NUMERICAL MODELLING OF FRACTURE AND FATIGUE PROCESSES

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ABSTRACT

This paper describes an interactive graphics computational system with a self-adaptive, integrated, two-dimensional finite element analysis code. This system is named **Quebra2D** and is able to handle both standard structural and fracture mechanics problems through different approaches: linear elastic fracture mechanics, elastoplastic fracture mechanics, the Delft lattice model and continuum damage models. The self-adaptive strategy is based on recursive spatial enumeration techniques: a binary tree partition for the boundary and the crack-line curves definition, and a quadtree partition for domain mesh generation. The system is capable of deciding where to refine an initial mesh, of redoing the analysis, and of repeating this procedure until a pre-defined convergence criterion is achieved. Cracks can be introduced arbitrarily by the user at any position in the model. The system regenerates the meshes automatically taking into account the new created crack surfaces. The self-adaptive procedure is also considered in the crack propagation process. This procedure takes into account the arbitrarily generated crack geometry and the finite element error estimation analysis. **Quebra2D** is integrated to second program to model fatigue processes. This program, named **ViDa**, is a general-purpose fatigue design software developed to predict both initiation and propagation fatigue lives under variable loading by all classical design methods, including load interaction effects. Fatigue crack propagation in complex two-dimensional structural components under constant and variable amplitude loading is numerically predicted and experimentally verified.

1. INTRODUCTION

This work describes a self-adaptive methodology for simulating crack or damage propagation in structural and equipment components. The adaptive procedure provides a regular mesh refinement for the free-boundary curves (including cracks) and is based on *a posteriori* error estimation. An *h*-refinement strategy is utilized in this process [1].

One of the main objectives in fatigue design is the prediction of crack growth rate with respect to the number of cycles of a cyclic solicitation. Traditionally, empirical models [2] are used for crack growth prediction. These models need the stress intensity factor history along the crack path, which is only available for typical (and simple) component geometry and loading [Fatigue manuals]. **Quebra2D** computes the stress intensity factor history for arbitrary component and crack geometries (2D).

The system has been implemented using the IUP user interface system [3] and the CD graphics system [4]. It has a flexible user interface that allows the visualization of the model and its results and responses at any time during the simulation.

Each step of the crack propagation simulation consists of:

- a finite element analysis of the initial mesh with the initial crack defined by the user;
- computation of stress intensity factors;
- determination of direction of crack extension and new tip location;
- updating of crack geometry;
- automatic remeshing;

Within the linear elastic fracture approach, **Quebra2D** allows the computation of the stress intensity factors through the following techniques: (a) the displacement correlation technique, (b) the modified crack closure integral and (c) J-integral with the equivalent domain integral method [5]. The crack direction of the next step of propagation is determined through the following interaction theories: (a) the maximum circumferential stress, (b) the maximum potential energy release rate and (c) the minimum strain energy density [5].

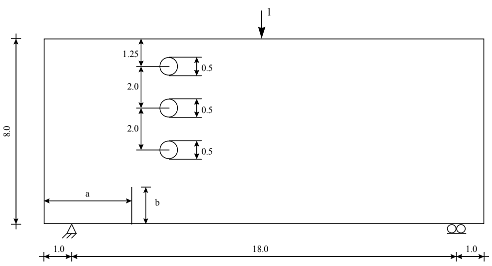
Quebra2D also allows its user to model crack evolution in concrete through a lattice model. In the "Delft lattice model" concrete is schematized as a network of two noded Bernoulli beams. Concrete is a heterogeneous material, and the most straightforward way to include effects from heterogeneity is through direct implementation of disorder. For concrete three material phases are distinguished, namely aggregate, matrix and interfacial transition zone (bond). Different stiffness and strength are assigned to the beams that fall in each phase. By removing in each loading step the beam element with the highest stress over strength ratio, fracture is simulated.

Fatigue crack propagation life prediction in intricate 2D structural components is a challenging problem, involving the calculation of the crack path, the associated stress intensity factors, and the crack propagation rates at each step [1, 2, 3]. The solution of this problem is achieved and experimentally validated by dividing it into two complementary global and local approaches, as described in detail in [4, 5]. First, self-adaptive finite elements are used to calculate, by means of three different methods, the (generally curved) fatigue crack path and the stress-intensity factors $\mathbf{K_I(a)}$ and $\mathbf{K_{II}(a)}$ under *simple* loading along the crack length **a**. This global method alone is not computationally efficient under variable amplitude loading, because it requires remeshing procedures and FE recalculations of the *whole* structure's stress/strain field at each load event. Instead, an analytical expression can be fitted to the discrete $\mathbf{K_I(a)}$ values calculated under *simple* loading and exported to a local approach program, where the direct integration of the crack growth rate equation can be efficiently used to calculate crack propagation under variable amplitude loading at each load event.

Two complementary software have been developed to implement this two-step hybrid methodology. The first one, named **Quebra2D**, is an interactive graphical program for simulating two-dimensional fracture processes based on a finite-element (FE) self-adaptive mesh-generation strategy. In this program, the crack increment direction and the stress-intensity factors are calculated using three different FE methods. The second program, named **ViDa**, is a general-purpose fatigue design software developed to predict both initiation and propagation fatigue lives under variable loading by all classical design methods, including load interaction effects. In particular, its crack propagation module accepts any stress-intensity factor expression, including the ones generated by the finite-element software.

2. LINEAR ELASTIC FRACTURE EXAMPLE

The following figures illustrate the automatic crack propagation simulation. Figure 1 shows the model with material properties and geometric information. Figure 2 shows the initial mesh with the defined crack. Figure 3 shows the experimental results. At the crack tip *quarter-point* singular elements have been used.



$E = 300\ ksi$

$\nu = 0.30$

$t = 0.5\ in$

$a = 5\ in$

$b = 1.5\ in$

Figure 1 - Model with atributtes and geometrics informations (dimensions in inches)

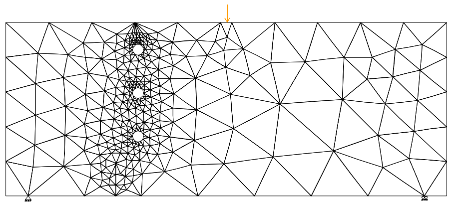


Figure 2 - Model with initial crack and mesh.

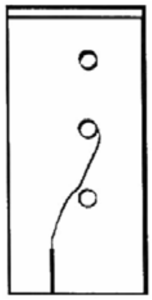


Figure 3 - Experimental trajectory observed.

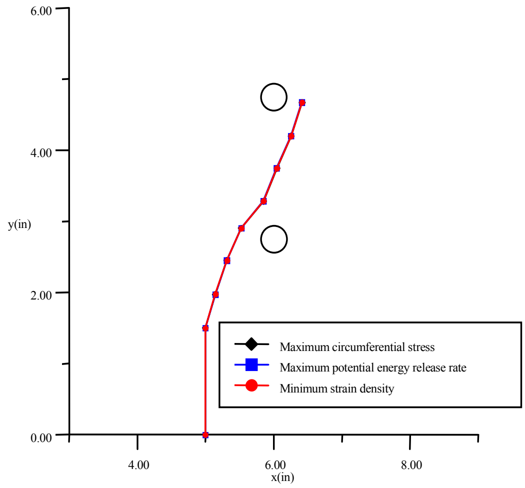


Figure 4 – Crack trajectory obtained with increment de 0.5in.

The predicted crack trajectory is shown in Figure 4. A detail of the final mesh obtained after propagation and adaptive refinement is depicted in Figure 5.

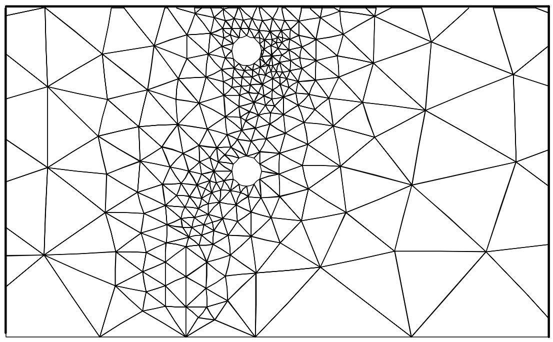


Figure 5 - Final mesh after the crack propagation and adaptive refinement.

3. A LATTICE MODEL APPLICATION

To illustrate the Lattice Model capability of **Quebra2D**, the well-known round-robin analysisy of anchor bolts embedded in concrete proposed by RILEM TC 90-FMA [6] has been analyzed. The geometry and the material properties used are shown in Fig. 6. Plane stress is assumed for this analysis. Two situations, with and without lateral confinement, lateral are considered ($k = 0$ e $k = \infty$). One region near to the anchor bolt is selected to be substituted by a more refined mesh, while in other areas finite elements (T6) are used. The material heterogeneity is then implemented in this selected area using the Weibull distribution. For this work, the two parameters for Weibull distribution are chosen as: the scale parameter equal to 2MPa and the shape parameter equal to 1MPa. The lattice beam length has been chosen to be 1mm and the beam height has been given from the relation *h*/*l* with the global Poisson ratio [6]. The connection between the two types of elements has been done by constraint equations.

After the application of a uniform unit vertical displacement on the upper side of the specimen, the same fracture criterion is used as described in [6], which is based on the effective stress.

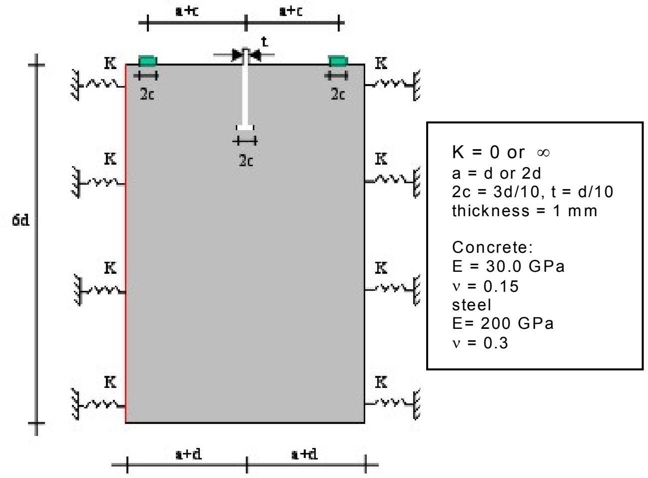
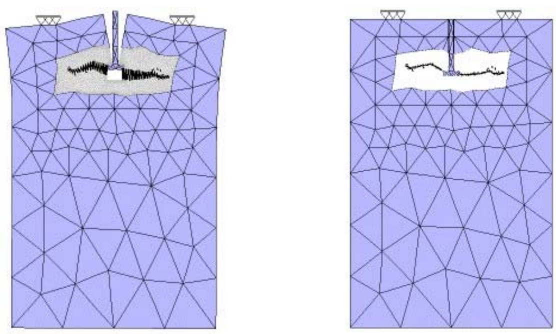


Figure 6 - Geometry and proposed material properties

Figure 7 shows the deformed configuration and the crack evolution for the case of $k = 0$. It may be observed that the cracks develop around the anchor bolt and propagate horizontally. Figure 8 presents the deformed configuration and the crack propagation for the case of $k = \infty$, i.e, with lateral confinement. Observe that now the cracks are developed initially around the anchor bolt and propagated horizontally until a certain stage. Then, due to the lateral confinement, they can not propagate horizontally any more and curve upward in direction to the upper supports. This fact is confirmed by the laboratory observations.

Table 1 shows a comparison of numeric results obtained by the system **Quebra2d** with the experimental data obtained by TUDelft [6]. The numeric results obtained are quite satisfatory and match in the variations of experimental results.



100 beams removed 100 beams removed

Figure 7 - Deformed structure and crack evolution for $k = 0$ (a=d=150mm)

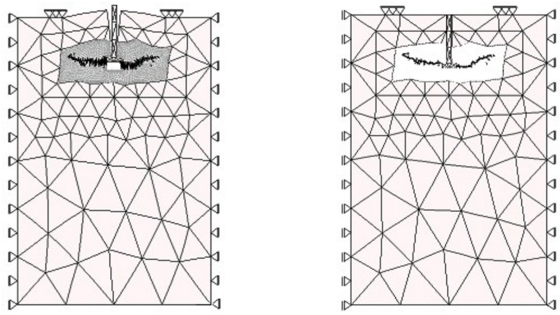


Figure 8 - Deformed structure and crack evolution for $k = \infty$ (a=d=150mm)

Table 1 - Comparison of predicted peak load with the experimental data

	Experimental	Quebra2D
a = d	44,5kN (2,08)	43,4kN
a = 2d	33,3kN (3,60)	31,8kN

*numbers in parentheses are standard deviations (kN) and d = 150mm

4. FATIGUE MODELLING

In this section, the integration between **Quebra2D** and **ViDa** is explored. This two-step methology include load interaction effects, such as crack retardation after tensile overloads and crack acceleration due to compressive underloads [7, 8, 9]. Elber-type crack retardation models are calibrated using regular compact tension (CT) specimens under variable amplitude loading, and the calibrated parameters are used to predict the fatigue lives of modified CT specimens, in which holes were machined to curve the crack propagation path (Figure 9).

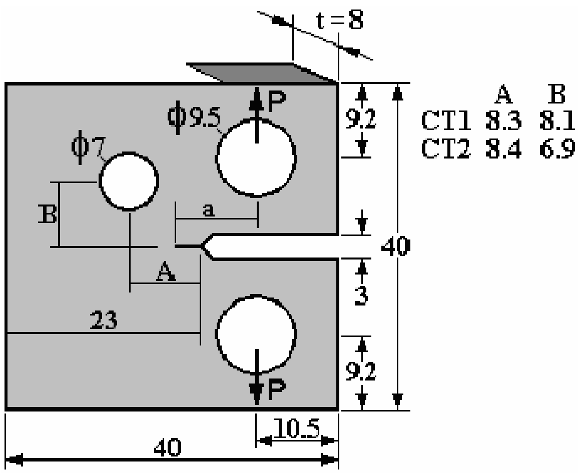


Figure 9 – Details of the modified CT specimens (mm).

Figure 10 shows a screen output of the **Quebra2D** software with a final meshed model. Figure 11 shows the calculated $\mathbf{K_I}$ values for the regular and the modified CT specimens, which can be easily exported to the **ViDa** software to predict the fatigue life including load interaction effects.

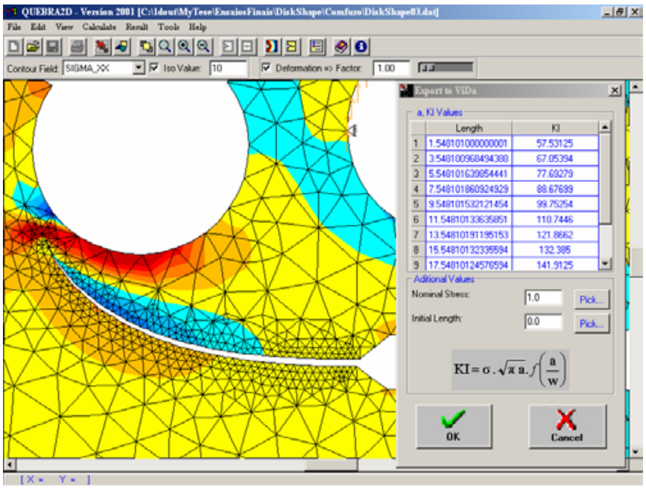


Figure 10 – Screen output of the **Quebra2D** software.

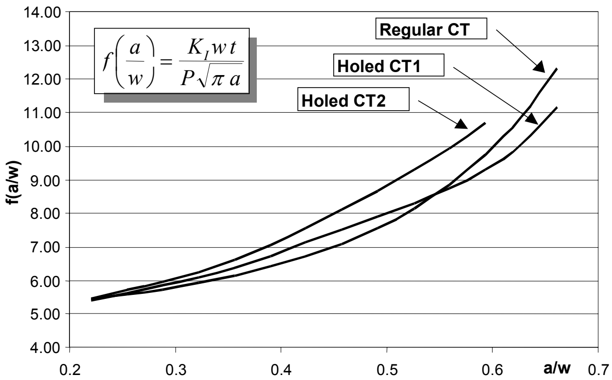


Figure 11 – **f(a/w)** curves for the standard and for the modified CT specimens.

All crack growth tests were performed at 20 and 30 Hz frequencies in a 250kN computer-controlled servo-hydraulic testing machine. The loads were regularly tuned to keep the specified stress intensity factors. In addition, a digital camera was used with an image analysis program to measure the crack size *and* path. After the tests the measured loads were inserted into the **ViDa** software, which used the FE calculated $\mathbf{K_I}$ values to predict the specimen life. Figure 12 shows a screen output of the **ViDa** software, which includes choices of several editable **da/dN** curves, $\mathbf{K_I}$ and $\mathbf{K_{II}}$ equations, and load interaction models. Note that the loading history can be represented by a sequential list of peaks and valleys (σ_{max}, σ_{min}), or else by the equivalent sequence of alternate and mean stresses and number of reversions (σ_a, σ_m, **2N**).

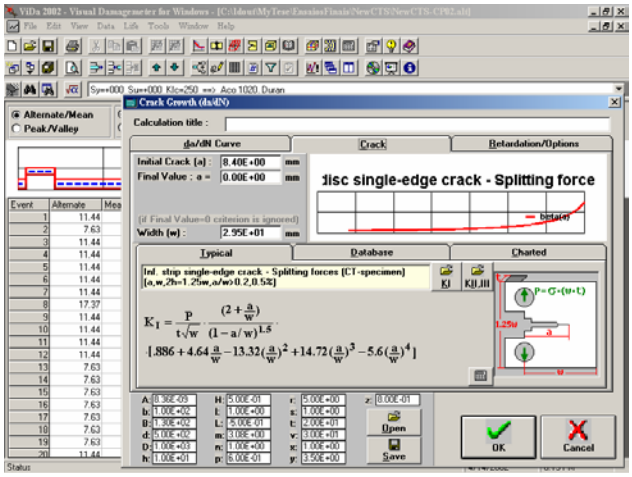


Figure 12 – Screen output of the **ViDa** software.

Two CT specimens (CT1 and CT2) were tested under simple (constant amplitude) loading. Using the calculated $\mathbf{K_I(a)}$, each load program was adjusted to maintain a quasi-constant stress-intensity range around $\mathbf{\Delta K_I \approx 20MPa\sqrt{m}}$, with $\mathbf{R = K_{min}/K_{max} = 0.1}$. These loading values induce a stage-II (Paris regime) crack growth in the 1020 steel **da/dN** curve.

The measured and the predicted curved crack paths present a good match (Figure 13). The fatigue lives of both specimens under simple loading are very well predicted in **ViDa** using **da/dN** crack propagation data measured in standard (straight cracked) CT specimens under pure Mode-I loading (Figures 14 and 15).

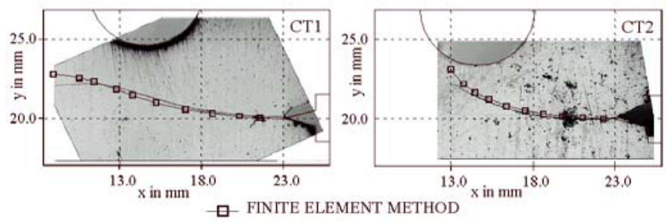


Figure 13 – Predicted and measured crack paths for the modified CT specimens (mm).

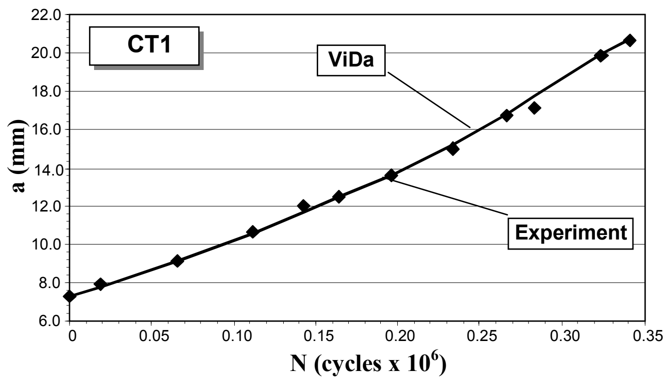


Figure 14 – Predicted and measured fatigue crack growth for the CT1 specimen.

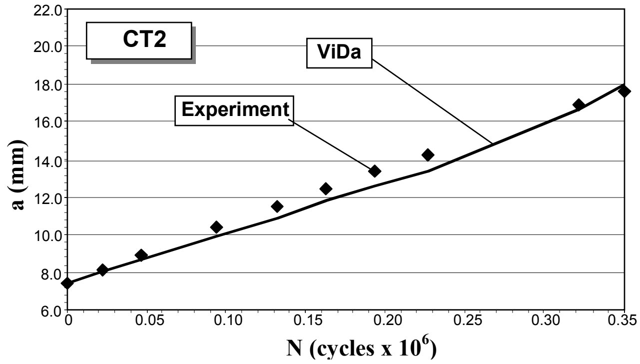


Figure 15 – Predicted and measured fatigue crack growth for the CT2 specimen.

Details of the modelling procedures and the variable amplitude loading simulations can be found in [7].

5. CONCLUSIONS

It has been shown that the interactive graphics computational system developed is capable of accurately predicting the crack propagation paths for linear elastic fracture problems.

The Lattice Model available in **Quebra2D** is an efficient tool to study the crack propagation in concrete structures. It allows one to monitor the geometrical input and the material properties, the finite element analysis and to visualize the numerical results, all in a graphics environment.

It has been observed that, to some extent, a rather coarse mesh can be used with statistically distributed beam properties to analyse the behaviour of a large-scale structure instead of using a much more refined mesh with an underlying aggregate structure. Combining the statistical distribution with the technique of selecting just the region with high concentration of tensile stress and keep the other regions with coarse continuum element, the efficiency in reducing the computational effort increases even relevantly. In this way the computational problems encountered in lattice-type fracture models for the analysis of large-scale structures can be dismissed.

A two-phase methodology was presented to predict fatigue crack propagation in generic 2D structures. First, self-adaptive finite elements were used to calculate the fatigue crack path and the stress-intensity factors along the crack length $\mathbf{K_I(a)}$ and $\mathbf{K_{II}(a)}$, at each propagation step. The computed $\mathbf{K_I(a)}$ was then used to calculate the propagation fatigue life by the local approach, considering overload-induced crack retardation effects.

Two complementary software have been developed to implement this methodology. The first is an interactive graphical program for simulating two-dimensional fracture processes based on a finite-element adaptive mesh-generation strategy. The second is a general-purpose fatigue design software developed to predict both initiation and propagation fatigue lives under variable loading by all classical design methods. Particularly, its crack propagation module accepts any stress-intensity factor expression, including the ones generated by the finite-element software.

Experimental results validated the proposed methodology. Moreover, the developed software demonstrated that effective and economical predictions of crack propagation paths and fatigue lives can be obtained for arbitrary two-dimensional structural components under variable amplitude loading

6. ACKNOWLEDGEMENTS

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