A Lagrange Multiplier Formulation for Considering Member Deformation Constraints into Matrix Structural Analysis

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This paper is concerned with the analysis of framed structures with inextensible and rigid members, i.e. members without axial and bending strains. Rigid and inextensible members may be useful in educational software because they capture the essence of the structural behavior with a reduced the number of variables. In addition, they allow a comparison with results obtained by hand calculation using classical structural analysis methods. There are three main approaches to constraint handling: transformation, penalty function and Lagrange multiplier methods [1,2,3]. The transformation methods, also known as master-slave elimination, use each constraint to eliminate one equilibrium equation, reducing the number of degrees of freedom. However, this approach does not allow the determination of axial forces in inextensible member or bending moments in rigid members. The implementation of the penalty function method is trivial, since physically it corresponds to assign a very large number to the axial stiffness (EA) of inextensible members and a very large number to the bending stiffness (EI) of rigid members. However, as the penalty factor increases, the stiffness matrix becomes increasingly ill-conditioned, leading to large solutions errors. This paper presents a methodology for considering structural member deformation constraints using Lagrange multipliers. It consists of adding strain constraints into the total potential energy minimization, leading to a quadratic programming problem. In addition, this approach is very suitable for computational implementation because it does not affect the generic characteristic of a matrix structural analysis. The solution gives rise to one Lagrange multiplier per constraint, which is essential for computing member internal forces. However, there are situations in which inextensible and rigid member constrains may be redundant, which prevents the determination of dependent Lagrange multipliers. In these cases, it is not possible to determine internal forces in the members with redundant constraints. Although not implemented, a special treatment is indicated for determining member internal forces in these situations.

References

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