Developments in semi-rigid joint moment versus rotation curves to incorporate the axial versus moment interaction

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ABSTRACT: Under certain circumstances, beam-to-column joints can be subjected to the simultaneous action of bending moments and axial forces. Although, the axial force transferred from the beam is usually low, it may, in some situations attain values that significantly reduce the joint flexural capacity. Few experimental tests are available and are usually described by their associated moment-rotation curves. An interesting question is how to incorporate these curves into a structural analysis, for the various required axial force load levels. The main aim of the present paper is the development of a consistent and simple approach to determine any moment versus rotation curve from experiments including the axial versus bending moment interaction.

1 INTRODUCTION

1.1 *Generalities*

Under certain circumstances, beam-to-column joints can be subjected to the simultaneous action of bending moments and axial forces. Although, the axial force transferred from the beam is usually low, it may, in some situations attain values that significantly reduce the joint flexural capacity. These conditions may be found in: vierendeel girder systems (widely used in building construction because they take advantage of the member flexural and compression resistances eliminating the need for extra diagonal members); regular sway frames under significant horizontal loading (seismic or extreme wind); irregular frames (especially with incomplete storeys) under gravity/horizontal loading; and pitched-roof frames.

On the other hand, with the recent escalation of terrorist attacks on buildings, the study of progressive collapse of steel framed building has been highlighted, as can be seen in Vlassis et al. (2006). Examples of these exceptional conditions are the cases where structural elements, such as central and/or peripheral columns and/or main beams, are suddenly removed, sharply increasing the joint axial forces. In these situations the structural system, mainly the connections, should be sufficiently robust to prevent the premature failure modes that may lead to progressive structural collapse.

Unfortunately, few experiments considering the bending moment versus axial force interactions have been reported. Additionally, the availabe experiments are associated with a small number of axial force levels and associated bending moment versus rotation curves, M-ϕ. Nevertheless, a question still remains on how to incorporate these effects into a structural analysis. There is a need for M-ϕ curves, associated with numerous axial force levels, which accurately represent the joint rotational stiffness.

This has led to the development of an approach to incorporate any moment versus rotation curve from tests including the axial versus bending moment interaction, as well as its evaluation and validation against experiments. This approach is not only restricted to the use of experiments, but can be applied to results obtained analytically, empirically, mechanically, and numerically.

As this approach is exclusively based on the use of M-ϕ curves, it can be easily incorporated into a nonlinear semi-rigid joint finite element formulation because the moment versus axial force interaction is associated with a specific M-ϕ curve. The nonlinear joint finite element formulation does not change. It only requires a rotational stiffness update procedure. This approach has been used to improve the joint finite element model proposed by Del Savio (2004) and Del Savio et al. (2004, 2005), which was initially based on the semi-rigid joint force independence.

1.2 *Component Method*

The component method consists of relatively simple joint mechanical models, based on a set of rigid and spring components. The component method, introduced in Eurocode 3 (2003), can be used to determine the joint's resistance and initial stiffness. Its application requires the identification of active com-

ponents; evaluation of the force-deformation response of each component; and the subsequent assembly of the active components for the evaluation of the joint moment versus rotation response.

The Eurocode 3 (2003) component method permits the evaluation of the semi-rigid joint's rotational stiffness and moment capacity when subjected to pure bending. However, this component method is still not able to calculate these properties when, in addition to the applied moment, an axial force is also present. Eurocode 3 (2003) suggests that the axial load may be disregarded in the analysis when its value is less than 10% of the beam's axial plastic resistance, but provides no information for cases involving larger axial forces. Even though, the Eurocode 3 (2003) component method has not considered the axial force, its general principles could be used to cover this situation, since it is based on the use of a series of force versus displacement relationships, which only depend on the axial force level, to characterize any component behaviour.

1.3 *Background: Experimental and theoretical models*

The study of the semi-rigid characteristics of beam to column connections and their effects on frame behaviour can be traced back to the 1930s, Li et al. (1995). Since then, a large amount of experimental and theoretical work has been conducted both on the behaviour of the connections and on their effects on the complete frame performances.

Despite the large number of experiments, they do not cover all possible connection ranges. As an alternative to tests, different methods have been proposed by researchers to predict bending moment versus rotation curves. These methods are usually classified as: empirical, analytical, mechanical (component-based approaches) and numerical (finite element).

Recently, several researchers have paid special attention to joint behaviour under combined bending moment and axial force. The investigators concluded that the presence of the axial force in the joints modifies their structural response and, therefore, should be considered. A number of experimental works deserve mention:

- Guisse et al. (1997) performed tests on six prototypes of column bases with extended endplates with bolts placed outside of the beam height and six tests on flush endplates with bolts inside the beam height. In these tests, the compressive axial force was first applied and kept constant during the test while the bending moment was subsequently increased up to failure.

- Wald et al. (2000) conducted two tests on beam-to-beam and beam-to-column joints. The loading system adopted a proportional increase of axial force and bending moment. However, a test without axial forces was not performed, making it difficult to assess the axial force influence on the joint response.

- Lima et al. (2004) and Simões da Silva et al. (2004) performed tests on fifteen prototypes, i.e. eight flush and seven extended endplate joints. All the tests adopted a loading strategy consisting of an initial application of the total axial force (tension or compression), held constant during the entire test, and the subsequent incremental application of the bending moment.

Regarding the theoretical models recently developed to predict the behaviour of beam-to-column joints under bending moment and axial force, it is possible to mention:

- Jaspart (1997, 2000), Finet (1994) and Cerfontaine (2001, 2004) have applied the principles of the component method to establish design predictions of the M-N interaction curves and initial stiffness.

- Simões da Silva & Coelho (2001), based on the same general principles, have proposed analytical expressions for the full non-linear response of a beam-to-column joint under combined bending and axial forces.

- Sokol et al. (2002) proposed an analytical model to predict the behaviour of joints subjected to bending moment and axial force for proportional loading.

Table 1 presents a summary of recent studies carried out to investigate joint behaviour when subjected to bending moment and axial force.

Table 1. Summary of studies of joints subjected to bending and axial force, Lima et al. (2004).

Authors	Analysis type
Finet (1994)	AM*
Jaspart (1997, 2000)	AM* and ET**
Cerfontaine (2001, 2004)	AM*
Simões da Silva & Coelho (2001)	AM*
Simões da Silva et al. (2001)	AM*
Lima (2003)	ET**
Wald & Svarc (2001)	AM*
Sokol et al. (2002)	AM*

* AM = Analytical model.
** ET = Experimental tests.

2 CORRECTION FACTOR

2.1 *Concepts of the Correction Factor*

The Correction Factor has initially been proposed by Del Savio et al. (2006) to consider the bending moment versus axial force interaction, by scaling original moment values present in the moment versus rotation curves (disregarding the axial force effect). This strategy shifts this curve up or down depending on the axial force level. However, as it only modifies the bending moment axis, it is not able to fully describe the bending moment versus rotation associated with different axial force levels. This fact is highlighted when the joint is subject to a tensile ax-

ial force, where there is a significant difference, principally, in terms of initial stiffness.

Aiming to improve the Correction Factor's basic idea, the Correction Factor was divided into two parts: one for the moment axis and another for the rotation axis. Both corrections are in principle independent, and do not depend on the moment versus axial force interaction diagram, as was the case for the initial idea presented by Del Savio et al. (2006). It is now only a function of the moment versus rotation curves for different axial force levels.

2.2 *Evaluation of the Correction Factor*

As previously noted, there are two corrections, one to the moment axis and another to the rotation axis.

As a general approach, the Correction Factor for the moment axis is evaluated in terms of the bending moment versus rotation curves considering the axial force effect. Using the design bending moment ratio and considering the axial force effect, the Correction Factor for the moment axis, CF_M, can be evaluated by:

$$CF_M = \frac{M_{\text{int}}}{M_{\max}} \qquad \begin{array}{l} M_{\text{int}} = f\left(Mx\phi(N_i)\right) \\ M_{\max} = f\left(Mx\phi(0.0)\right) \end{array} \tag{1}$$

where $Mx\phi$ = bending moment versus rotation curve; M_{int} = design bending moment for the $M\text{-}\phi(N_i)$ considering the axial force; M_{max} = design bending moment for cases without axial forces; and N_i = axial force present in the i interaction. M_{int} and M_{max} can be determined according to Eurocode 3 part 1.8 (2003), through the intersection between two straight lines, one parallel to the initial stiffness and another parallel to the $M\text{-}\phi$ post limit stiffness, Figure 1.

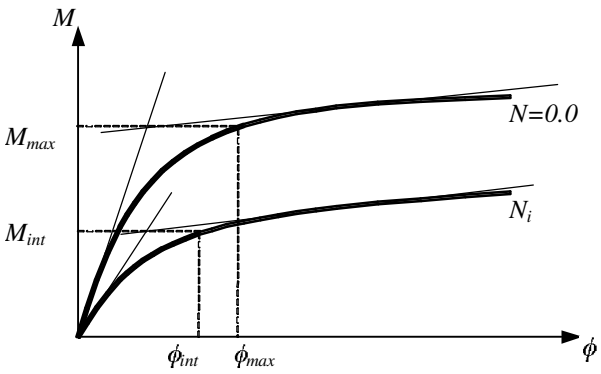

Figure 1. Correction Factors parameters for the moment and rotation axis.

Similarly, the rotation axis Correction Factor, CF_ϕ, is evaluated using the design rotation ratio, i.e.:

$$CF_\phi = \frac{\phi_{\text{int}}}{\phi_{\max}} \qquad \begin{array}{l} \phi_{\text{int}} = f\left(Mx\phi(N_i)\right) \\ \phi_{\max} = f\left(Mx\phi(0.0)\right) \end{array} \tag{2}$$

where ϕ_{int} = design rotation related to M_{int}; and ϕ_{max} = rotation related to M_{max}. Both design rotations are found by tracing a horizontal straight line at the design moment level until it reaches the $M\text{-}\phi$ curve. At this point a vertical straight line is drawn until it intersects the rotation axis, Figure 1.

With the Correction Factors evaluated for both the moment (Equation 1) and rotation (Equation 2) axes, they are incorporated into the joint structural response considering the moment versus axial force interaction, modifying the $M\text{-}\phi$ curve for the zero axial force case, i.e.:

$$\begin{array}{l} Mx\phi(N=0) \rightarrow Mx\phi(N_i) \\ Mx\phi(N_i) = Mx\phi(M_{N=0} \cdot CF_M, \phi_{N=0} \cdot CF_\phi) \end{array} \tag{3}$$

Basically, the $M\text{-}\phi$ point coordinates, for the case without axial forces, are multiplied by the Correction Factors, where CF_M and CF_ϕ, multiply the moment and the rotation axis coordinates, respectively.

However, using only a pair of Correction Factors throughout the whole $M\text{-}\phi$ curve, for the case without axial forces, does not provide a good approximation to $M\text{-}\phi$ curve considering the axial force, because it is very sensitive to the adopted initial stiffness and post-limit stiffness angles.

This motivated the division of the $M\text{-}\phi$ curve into three segments with different pairs of Correction Factors. This division was made for two-third, one, and 1.1 times the design moment, as shown in Figure 2.

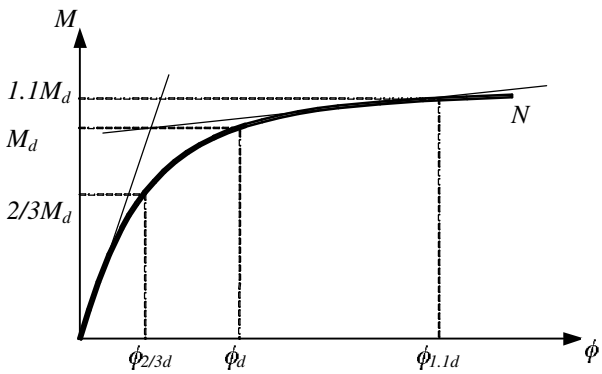

Figure 2. Correction Factor strategy approach using a three segment division of the $M\text{-}\phi$ curve.

With this division, the Correction Factors cannot be applied as presented in Equation 3. This is justified, in fact, because they would provoke two abrupt variations of stiffness throughout the approximate $M\text{-}\phi$ curve at around the point of intersection of the approximate curve with the vertical lines at the points $\phi_{2/3d}$ and ϕ_d, Figure 3. This is due to the use of three different pairs of Correction Factors.

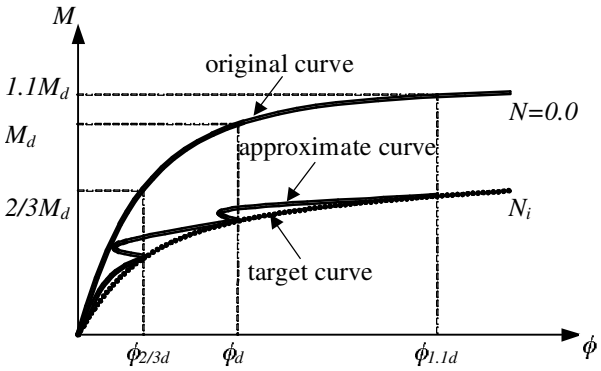

Figure 3. Approximate M-ϕ curve using three Correction Factor pairs.

To avoid the problem of abrupt alterations of stiffness presented in Figure 3, it is proposed, in Figure 4, to use a tri-linear representation of the M-ϕ curve.

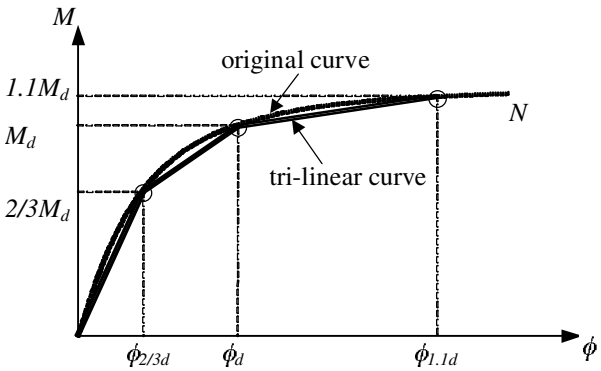

Figure 4. Tri-linear representation of the M-ϕ curve approach.

With this tri-linear approach, Figure 4, the moment levels of the required M-ϕ curve, associated to a certain axial force level (N_i), can be evaluated by:

$$
\begin{aligned}
&M_p = \left(M_{N,p} - M_{0,p}\right)\frac{N_i}{N} + M_{0,p} \\
&p = 2/3M_d; M_d; 1.1M_d \\
&0 < N_i \le N \rightarrow \textit{tension axial force} \\
&N \le N_i < 0 \rightarrow \textit{compression axial force}
\end{aligned}
\tag{4}
$$

where M_p = evaluated moment for the new M-ϕ; $M_{N,p}$ = moment on the reference M-ϕ curve considering the axial force; $M_{0,p}$ = moment on the reference M-ϕ curve without axial forces; and N = axial force load level associated to the reference M-ϕ curve.

Likewise, the rotations of the evaluated M-ϕ curve, for the associated N_i, can be calculated by:

$$
\begin{aligned}
&\phi_p = \left(\phi_{N,p} - \phi_{0,p}\right)\frac{N_i}{N} + \phi_{0,p} \\
&p = 2/3M_d; M_d; 1.1M_d \\
&0 < N_i \le N \rightarrow \textit{tension axial force} \\
&N \le N_i < 0 \rightarrow \textit{compression axial force}
\end{aligned}
\tag{5}
$$

where ϕ_p = evaluated rotation for the new M-ϕ curve; $\phi_{N,p}$ = rotation on the reference M-ϕ curve considering the axial force; and $\phi_{0,p}$ = rotation on the reference M-ϕ curve without axial force effects.

3 APPLICATION OF THE CORRECTION FACTOR

3.1 *Uses and input of the Correction Factor*

The main focus of the proposed Correction Factor was to determine M-ϕ curves for any axial force level from the M-ϕ curve for zero axial force. The quality of the approximations obtained will depend on quality of M-ϕ used as input to the method. This method requires three M-ϕ curves, one disregarding the axial force effect and two considering the compression and tension axial force effects.

3.2 *Example of application and validation*

The goal of this section is to demonstrate how to use the Correction Factor method to obtain M-ϕ curves for any axial force level, as well as to validate it, using experimental tests carried out by Lima et al. (2004) and Simões da Silva et al. (2004), on eight flush endplate joints. The geometric properties of the flush endplate analysed, the M-ϕ curves describing the experimental behaviour of each test, and the bending moment versus axial force interaction diagram are shown in Figures 5-7, respectively.

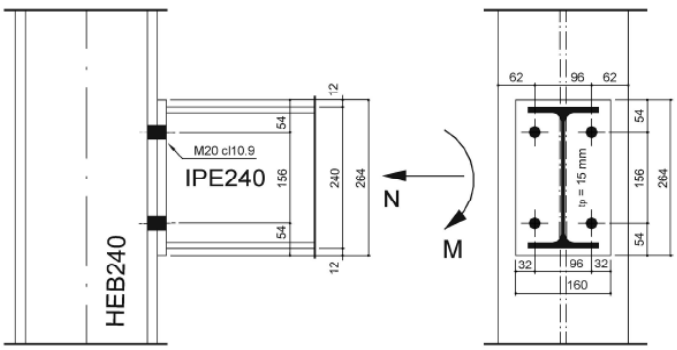

Figure 5. Flush endplate joint layout, Simões da Silva et al. (2004).

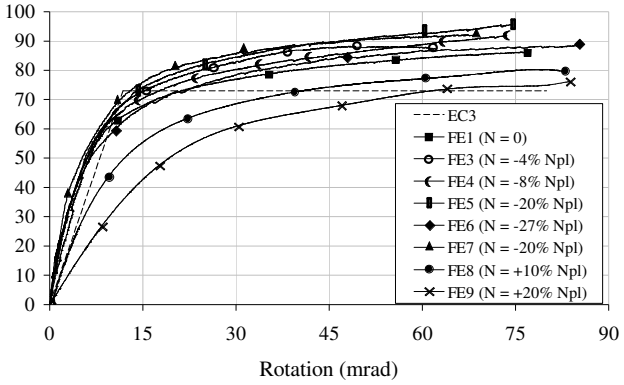

Figure 6. Experimental moment versus rotation curves, Simões da Silva et al. (2004).

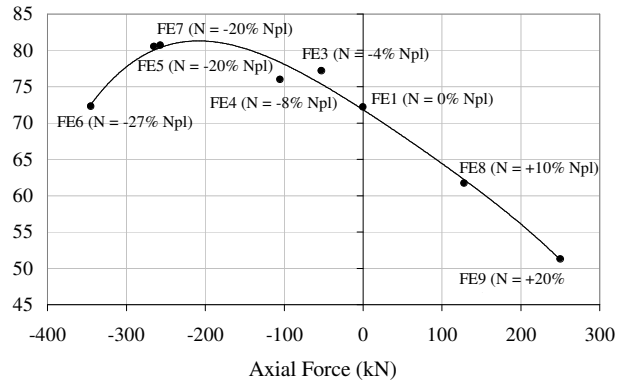

Figure 7. Flush endplate bending moment versus axial force interaction diagram, Simões da Silva et al. (2004).

The experimental data, Figure 6, provides the necessary input for the Correction Factor method. The minimum input is composed of two M-ϕ curves, disregarding and considering either the tension or compression axial force. The flush endplate joint, tested by Simões da Silva et al. (2006), exhibited a decrease in the moment resistance for the tensile axial forces whilst achieving the highest moment resistance for the compression axial force of 20% of the beam plastic resistance (see Figure 7, FE7). Therefore, three M-ϕ curves: FE1 ($N = 0$); FE7 ($N = -257$ *kN, -20% Npl*), and FE9 ($N = 250$ *kN, +20% Npl*), where *Npl* is the beam axial plastic resistance, have been used. These three experimental curves and their tri-linear approximations are shown in Figure 8.

Tri-linear M-ϕ curves, Figure 8, are used to define paths between each curve at points $2/3M_d$, M_d and $1.1M_d$, Figure 9. These paths will be used to guide the Correction Factor throughout the range of axial force levels to determine the required set of M-ϕ curves.

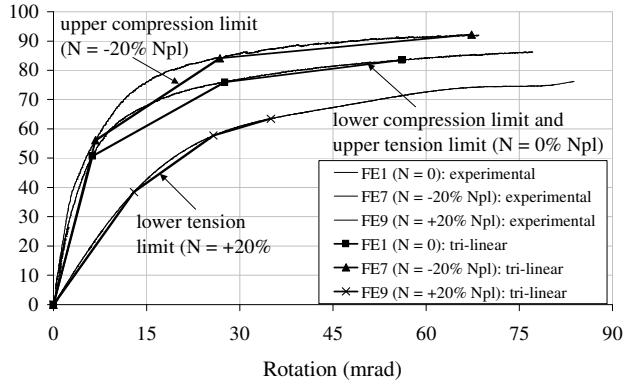

Figure 8. Tri-linear approach for the experimental M-ϕ curves.

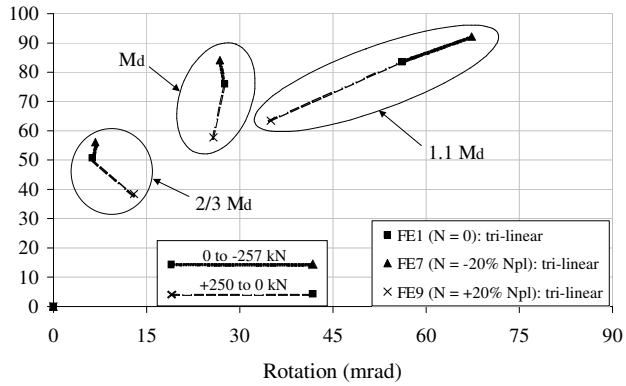

Figure 9. Paths used to define the way to find any M-ϕ curve contained within these limits imposed on the Correction Factor.

Subsequently, Figures 10-12 show the results obtained using the Correction Factor approach for three experimental M-ϕ curves: FE8, FE3 and FE4. The evaluated M-ϕ curve requires two M-ϕ reference curves defining the maximum and minimum limit for the associated axial force level. Figure 13 gives the complete results of the Correction Factor approach.

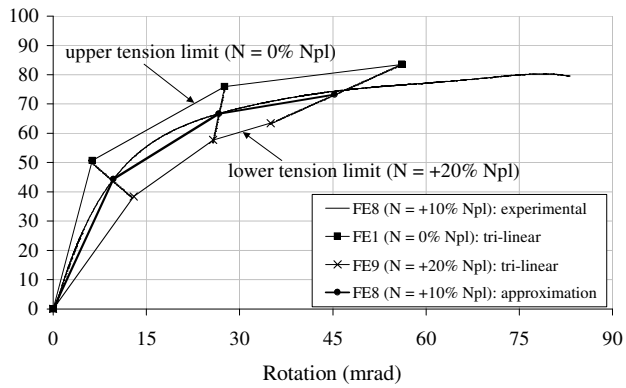

Figure 10. FE8 M-ϕ curve approximation, considering a tension force of 10% of the beam axial force plastic resistance.

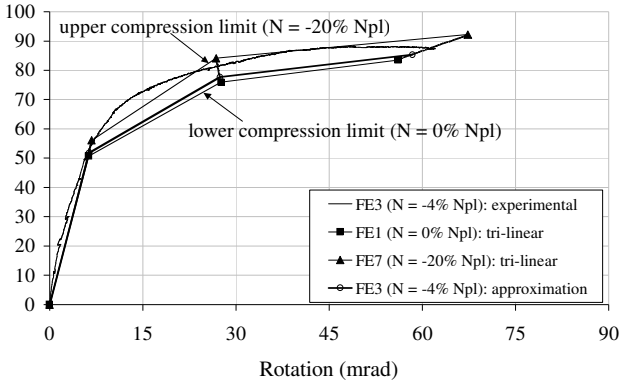

Figure 11. FE3 M-ϕ curve approximation, considering a compression force of 4% of the beam axial force plastic resistance.

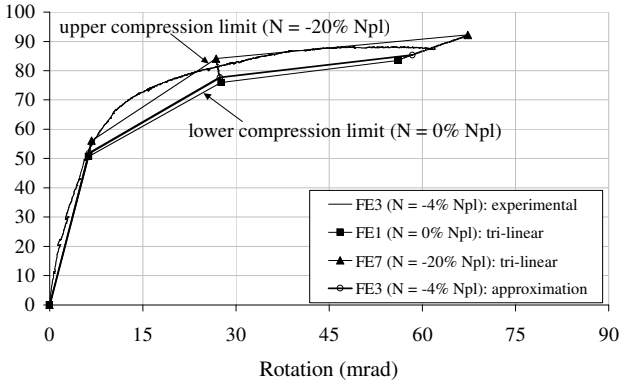

Figure 12. FE4 M-ϕ curve approximation, considering a compression force of 8% of the beam axial force plastic resistance.

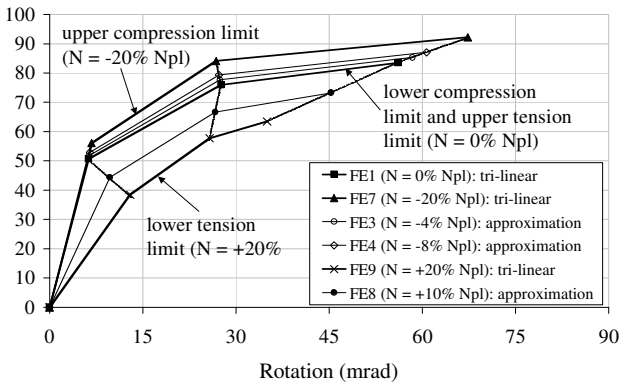

Figure 13. Final Correction Factor approach curves.

4 RESULTS AND DISCUSSION

Three experimental M-ϕ curves, by Lima et al. (2004) and Simões da Silva et al. (2004), were subjected to the Correction Factor concept, as can be seen in Figures 10-11.

Figure 10 illustrates an approximation for FE8 M-ϕ curve that considers a tension force of 10% of the beam axial force plastic resistance. This approximation was obtained from two tri-linear M-ϕ curves, disregarding and considering a tension force of 20% of the beam axial force plastic resistance. This approximation was very close to FE8 M-ϕ test curve.

Figures 11 and 12, present approximations for FE3 and FE4 M-ϕ curves that consider compression forces of 4% and 8% of the beam axial force plastic resistance, respectively. These approximations were obtained from two tri-linear M-ϕ curves, disregarding and considering a compression force of 20% of the beam axial force plastic resistance. The approximation acquired for FE4 M-ϕ curve, Figure 12, was relatively close to the experimental curve. However, for FE3 M-ϕ curve, Figure 11, the obtained estimation was not as good. This was due to the differentiable behaviour of this experimental curve when compared to the others. It is possible to observe in Figure 7 that there is an increase in the flush endplate joint moment capacity from FE1 M-ϕ curve ($N = 0\%\ Npl$) to FE7 M-ϕ curve ($N = -20\%\ Npl$). However, within this range, with 4% beam compression force plastic resistance the final moment is larger than the maximum moment obtained with the 8% test. A possible reason for that could be related to problems with this specific experimental test, i.e., measuring errors, eccentricities in the construction and assembly of this test, etc.

5 CONCLUSIONS

The main goal of this work was to present an approach to determine any moment versus rotation curve from experimental tests, including the axial versus bending moment interaction. It can also be applied to results obtained analytically, empirically, mechanically, and numerically. Due to its simplicity and the fact that its basis is M-ϕ curves that already consider the moment versus axial force interaction, it can be easily incorporated into a nonlinear semi-rigid joint finite element formulation. The use of the proposed approach does not change the basic formulation of the non-linear joint finite element, only requiring a rotational stiffness update procedure.

This approach is a simple and accurate way of introducing semi-rigid joint experimental test data into structural analysis, through of M-ϕ curves.

Application and validation of this approach to obtain M-ϕ curves for three axial force level, using experimental tests carried out by Lima et al. (2004) and Simões da Silva et al. (2004), on eight flush endplate joints, were performed with results close to the experiments.

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