STRUCTURAL EVALUATION OF SEMI-RIGID STEEL PORTAL FRAMES

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ABSTRACT

Over the last few years various investigations related to the development of finite element formulations for semi-rigid steel portal frame design including the geometric non-linear response have been produced. On the other hand, it is suggested that, for a more precise semi-rigid connection modelling, the axial and shear force effects should be incorporated to the usual bending deformations. These facts motivated the conception and development of a joint finite element that could accurately represent these effects as well as contemplated a possible interaction amongst these stiffness components. This element also enables joint elasto-plastic analysis to be performed based on their associated moment versus rotation curves. Consequently, the main aim of this paper is to describe in detail the proposed joint finite element and also its linear and non-linear formulations that incorporated the transversal deformations due to shear stresses. In order to validate and calibrate this element, and its associate formulation, a three storey semi-rigid steel portal frame proposed by Keulen et al. [4] was analysed and investigated.

1 INTRODUCTION

Various investigators have been developing geometric non-linear finite element systems for semi-rigid portal frame modelling. One of the most traditional approaches was proposed by Chan & Chui [5], using a hybrid finite element to characterise the semi-rigid joint response. This finite element was generated from a beam element with two additional rotation springs statically condensate at element ends. A similar deduction to Chan and Chui [5] will be presented for the determination of the stiffness matrix of the 2D beam finite element with condensed springs on its extremities as well as the equivalent nodal loading vector for uniformly distributed loads.

One of the objectives of the present investigation is to consider the additional effects of the transversal forces on the usual flexural deformations of steel portal frames with semi-rigid connections. Additionally, the shear-bending interaction is also contemplated since this was not included in the original 2D beam finite element with condensed springs on its extremities where only the rotation stiffness was considered. The non-linear elasto-plastic behaviour of this joint element is considered through its associated moment versus rotation curve. Elasto-plastic analyses of the global structure are performed introducing joint elements in points where probable plastic hinge could occur.

The present paper presents the proposed joint finite element in terms of its linear and non-linear formulations that considered the transversal deformations due to the shear stress effect. These effects are not coupled in the linear formulation. On the other hand, in the non-linear case the joint finite element is based on Timoshenko beam theory leading to a stiffness matrix with coupled stiffnesses. This formulation was developed under an Updated Lagrangian referential with an approach considering corotational displacements.

The present element was implemented in the FEMOOP (*Finite Element Method – Object Oriented Program*) software. A more detailed description of the element implementation scheme and semi-rigid joint pre-processing and post-processing tools can be found in Del Savio et al. [1]. In order to calibrate and validate the non-linear joint finite element formulation, a three storey semi-rigid steel portal frame, Keulen et al. [4], is analysed and investigated.

2 PLANE BEAM ELEMENT WITH SPRINGS ON ITS EXTREMITIES

2.1 Stiffness Matrix

The plane beam finite element and the adopted coordinate system are illustrated in Fig. 1, where the left and right elastic rotation springs are represented by: C_L and C_R. The schematic representation of the springs and hinges, Fig. 1, considers that the distance from the hinge to the nearest beam extremity is infinitely small.

Fig. 1: 2-D Beam element with springs on its extremities. Global system.

Fig. 2: 2-D Beam element with springs on its extremities. Auxiliary Global system.

It is not possible to directly obtain the element stiffness matrix since the reference system coordinate number is insufficient to fully describe the actual element displacements. This fact led to the adoption of an auxiliary global coordinate system, Fig. 2. The element stiffness matrix is obtained in this system, and a static condensation of the internal degrees-of-freedom (DOF) 7 and 8 (Fig. 2) is made, leading to the Fig. 1 desired stiffness matrix. The Fig. 2 beam stiffness matrix was simply obtained with the aid of the local auxiliary coordinate system, Fig. 3, where the beam element of Fig. 2 was decomposed in three well-known elements: two springs and a conventional beam element.

Fig. 3: 2-D Beam element with springs on its extremities. Auxiliary Local system.

Using these three stiffness matrices the local auxiliary system element stiffness matrix is:

$$K_{Local\ Auxiliar} = \begin{bmatrix} k_L & 0 & 0 \\ 0 & k_N & 0 \\ 0 & 0 & k_R \end{bmatrix} \tag{1}$$

where k_L , k_R and k_N are the stiffness matrices of the left and right springs and the 2-D beam element. This simple idea led to the beam stiffness matrix in terms of the auxiliary global coordinate system:

$$K_{Global\ Auxiliar} = A^T \cdot K_{Local\ Auxiliar} \cdot A \tag{2}$$

where A is the cinematic incidence matrix that transforms the displacements from Fig. 2 coordinate system (auxiliary global system) into Fig. 3 coordinate system (local auxiliary system). Having the element stiffness matrix in terms of the global auxiliary system of Fig. 2 in hand, the equivalent stiffness matrix for Fig. 1 system can be obtained using static condensation principles applied to the 7 and 8 internal DOF of Fig. 2. This strategy enables only the external DOF to be directly considered through which the compatibility of displacements and the substructure equilibrium against the other structures parts are enforced. The matrix representation of the substructure equilibrium, Fig. 2, is expressed in terms of:

$$\begin{bmatrix} k_{e,e} & k_{e,i} \\ k_{i,e} & k_{i,i} \end{bmatrix} \cdot \begin{Bmatrix} d_e \\ d_i \end{Bmatrix} = \begin{Bmatrix} p_e \\ p_i \end{Bmatrix} \tag{3}$$

where the subscripts e and i are respectively related to the external and internal parts. In this equation p_e and p_i are force vectors applied to the internal/external dofs while d_e and d_i are internal/external displacements. Finally, $k_{e,e}$, $k_{e,i}$, $k_{i,e}$ and $k_{i,i}$ are sub-matrices of the Fig. 2 element stiffness matrix $K_{Global\ Auxiliar}$. Fig. 1 element stiffness matrix after the static condensation is:

$$K = \left\lfloor k_{e,e} - k_{e,i} \cdot \left(k_{i,i}\right)^{-1} \cdot k_{i,e} \right\rfloor \tag{4}$$

2.2 Equivalent Nodal Loads

Due to the static condensation of the internal nodes (7 e 8) the equivalent nodal load vector was also affected. The same method earlier described is used applying the static condensation principle to the equivalent nodal loads. From a uniformly distributed load acting on Fig. 3(b) element, and knowing beforehand its equivalent load, the equivalent load vector for the elements of Fig. 3 can be determined. When this is applied to Fig. 2 element DOF the following nodal load vector is generated:

$$S_{Global\ Auxiliar} = \left\{0 \quad -qL/2 \quad 0 \quad 0 \quad -qL/2 \quad 0 \quad -qL^2/12 \quad qL^2/12\right\}^T \tag{5}$$

Condensating the internal DOF (7 and 8), Fig. 2, this expression can be written in terms of the equivalent loads, i.e.:

$$S = p_e - k_{e,i} \cdot \left(k_{i,i}\right)^{-1} \cdot p_i \tag{6}$$

This yields the expression for the equivalent nodal load vector for Fig. 1 element when submitted to a uniformly distributed load.

3 JOINT FINITE ELEMENT FORMULATION

The proposed joint finite element, Fig. 4, has the main objective of modelling a semi-rigid connection in linear and non-linear structural analysis.

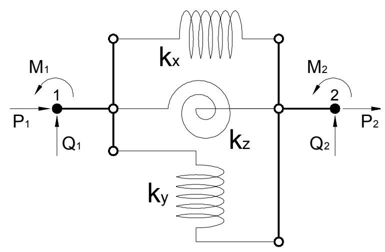

Fig. 4: Joint finite element

3.1 Linear Formulation

The stationery potential energy principle was used to formulate the joint element stiffness matrix and the corresponding equilibrium equation. The total potential energy functional, *Π*, is expressed in two parts:

$$\Pi = U + V \tag{7}$$

where *U* is the strain energy and *V* is the potential of the external forces. The strain energy, *U*, can be expressed in terms of the spring stiffnesses and their respective relative displacements as:

$$U = \frac{1}{2}k_x\Delta_x{}^2 + \frac{1}{2}k_y\Delta_y{}^2 + \frac{1}{2}k_z\Delta_z{}^2 \tag{8}$$

in which: $\Delta_x = u_1 - u_2$, $\Delta_y = v_1 - v_2$ and $\Delta_z = \theta_1 - \theta_2$. The potential of the external forces is:

$$V = (P_1u_1 + Q_1v_1 + M_1\theta_1) - (P_2u_2 + Q_2v_2 + M_2\theta_2) \tag{9}$$

where the subscripts *1* and *2* are related to the initial a final nodes, while *u*, *v*, *θ* represent respectively the axial displacement, transversal displacement, and rotation and *P*, *Q* and *M* represent respectively the axial, transverse and moment forces. Applying the total potential energy principle, the equilibrium equations can be determined from the stationary condition of the functional *Π*. The stiffness matrix and the internal loading vector can be derived from the strain energy, equation (8):

$$k_{ij} = \frac{\partial^2 U}{\partial d_i \partial d_j}, \quad d_i = u_1, v_1, \theta_1, u_2, v_2, \theta_2 \tag{10}$$

$$F_i = \frac{\partial U}{\partial d_i}, \quad F_i = P_1, Q_1, M_1, P_2, Q_2, M_2 \tag{11}$$

3.2 Non-Linear Formulation

The non-linear formulation for the joint element was developed in a Total Lagrangian referential using a corotational approach for the displacements. The stiffness matrix and the internal load vector are evaluated on the total natural displacement space, i.e., referenced to a continuously updated axis system, Fig. 5. The total natural displacement vector has 3 parts:

$$u_n = \begin{bmatrix} u_{n2} & \theta_{n1} & \theta_{n2} \end{bmatrix}^T \tag{12}$$

These values are calculated from the below expressions, as can be seen in Fig. 5:

$$u_{n2} = {}^{t+\Delta t}L - {}^{0}L, \;\; \theta_{n1} = \theta_1 - \theta_r, \;\; \theta_{n2} = \theta_2 - \theta_r \tag{13}$$

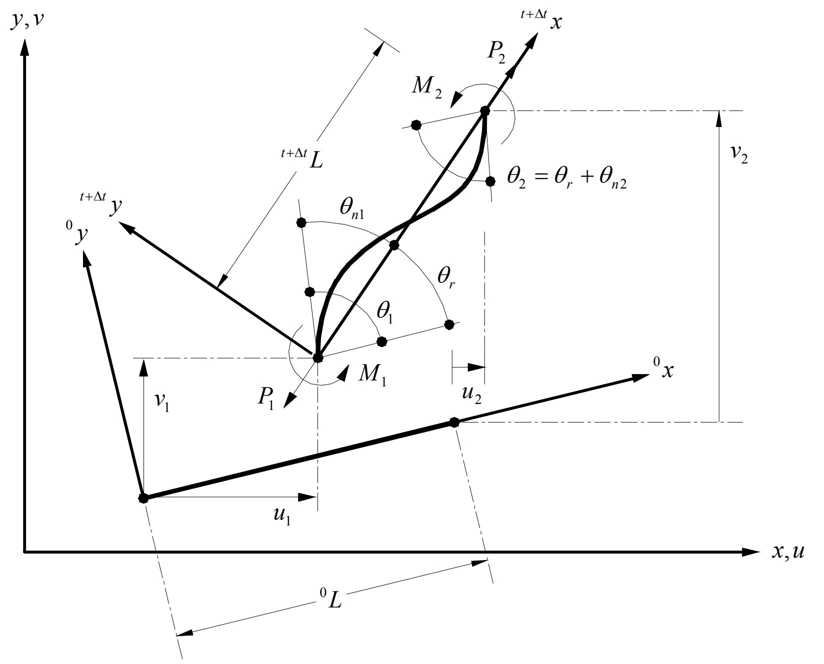

Fig. 5: Geometrical relationships of the corotational referential

where ^0L is the initial elementary length and $^{t+\Delta t}L$ is the elementary length in the current equilibrium configuration, i.e.:

$$^0L = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \tag{14}$$

$$^{t+\Delta t}L = \sqrt{(x_2 + u_2 - x_1 - u_1)^2 + (y_2 + v_2 - y_1 - v_1)^2} \tag{15}$$

The rigid body rotation, θ_r can be evaluated through:

$$\theta_r = \arctan\left(\frac{(x_2 - x_1)(v_2 - v_1) - (y_2 - y_1)(u_2 - u_1)}{(x_2 - x_1)(x_2 - x_1 + u_2 - u_1) + (y_2 - y_1) + (y_2 - y_1 + v_2 - v_1)}\right) \tag{16}$$

A classic Timoshenko beam element, with two nodes was used to define the total potential energy in terms of total natural displacements. Linear interpolations were used to evaluate the displacements u and v, and the rotation θ:

$$u = \frac{x}{L}u_{n2}, \;\; v = 0, \;\; \theta = \left(1 - \frac{x}{L}\right)\theta_{n1} + \frac{x}{L}\theta_{n2} \tag{17}$$

The curvature k, the shear strain γ, and the axial deformation ε are defined by:

$$k = \frac{\partial \theta}{\partial x} = \frac{\theta_{n1} - \theta_{n2}}{L} \tag{18}$$

$$\gamma = \frac{\partial v}{\partial x} - \theta = -\left(1 - \frac{x}{L}\right)\theta_{n1} - \frac{x}{L}\theta_{n2} \tag{19}$$

$$\varepsilon = \frac{\partial u}{\partial x} - ky = \frac{u_{n2}}{L} - \frac{\theta_{n2} - \theta_{n1}}{L}y \tag{20}$$

The strain energy, U, can be expressed in terms of the spring stiffnesses and the total natural displacements as:

$$U = \frac{1}{2}\int_{0}^{L}\left(k_x \varepsilon_{xx}{}^{2} + k_y \gamma^2 + k_z k^2\right)dx \tag{21}$$

The external forces work can be written as:

$$V = \left(M_{n1}\theta_{n1}\right) - \left(P_{n2}u_{n2} + M_{n2}\theta_{n2}\right) \tag{22}$$

Using the strain energy equation (21) the stiffness matrix and the internal load vector could be obtained in terms of corotational coordinates:

$$^{t+\Delta t}K_{nij} = \frac{\partial^2 U}{\partial d_{ni}\partial d_{nj}}, \quad d_{ni} = \theta_{n1}, u_{n2}, \theta_2 \tag{23}$$

$$^{t+\Delta t}F_{ni} = \frac{\partial U}{\partial d_{ni}}, \quad F_{ni} = M_{n1}, P_{n2}, M_{n2} \tag{24}$$

Finally, the internal load vector and the element stiffness matrix, in global coordinates, can be obtained by applying successive differentiations i.e.:

$$^{t+\Delta t}F_i = \left\{\frac{\partial U}{\partial d_{ni}}\frac{\partial d_{ni}}{\partial d_i}\right\}, \; d_{ni} = u_{n2}, \theta_{n1}, \theta_{n2}, \; d_i = u_1, v_1, \theta_1, u_2, v_2, \theta_2 \tag{25}$$

$$^{t+\Delta t}K = \left\{\frac{\partial^2 U}{\partial d_{ni}\partial d_{nj}}\frac{\partial d_{ni}\partial d_{nj}}{\partial d_l \partial d_m} + \frac{\partial U}{\partial d_{ni}}\frac{\partial^2 d_{ni}}{\partial d_l \partial d_m}\right\} \tag{26}$$

4 NUMERICAL EXAMPLE

In this section comparisons were made with Keulen et al. results [4], for a three storey steel portal frame using full moment versus rotation curves and a bi-linear approximation. This approximation uses a well-known simplified joint representation named half initial secant stiffness method. The portal frame spanned 7.2 metres, with 10.8 metres of height, Fig. 6, [8]. The beam and column sections used, respectively, IPE360 and HEA260 steel profiles. The beam-to-column connections are bolted flush endplates, Fig. 7. The horizontal load *aF* represents wind load, Fig. 6, but is also used to take into account imperfections and second order effects. The *a*-factor is taken as 0.15. The beam transverse load *w* is equal to 1/6 of the vertical joint load *F*.

In order to evaluate the non-linear joint finite element formulation two geometric non-linear analysis of the mentioned structure where performed including the joint plastification. The difference between the analyses was related to the considered moment versus rotation curve associated with the joint structural response. The first analysis used a complete moment versus rotation curve while the second adopted a bi-linear approximation, Fig. 8.

The Keulen et al. [4] considered two analyses: a *Reference Analysis* using ANSYS version 5.5. To obtain the reference solution, a second-order elastic-plastic frame analysis is used. Plastic hinges in the beams are modelled using rotational spring elements at locations where plastic hinges are expected to occur. These rotational spring elements have a rigid-plastic characteristic neglecting the influence of normal and shear forces on the plastic moment capacity. Fig. 8 illustrates the spring properties used for the base joints, eaves joints and the beam springs. The second analysis, *Half Initial Secant Stiffness Approach*, modelled the portal frame using a bi-linear moment versus rotation curve considering half of the joint secant initial stiffness.

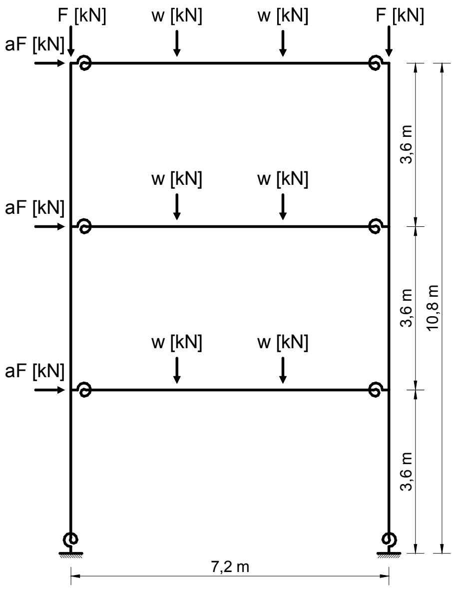

Fig. 6: Multistorey frame [4]

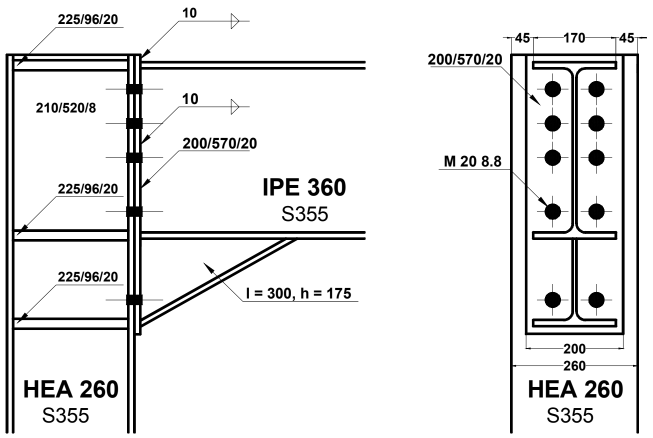

Fig. 7: Beam to column joint details [4]

It can be observed in Figures 9 to 11 that the lateral displacements at the third, second and first storey level, obtained with the proposed formulation, are very similar to Keulen et al [4], thus demonstrating the adequacy of the proposed connection element, Del Savio [1].

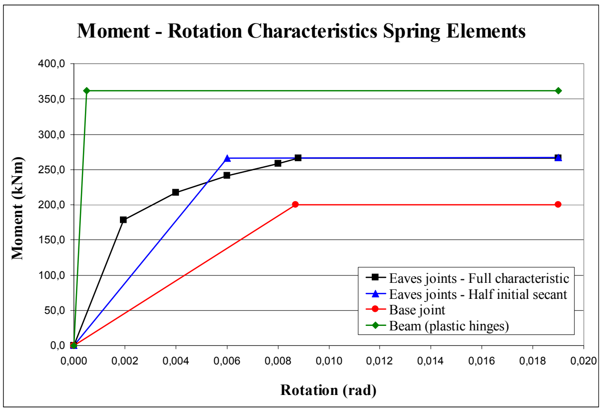

Fig. 8: Moment versus rotation curves [4]

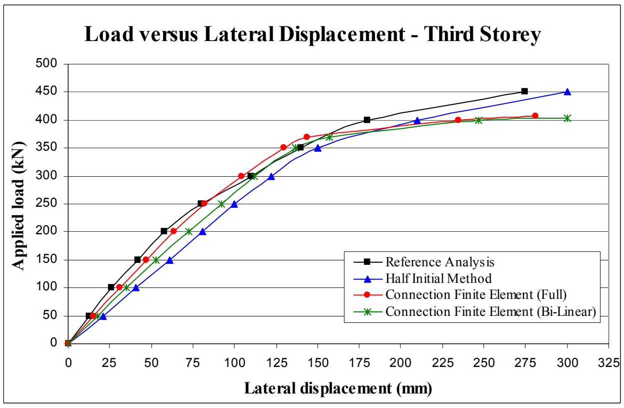

Fig. 9: Load versus deformation diagram

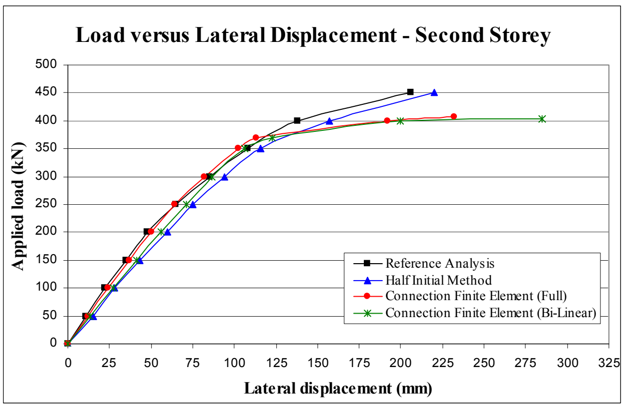

Fig. 10: Load versus deformation diagram

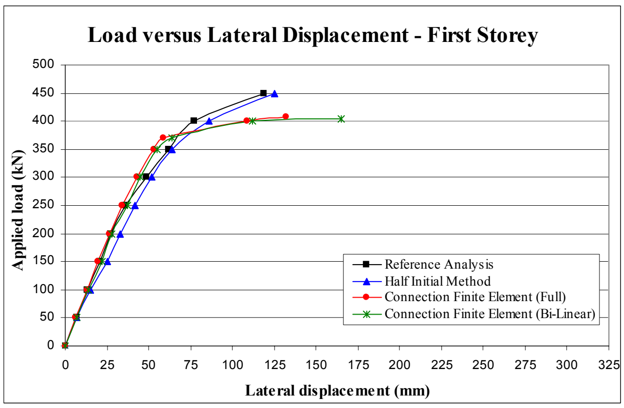

Fig. 11: Load versus deformation diagram

5 FINAL CONSIDERATIONS

Linear and non-linear formulations for the proposed joint finite element have been tested with success, Del Savio [2]. Typical application example of the joint element use in single storey steel frames with semi-rigid connection can be found in Del Savio et al. [1]. In order to validate the joint finite element stiffness matrix modification procedure in a non-linear analysis, a more complex structure was investigated, Keulen et al. [4].

The modification of the connection element stiffness matrix, Del Savio [2], in terms of the lateral displacements of the three storeys, when used in a non-linear structural analysis, produced results close to the *Reference Analysis* [4], (second order elasto-plastic analysis), and better than the *Half Initial Secant Stiffness Approach* [4]. This is evident in Figs. 9 to 11, up to the onset of yielding of the structure. From this point on, as expected, there is a slight difference of results. This difference of results was expected since only the geometric and joints non-linearities were considered in the present analysis.

The non-linear semi-rigid joint finite element formulation presented in the present paper was developed considering the shear force influence (Timoshenko). However, this effect was not considered in the calibration examples because their main aim was to only investigate the influence of the joint rotation stiffness over the global portal frame structural response. This fact also explains why the normal force effect was also not taken into account in the analysis. The shear and normal stiffness were not considered by the use of a rigid link for their associate degrees of freedom. This strategy did not modify their values inside the implemented joint element. Alternatively, only the bending moment transmission capacity was altered, according to the moment versus rotation curve adopted in the joint element.

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REFERENCES

[1] Del Savio, A.A., Andrade, S.A. de, Vellasco, P.C.G.S., Martha, L.F.C.R, A Non-Linear System for Semi-Rigid Steel Portal Frame Analysis, *Proceedings of the Seventh International Conference on Computational Structures Technology - CST2004*, v.1, p.1-12., 2004.

[2] Del Savio, A.A., *Computer Modelling of Steel Structures with Semi-rigid Connections*, MSc. Dissertation, Civil Eng. Depart. – PUC-Rio, Brazil, (in Portuguese), 152p., 2004.

[3] Martha, L.F.C.R, Parente Jr, E., An Object-Oriented Framework for Finite Element Programming, *Proceedings of the Fifth World Congress on Computational Mechanics*, IACM, Vienna, Austria, Jul. 2002.

[4] Keulen, D.C. van, Nethercot, D.A., Snijder, H.H., Bakker, M.C.M.. Frame analysis incorporating semi-rigid joint action: Applicability of the half initial secant stiffness approach, *Journal of Constructional Steel Research*, v. 59, 1083-1100, 2003.

[5] Chan, S. L. and Chui, P. P. T., *Non-Linear Static and Cyclic Analysis of Steel Frames with Semi-Rigid Connections*, Elsevier Science Ltd, 336p., 2000.

KEYWORDS

Semi-rigid connections, semi-rigid connection modelling, non-linear finite element analysis, semi-rigid steel portal frames, joint finite element non-linear formulation.