function



$$g(\mathbf{x}, \boldsymbol{\eta}) = \delta_{max} - \delta(\mathbf{x}, \boldsymbol{\eta})$$

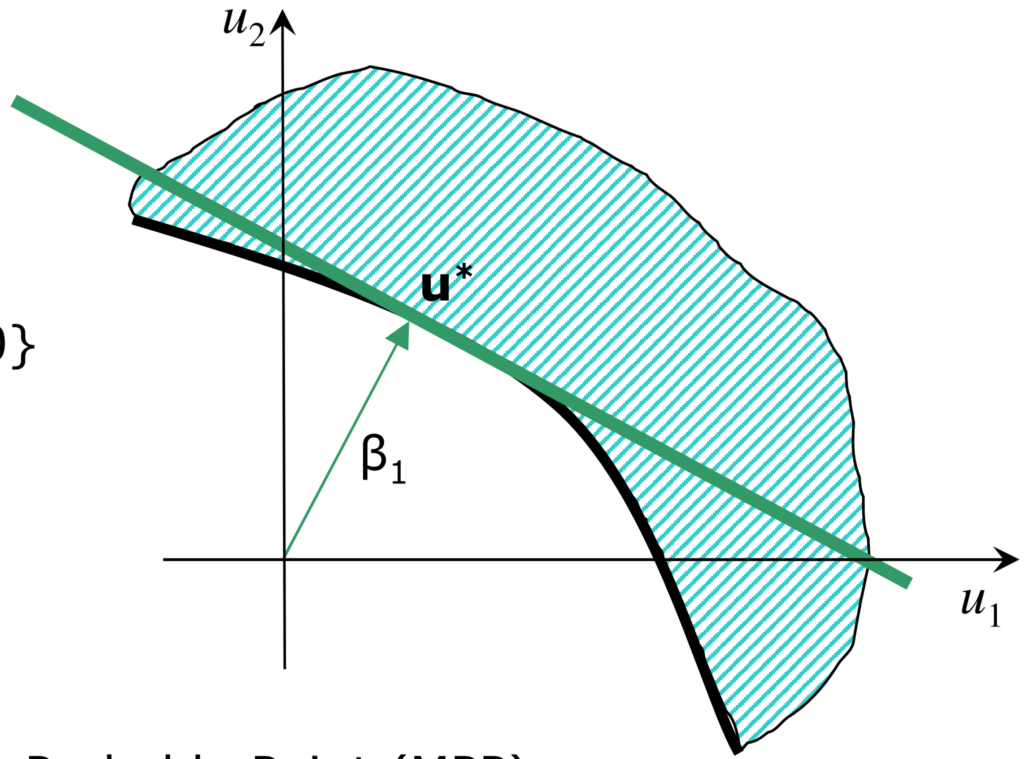
Transformation from the original to the normal standard space of the random variables

This transformation is performed using the Nataf Transformation.

Reliability Analysis

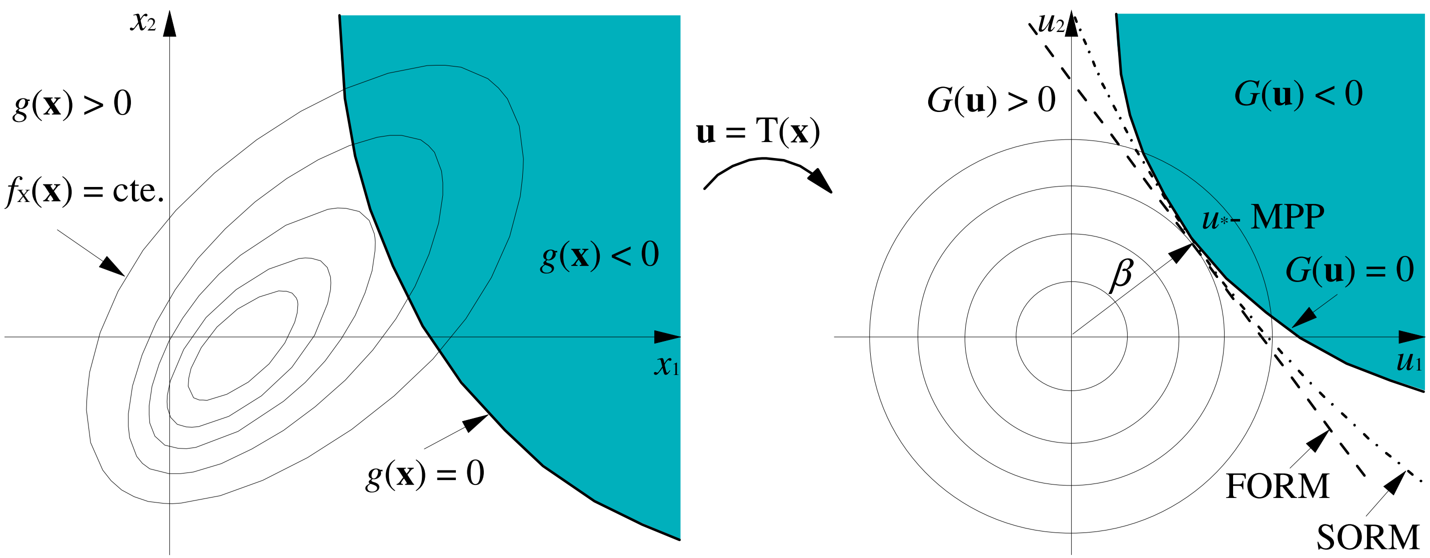
Analitycal Method

FORM:
$\beta \cong \beta_1 = \min \{ ||\mathbf{u}|| \; , \; g(\mathbf{u}, \eta) = 0 \}$



$\mathbf{u}^*$ - Most Probable Point (MPP)

FORM/SORM



original space - x

standard normal space - u

RBDO

Deterministic Design optimization (DDO)

$$
\begin{array}{lll}
\textit{Minimize} & f(\mathbf{h},\mathbf{p}) & \\
\textit{subjecto to} & g_i^D(\mathbf{h},\mathbf{p}) \geq 0 & i = 1...nr \\
 & \mathbf{h}^l \leq \mathbf{h} \leq \mathbf{h}^u & i = 1...n
\end{array}
$$

Reliability-Based Design Optimization (RBDO)

$$
\begin{array}{lll}
\textit{Minimize} & f(\mathbf{h},\mathbf{p}) & \\
\textit{subject to} & g_i^R(\mathbf{x},\boldsymbol{\eta}) \geq 0 & i = 1...nr \\
 & \mathbf{h}^l \leq \mathbf{h} \leq \mathbf{h}^u & i = 1...n
\end{array}
$$

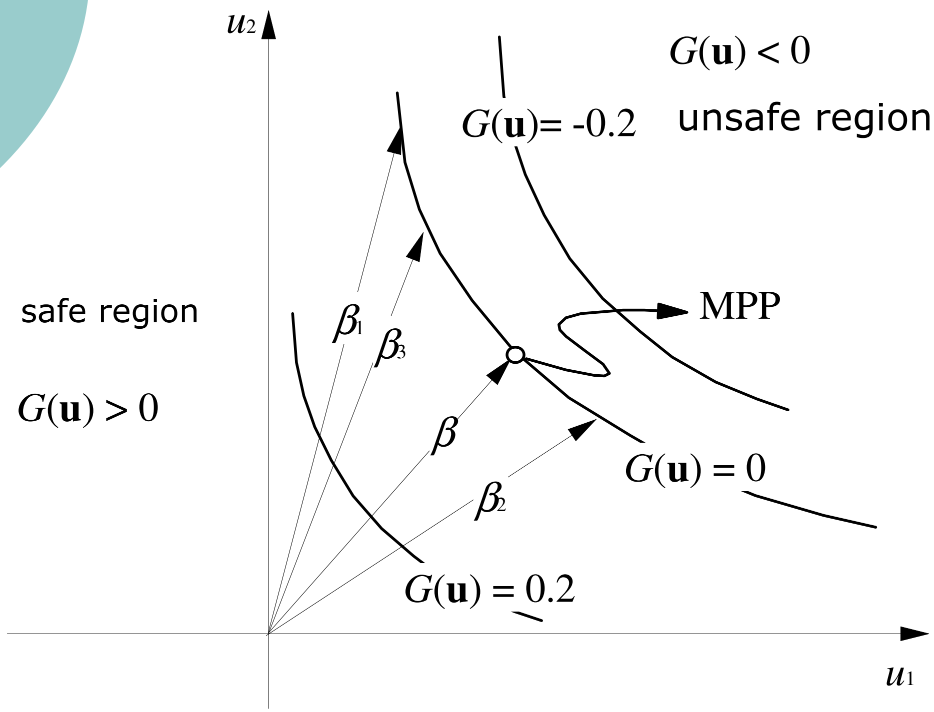
$$
\left.
\begin{array}{ll}
g_i^R = P_{t_i} - P_i, & i = 1...nr \\
 & \\
g_i^R = \beta_i - \beta_{t_i}, & i = 1...nr
\end{array}
\right\} \underline{\text{RIA}}
$$

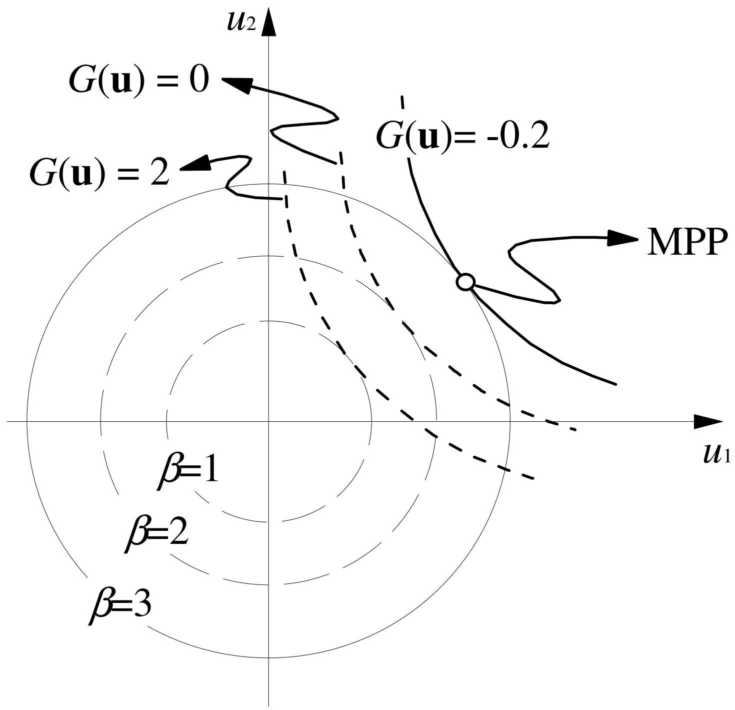
$$
\left.
g_i^R = G_i^D(\mathbf{u}^*_{i\,\beta=\beta_t}, \boldsymbol{\eta}), \qquad i = 1...nr
\right\} \underline{\text{PMA}}
$$

MPP Calculation

RIA minimize $\|\mathbf{u}\|$
subjecto to $G(\mathbf{u}, \boldsymbol{\eta}) = 0$

PMA minimize $G_i^D(\mathbf{u}, \boldsymbol{\eta})$
subject to $\|\mathbf{u}\| = \beta_{t_i}$





Search for the point on G(u) = 0 closest to the origin

Search the smallest value of G for a given target radius β

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Algorithms for Searching the MPP

RIA

Minimize $\|\mathbf{u}\|$
Subject to $G_i(\mathbf{u}, \eta) = 0$

HLRF
iHLRF
SQP
IP
…

PMA

Minimize $G_i(\mathbf{u}, \eta)$
Subject to $\|\mathbf{u}\| = \beta_{t_i}$

SQP
IP
…

RIA – Reliability Index Algorithm

PMA – Performance Measure Algorithm

Minimize $\|\mathbf{u}\|$
Subject to $G(\mathbf{u}) \leq 0$

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4. RBDO – Sensitivities

the sensitivities are performed analytically both for the PMA and RIA formulations

PMA $(\mathbf{u}_i^* = \mathbf{u}_{i\,\beta=\beta_{t_i}}^*)$

$$g_i^R = G_i^D(\mathbf{u}_{i\,\beta=\beta_t}^*, \boldsymbol{\eta}), \qquad i = 1 \dots nr$$

$$\frac{\mathrm{d}g_i^R(\mathbf{h},\mathbf{p})}{\mathrm{dh}} = \frac{\mathrm{d}g_i^P(\mathbf{h},\mathbf{p})}{\mathrm{dh}} = \boxed{\frac{\mathrm{d}G_i(\mathbf{u}_i^*,\boldsymbol{\eta})}{\mathrm{dh}}} \left\{ \begin{aligned} \frac{\mathrm{d}G_i(\mathbf{u}_i^*,\boldsymbol{\eta})}{\mathrm{dh}} &= \frac{\mathrm{d}G_i(\mathbf{u}_i^*,\boldsymbol{\eta})}{\mathrm{d}\boldsymbol{\eta}} \\ \frac{\mathrm{d}G_i(\mathbf{u}_i^*,\boldsymbol{\eta})}{\mathrm{dh}} &= \nabla_{\mathbf{u}} G_i(\mathbf{u}_i^*,\boldsymbol{\eta}) \frac{\partial T(\mathbf{x}_i^*,\boldsymbol{\theta})}{\partial \boldsymbol{\theta}} \end{aligned} \right.$$

RIA $(\mathbf{u}_i^* = \mathbf{u}_{i\,G(\mathbf{u})=0}^*)$

$$g_i^R = \beta_i - \beta_{t_i}, \qquad i = 1 \dots nr$$

$$\frac{\mathrm{d}g_i^R(\mathbf{h},\mathbf{p})}{\mathrm{dh}} = \boxed{\frac{\mathrm{d}\beta_i(\mathbf{h},\mathbf{p})}{\mathrm{dh}}} = \frac{1}{\|\nabla_{\mathbf{u}} G_i(\mathbf{u}_i^*,\boldsymbol{\eta})\|} \boxed{\frac{\mathrm{d}G_i(\mathbf{u}_i^*,\boldsymbol{\eta})}{\mathrm{dh}}}$$

$$g_i^R = P_{t_i} - P_i, \qquad i = 1 \dots nr$$

$$\frac{\mathrm{d}P_{f\,i}(\mathbf{x},\boldsymbol{\eta})}{\mathrm{dh}} = -\varphi\left(-\beta_i(\mathbf{h},\mathbf{p})\right) \boxed{\frac{\mathrm{d}\beta_i(\mathbf{h},\mathbf{p})}{\mathrm{dh}}}$$

RBDO of Space Trusses

Minimize	$f(\mathbf{h}, \mathbf{p})$	$\mathbf{h} \in \Re^n$
subject to	$g_i^R(\mathbf{h}, \mathbf{p}) \geq 0$	$i = 1 \ldots l$
	$h_i^l \leq h_i \leq h_i^u$	$i = 1 \ldots n$

$$f(\mathbf{h}, \mathbf{p}) = V(\mathbf{h}, \mathbf{p}) = \sum_{i=1}^{ne} A_i(\mathbf{h}, \mathbf{p}) l_i(\mathbf{h}, \mathbf{p})$$

$$\left.\begin{aligned} & g_i^R(\mathbf{x}, \boldsymbol{\eta}) = P_{t_i} - P_{f_i}(\mathbf{x}, \boldsymbol{\eta}), \qquad i = 1 \ldots nr \\ & \beta_{t_i} = -\Phi^{-1}\left(P_{t_i}\right) \\ & g_i^R(\mathbf{x}, \boldsymbol{\eta}) = \beta_i(\mathbf{x}, \boldsymbol{\eta}) - \beta_{t_i}, \qquad i = 1 \ldots nr \end{aligned}\right\} \text{RIA}$$

$$\left.g_i^R(\mathbf{x}, \boldsymbol{\eta}) = g_i^P(\mathbf{x}, \boldsymbol{\eta}) = G_i(\mathbf{u}^*_{i\,\beta=\beta_{t_i}}, \boldsymbol{\eta}),\right\} \text{PMA}$$

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RBDO of Space Trusses

Performance Functions

- maximum allowed displacement

$$g(\mathrm{h},\mathrm{p}) = 1 - \frac{|q_i(\mathrm{h},\mathrm{p})|}{q^a}$$

- maximum allowed yield / critical stress

yield stress

$$g(\mathrm{h},\mathrm{p}) = 1 - \frac{|\sigma_i(\mathrm{h},\mathrm{p})|}{\sigma^a} \quad \left\{ \begin{array}{l} \sigma^a = \min[\sigma^0, \sigma^{cr}] \\ \sigma^{cr} = \dfrac{\pi^2 E I_{min}}{Al} \end{array} \right.$$

Euler critical stress

- global critical load

$$g(\mathrm{h},\mathrm{p}) = \lambda_{cr}(\mathrm{h},\mathrm{p}) - \lambda^a$$

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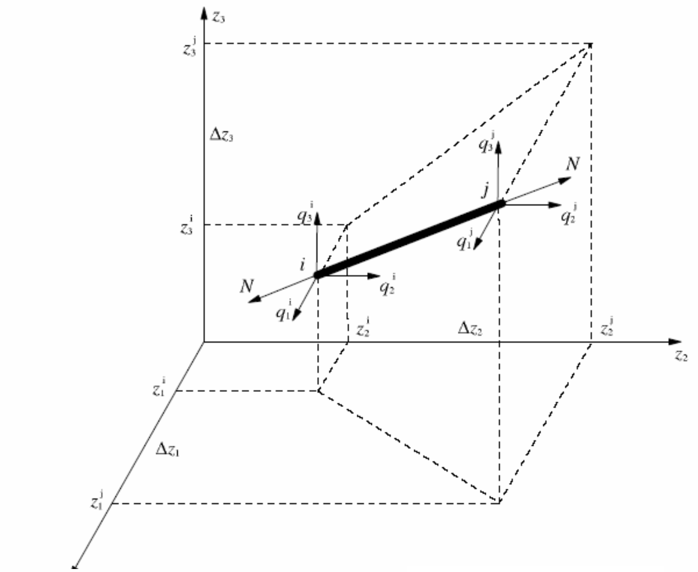
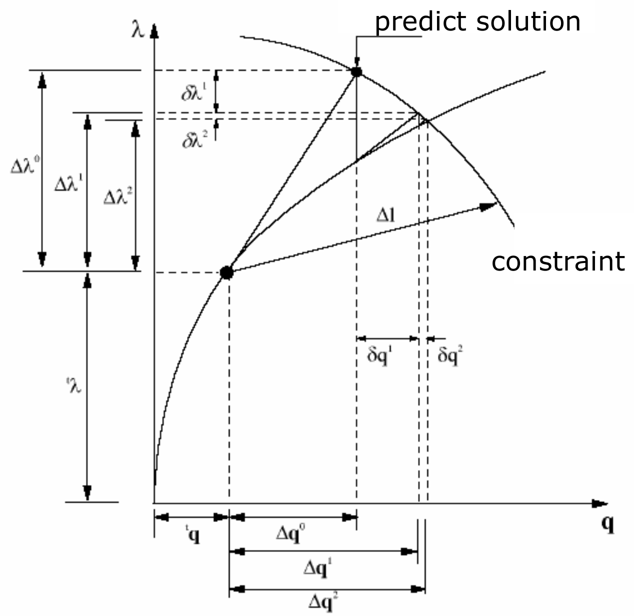
Structural Analysis

$$\mathbf{FEA}\begin{cases}\boxed{\begin{matrix}q_i(\mathbf{h},\mathbf{p})\\[4pt] \sigma_i(\mathbf{h},\mathbf{p})\end{matrix}}\\[8pt] \lambda_{cr}(\mathbf{h},\mathbf{p})\end{cases}$$



- Referencial – Total Lagrangian
- incremental-iterative scheme



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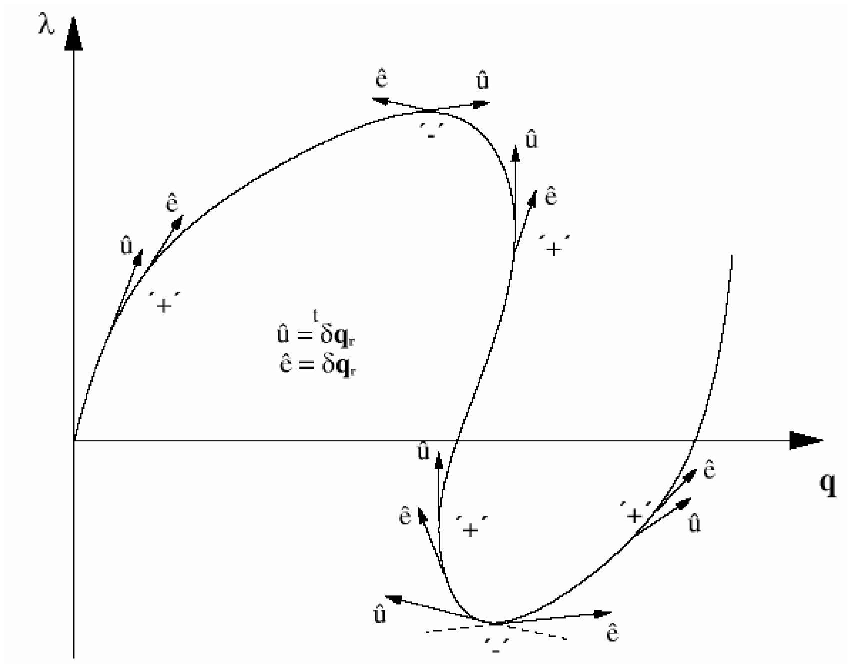
Structural Analysis

$$\mathbf{FEA}\begin{cases} q_i(\mathbf{h},\mathbf{p}) \\ \sigma_i(\mathbf{h},\mathbf{p}) \\ \boxed{\lambda_{cr}(\mathbf{h},\mathbf{p})} \end{cases}$$

- Indirect method (GSP)



Examples of RBDO

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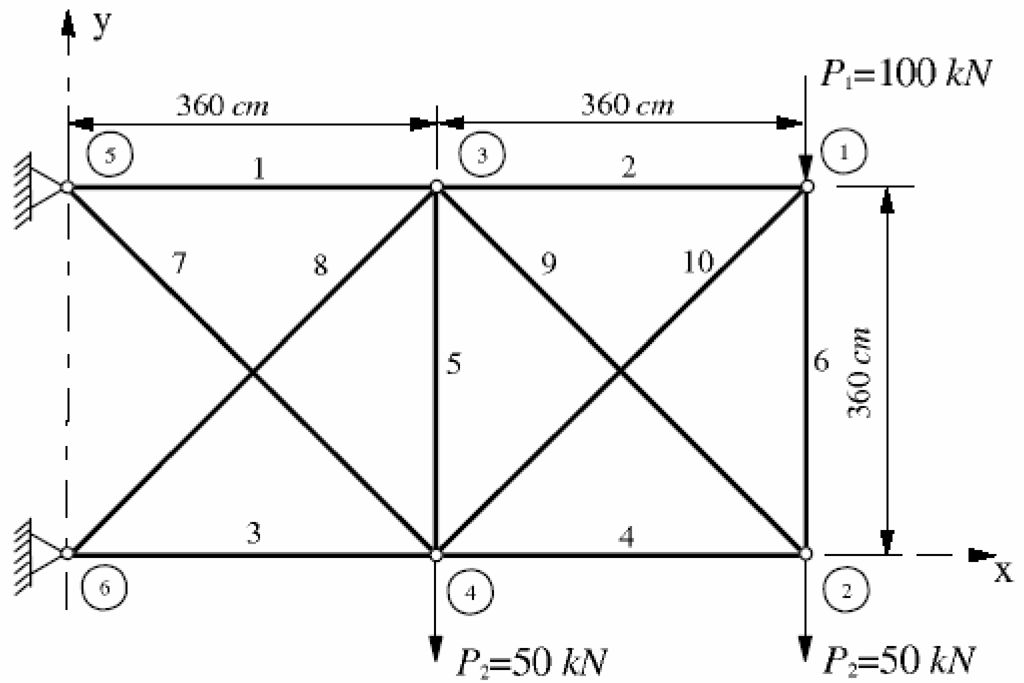
Example 1- 10 bar Plane Truss



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RIO

10 bar Plane Truss (cont.)

X	Description	Type	Momen.	Inital value	h	Pres. work	Stocki et. al [73]
X_1	Cross-section area - el. 1 constant δ (5%)	LN	$E(X_1)$	20.00	h_1	29.22	28.85
			$\sigma(X_1)$	1.0			
X_2	Cross-section area - el. 2 constant δ (5%)	LN	$E(X_2)$	20.00	h_2	13.04	18.57
			$\sigma(X_2)$	1.0			
X_3	Cross-section area - el. 3 constant δ (5%)	LN	$E(X_3)$	20.00	h_3	40.34	40.21
			$\sigma(X_3)$	1.0			
X_4	Cross-section area - el. 4 constant δ (5%)	LN	$E(X_4)$	20.00	h_4	25.30	16.40
			$\sigma(X_4)$	1.0			
X_5	Cross-section area - el. 5 constant δ (5%)	LN	$E(X_5)$	20.00	h_5	5.09	5.09
			$\sigma(X_5)$	1.0			
X_6	Cross-section area - el. 6 constant δ (5%)	LN	$E(X_6)$	20.00	h_6	5.09	5.09
			$\sigma(X_6)$	1.0			
X_7	Cross-section area - el. 7 constant δ (5%)	LN	$E(X_7)$	20.00	h_7	5.09	6.03
			$\sigma(X_7)$	1.0			
X_8	Cross-section area - el. 8 constant δ (5%)	LN	$E(X_8)$	20.00	h_8	24.22	24.60
			$\sigma(X_8)$	1.0			
X_9	Cross-section area - el. 9 constant δ (5%)	LN	$E(X_9)$	20.00	h_9	10.70	5.75
			$\sigma(X_9)$	1.0			
X_{10}	Cross-section area - el. 10 constant δ (5%)	LN	$E(X_{10})$	20.00	h_{10}	30.41	35.53
			$\sigma(X_{10})$	1.00			
X_{11}	Young modulus for all elements	LN	$E(X_{11})$	21000.0			
			$\sigma(X_{11})$	1050.0			
X_{12}	Yield stress for all elements	LN	$E(X_{12})$	21.0			
			$\sigma(X_{12})$	1.0			
X_{13}	x coordinate - node 1	N	$E(X_{13})$	720.0			
			$\sigma(X_{13})$	2.0			
X_{14}	y coordinate - node 1	N	$E(X_{14})$	360.0	h_{11}	150.0	150.0
			$\sigma(X_{14})$	2.0			
…			…			…	
X_{19}	x coordinate - node 4	N	$E(X_{19})$	360.0			
			$\sigma(X_{19})$	2.0			
X_{20}	y coordinate - node 4	N	$E(X_{20})$	0.0	h_{14}	22.11	35.3
			$\sigma(X_{20})$	2.0			
X_{21}	Nodal load P_1	GU	$E(X_{21})$	100.0			
			$\sigma(X_{21})$	20.0			
X_{22}	Nodal load P_2	LN	$E(X_{22})$	50.0			
			$\sigma(X_{22})$	2.5			

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10 bar Plane Truss (cont.)

Minimize $V(\mathbf{h}, \mathbf{p})$

subject to

$$\begin{array}{ll} \beta_i(\mathbf{x}, \boldsymbol{\eta}) \geq 3.7 & i = 1, \ldots, 11 \\ 5.09 \text{ cm}^2 \leq h_k \leq 76.4 \text{ cm}^2 & k = 1, \ldots, 10 \\ 150.0 \text{ cm} \leq h_k \leq 450.0 \text{ cm} & k = 11, 13 \\ -100.0 \text{ cm} \leq h_k \leq 100.0 \text{ cm} & k = 12, 14 \end{array}$$

vertical displacement of node $2\ (q_2^2)$

$$g_1(\mathbf{q}(\mathbf{h}, \mathbf{p})) = 1 - \frac{|q_2^2(\mathbf{h}, \mathbf{p})|}{q^a}$$

where $q^a = 3.5$ cm.

maximum allowed yield / critical stress

$$g_i(\mathbf{h}, \mathbf{p}) = 1 - \frac{|\sigma_{i-1}(\mathbf{h}, \mathbf{p})|}{\sigma_{i-1}^a(\mathbf{p})} \quad i = 2, \ldots, 11,$$

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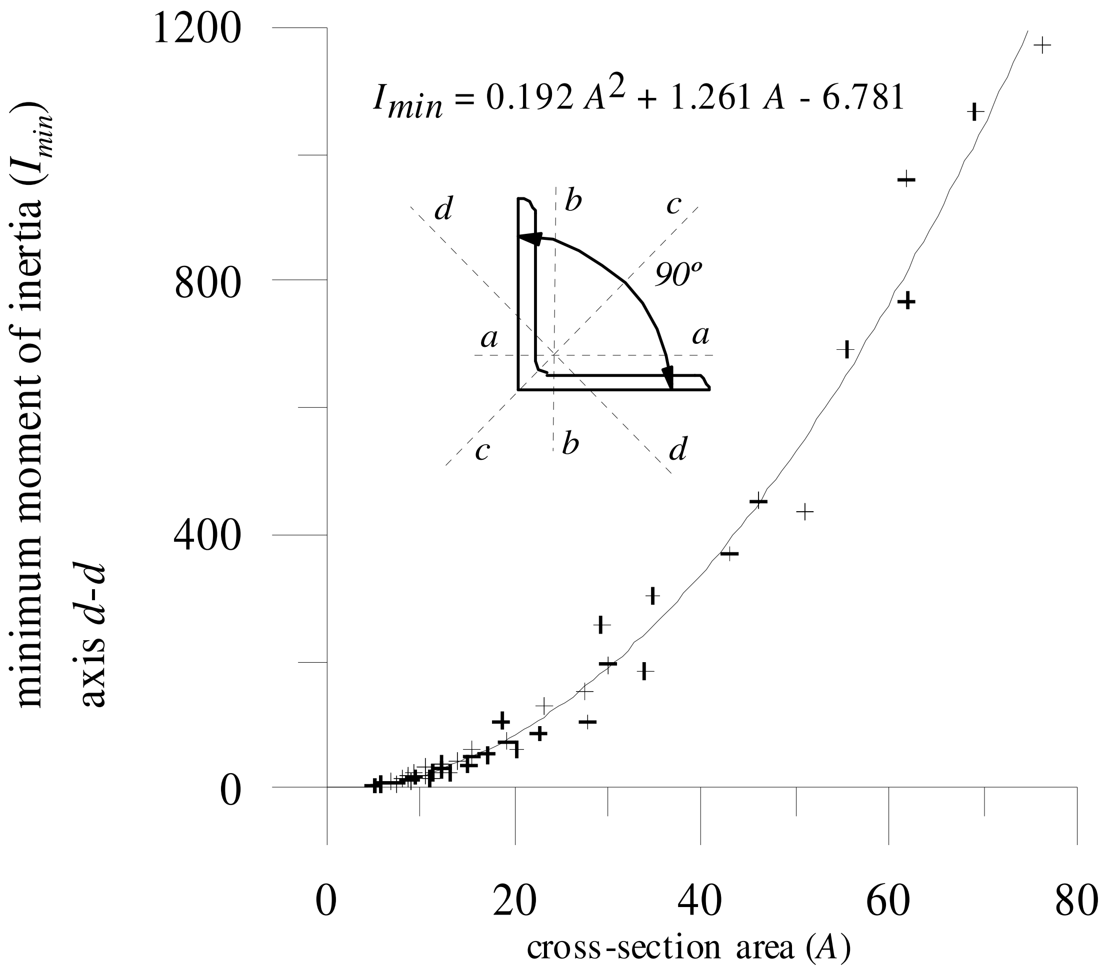
10 bar Plane Truss (cont.)



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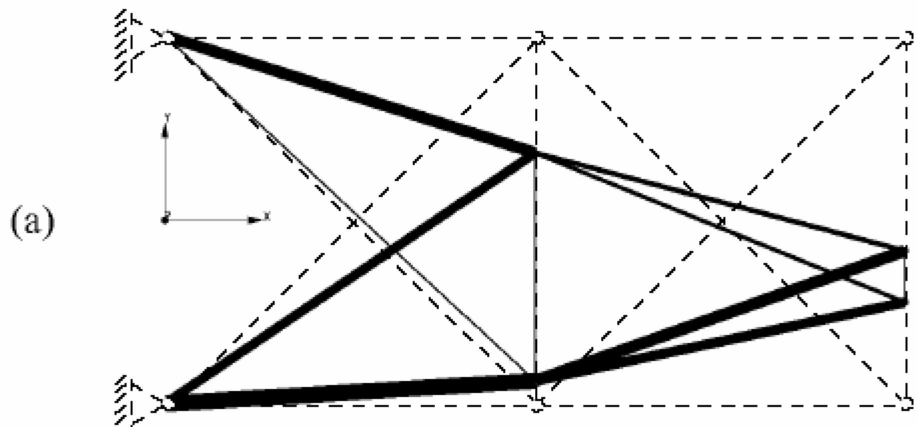
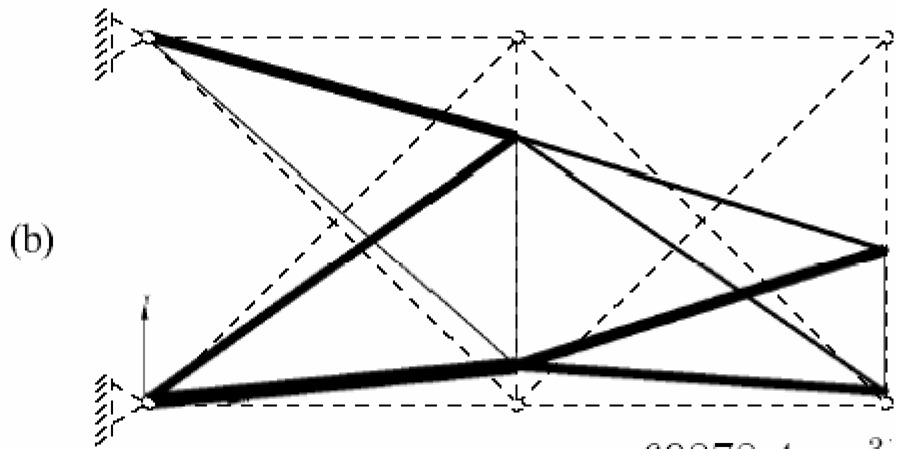
10 bar Plane Truss (cont.)



69988.61 cm^3



69878.4 cm^3

70330.13 cm^3

h	pres. work	Stocki et. al [73]
h_1	29.22	28.85
h_2	13.04	18.57
h_3	40.34	40.21
h_4	25.30	16.40
h_5	5.09	5.09
h_6	5.09	5.09
h_7	5.09	6.03
h_8	24.22	24.60
h_9	10.70	5.75
h_{10}	30.41	35.53
h_{11}	150.0	150.0
h_{12}	98.74	10.1
h_{13}	245.83	259.9
h_{14}	22.11	35.3

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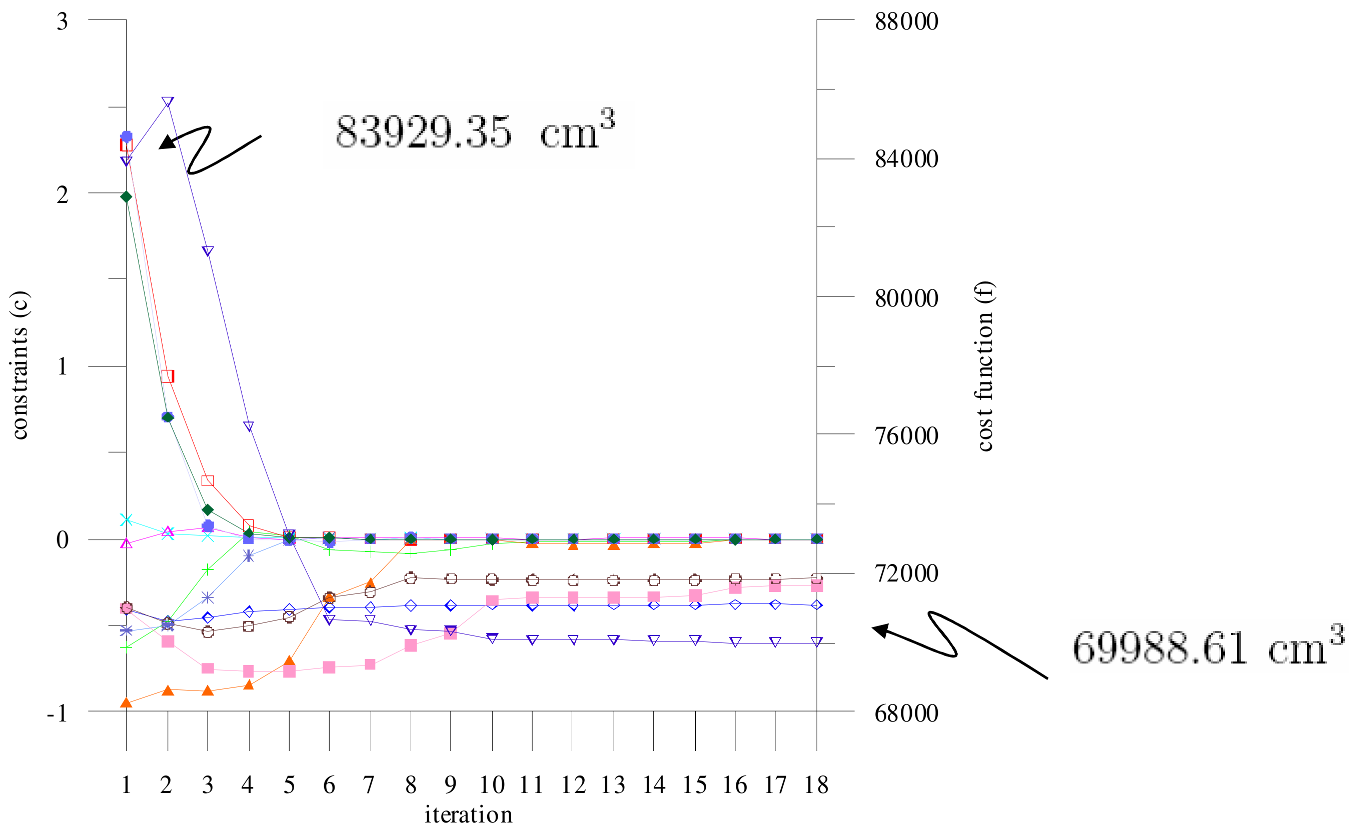
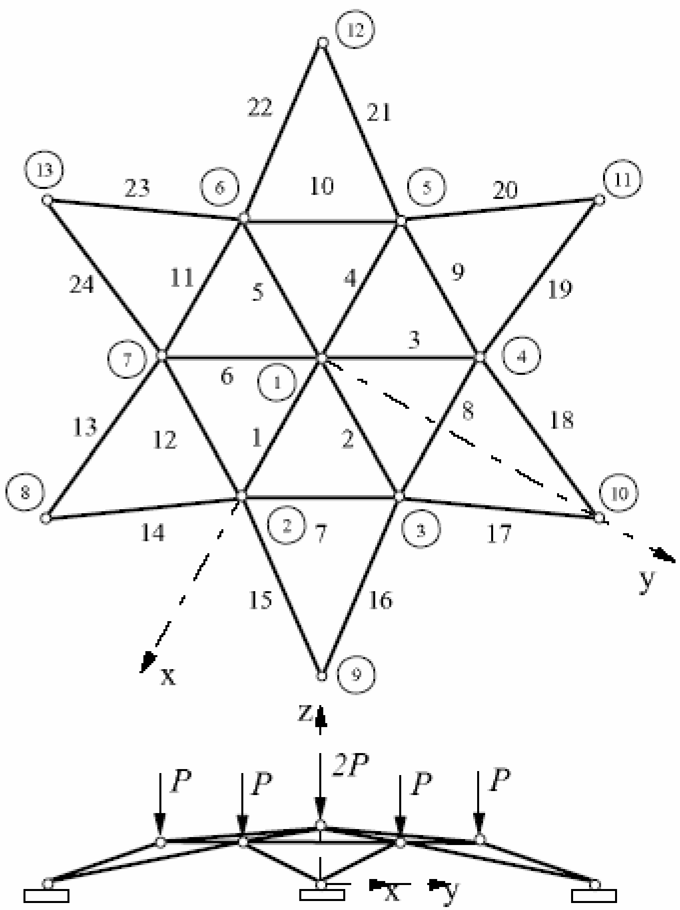
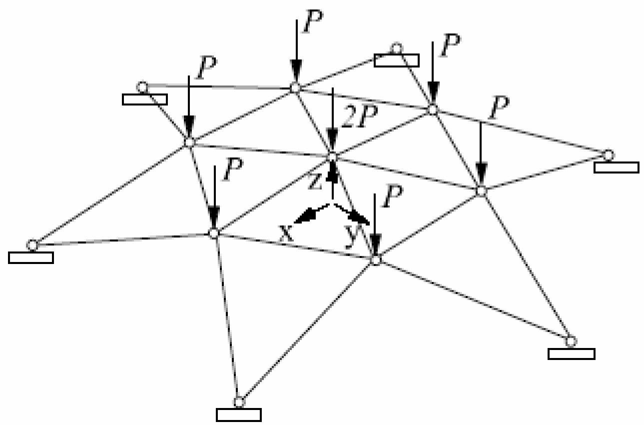
10 bar Plane Truss (cont.)



In the present paper, 18 iterations, 1875 structural analyses and 875 sensitivity analyses were necessary to reach the convergence to the solution.

Example 2- 24 bar Dome





24 bar Dome (cont.)

X	Description	Type	Momen.	Initial value	h	Pres. work	Stocki et. al [73]
X_1	Cross-section area - el. 1-6 constant δ (5%)	LN	$E(X_1)$	20.00	h_1	19.398	19.371
			$\sigma(X_1)$	1.0			
X_2	Cross-section area - el. 7-12 constant δ (5%)	LN	$E(X_2)$	20.00	h_2	13.655	13.712
			$\sigma(X_2)$	1.0			
X_3	Cross-section area - el. 13-24 constant δ (5%)	LN	$E(X_3)$	20.00	h_3	14.239	14.239
			$\sigma(X_3)$	1.0			
X_4	Young modulus for all elements	LN	$E(X_4)$	21000.0			
			$\sigma(X_4)$	1050.0			
X_5	Yield stress for all elements	LN	$E(X_5)$	21.0			
			$\sigma(X_5)$	1.0			
X_6	x coordinate - node 1	N	$E(X_6)$	0.0			
			$\sigma(X_6)$	1.0			
X_7	y coordinate - node 1	N	$E(X_7)$	0.0			
			$\sigma(X_7)$	1.0			
X_8	z coordinate - node 1	N	$E(X_8)$	82.16			
			$\sigma(X_8)$	1.0			
	...		...			...	
X_{24}	x coordinate - node 7	N	$E(X_{24})$	125.0			
			$\sigma(X_{24})$	1.0			
X_{25}	y coordinate - node 7	N	$E(X_{25})$	-216.51			
			$\sigma(X_{25})$	1.0			
X_{26}	z coordinate - node 7	N	$E(X_{26})$	62.16			
			$\sigma(X_{26})$	1.0			
X_{27}	Nodal load P	GU	$E(X_{27})$	20.0			
			$\sigma(X_{27})$	3.0			

24 bar Dome (cont.)

Minimize $V(\mathbf{h}, \mathbf{p})$

subject to

$$\beta_i(\mathbf{x}, \boldsymbol{\eta}) \geq 3.7 \qquad i = 1, \ldots, 25$$

$$\beta_{26}(\mathbf{x}, \boldsymbol{\eta}) \geq 4.7$$

$$3.13 \text{ cm}^2 \leq h_k \leq 30.76 \text{ cm}^2 \quad k = 1, \ldots, 10$$

$$g_1(\mathbf{q}(\mathbf{h}, \mathbf{p})) = 1 - \frac{|q_3^1(\mathbf{h}, \mathbf{p})|}{q^a}$$

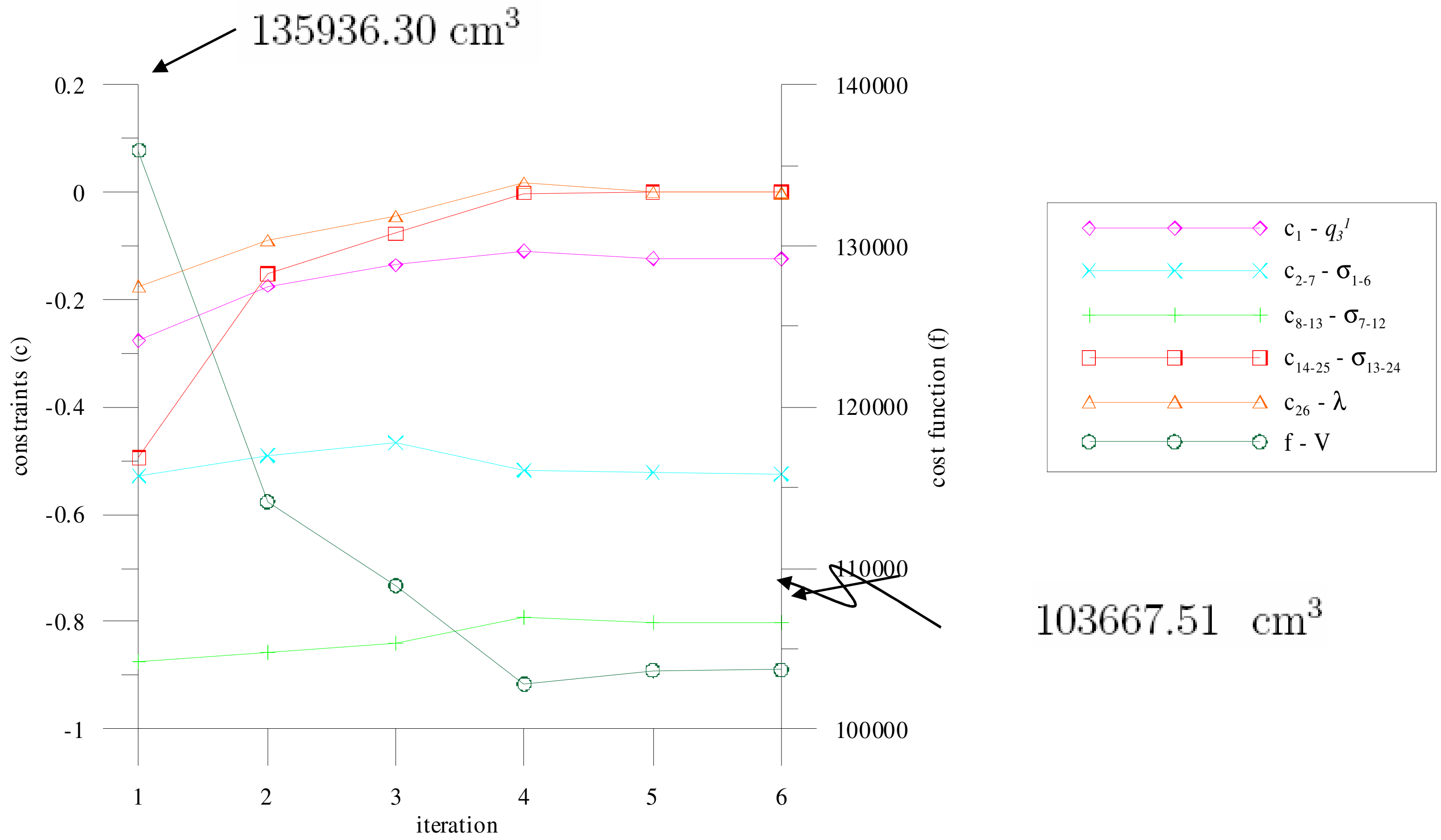
$$g_i(\mathbf{h}, \mathbf{p}) = 1 - \frac{|\sigma_{i-1}(\mathbf{h}, \mathbf{p})|}{\sigma_{i-1}^a(\mathbf{p})} \quad i = 2, \ldots, 25,$$

$$I_{min}(A) = \frac{\left(\left(\frac{d_e}{d_i}\right)^2 + 1\right) A^2}{4\pi \left(\left(\frac{d_e}{d_i}\right)^2 - 1\right)}$$

d_e/d_i :was fixed in 1.25

$$g_{26}(\mathbf{h}, \mathbf{p}) = \lambda_{cr}(\mathbf{h}, \mathbf{p}) - 1$$

24 bar Dome (cont.)



In the present work, 6 iterations, 1671 structural analyses and 831 sensitivity analyses were necessary to reach the convergence to the solution.

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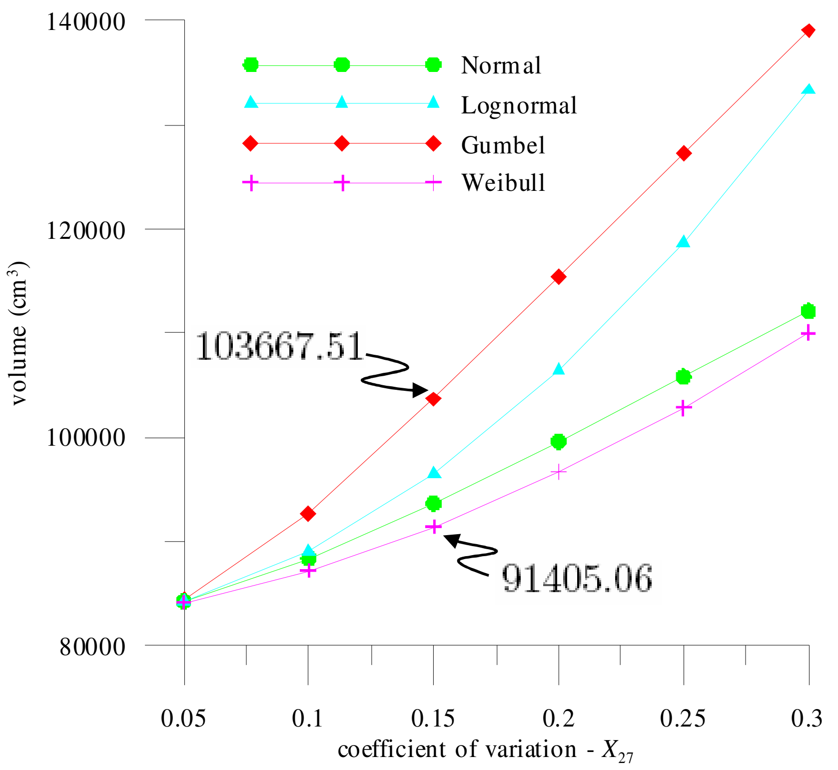
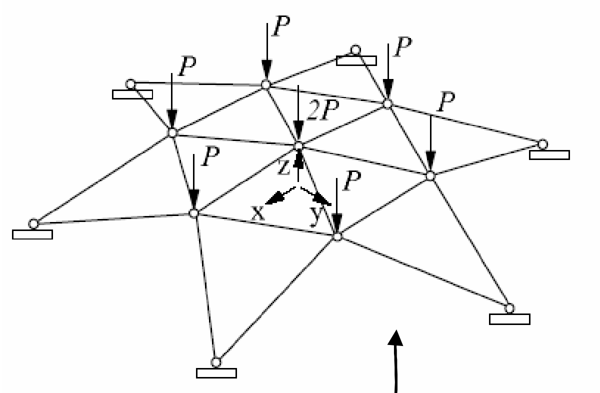
24 bar Dome (cont.)





load (P)

$\delta(P) = \sigma(P) / \mu(P)$

Final Comments

- A computational tool which performs the RBDO of space trusses is presented.
The design variables are the mean values of the nodes coordinates and/or the areas of the cross sections.
Geometric nonlinear behaviour is considered in the structural analysis.
Sensitivities analyses are performed analytically.
The tool integrates modules of mathematical programming algorithms, geometric nonlinear structural analysis, sensitivity analysys and reliability analysis.
Two basic formulations for the RBDO problem were implemented, namely, the RIA and the PMA. The PMA presented a better performance.

- The performance of the computer program became much more efficient when information of the anterior reliability analysis is used in a new iteration of the optimization process leading to a significant reduction in the number of total iterations.

- Nonlinear structures were successfully optimized even when presenting great displacements and limit points.

Joint Probability Function