2D Computer Graphics

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Summer 2020

IMPA

Gradients in SVG [SVG, 2011]

Color ramp

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that maps the interval $[0,1]$ to colors with transparency

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$c(t)$ is linear by parts

$$c(t) = \frac{(t_{i+1} - t)\, c_i + (t - t_i)\, c_{i+1}}{t_{i+1} - t_i}, \quad t_i \leq t < t_{i+1}$$

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$$\begin{aligned}
\mathrm{pad}(t) &= \min\big(1, \max(0,t)\big)\\
\mathrm{repeat}(t) &= t - \lfloor t \rfloor\\
\mathrm{reflect}(t) &= 2\big|\tfrac{1}{2}\,t - \lfloor \tfrac{1}{2}\,t + \tfrac{1}{2}\rfloor\big|
\end{aligned}$$

Linear gradient mapping

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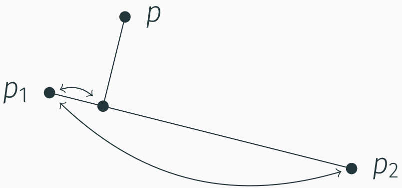

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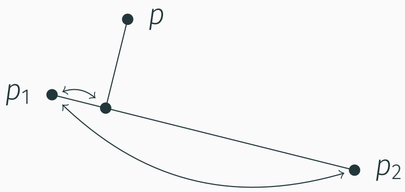

It computes the normalized projected length of $p - p_1$ into $p_2 - p_1$

$$\ell(p) = \frac{\langle p - p_1,\, p_2 - p_1 \rangle}{\langle p_2 - p_1,\, p_2 - p_1 \rangle}$$

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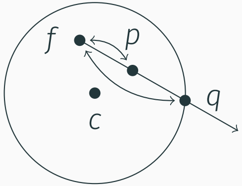

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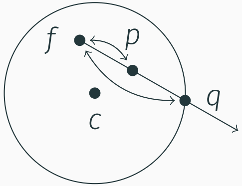

It computes the length ratio of from point p to f and q to f

$$r(p) = \frac{\|p - f\|}{\|q - f\|}$$

where q is the intersection between the ray from focal point f to point p and the circle centered at c with radius r

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Similarly, every paint includes a transformation T_p that maps points from *paint coordinates* (where the color is computed) to *scene coordinates* (where the color is painted)

If you want to apply a transformation T to a shape and want its paint to move with it, simply compose

$$T'_o = T \circ T_o$$
$$T'_p = T \circ T_p$$

A *linear gradient* is a function

$$\mathsf{R}^2 \to \mathrm{sRGBA}$$

formed by the composition of a paint transform T_p, a linear gradient mapping ℓ, a wrapping function s, and a color ramp c

$$p \mapsto c\Big(s\big(\ell(T_p^{-1}\, p)\big)\Big)$$

Gradient paints

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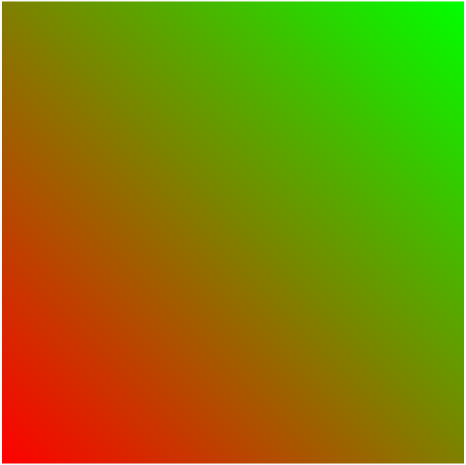
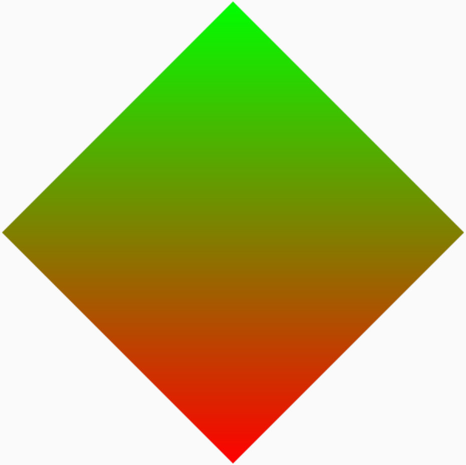
Show in Inkscape

Examples





A *radial gradient* is a function

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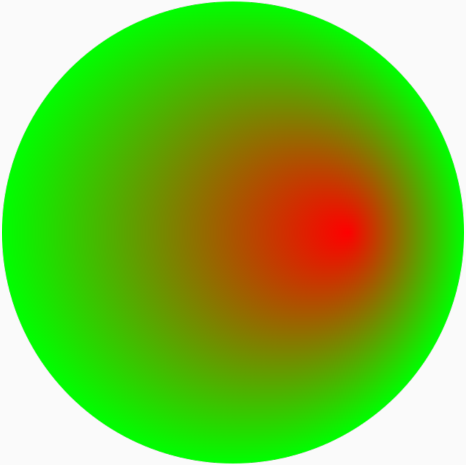

How to efficiently evaluate a ramp

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How to efficiently evaluate linear and radial mappings?

- How many parameters are really needed?

Gradients in PostScript and PDF

Shading types

Type 1: Function-dictionary-based shading

- Basically texture mapping
- Show EPS file
- Will discuss in following classes

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Type 2: Axial shading

- Same as linear gradient
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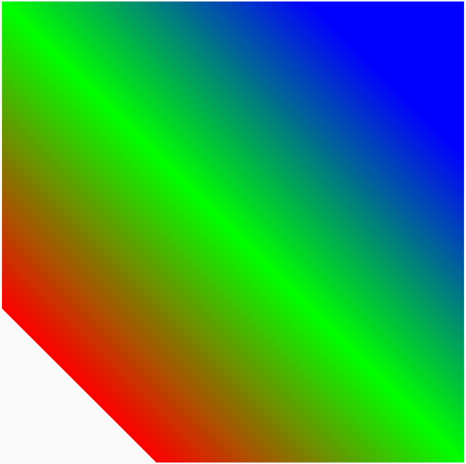

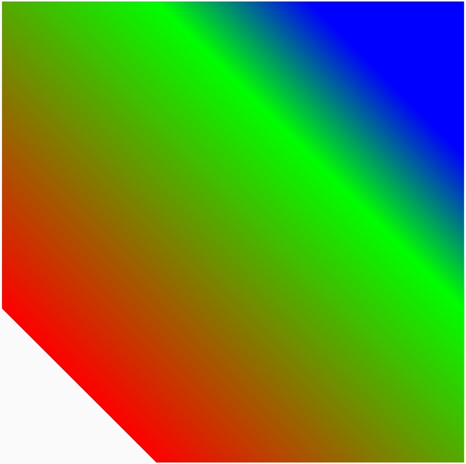

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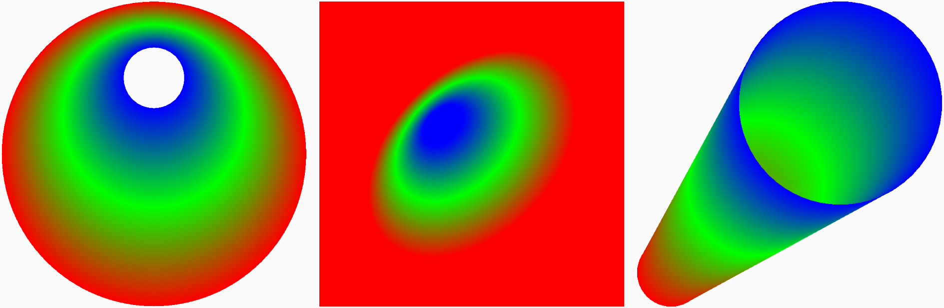

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$$p(s,t) \mapsto c(s,t)$$

with

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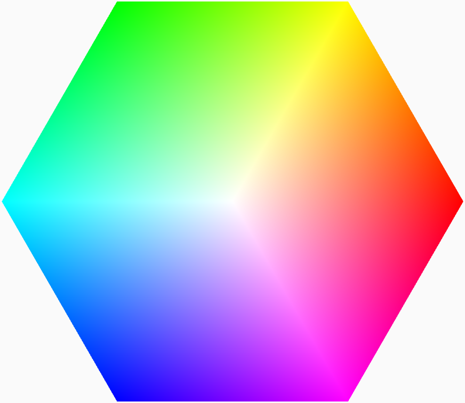
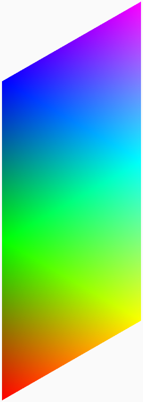
Type 5: Lattice-form Gouraud-shaded triangle mesh

- Same, but for a “regular” grid of triangles

Examples





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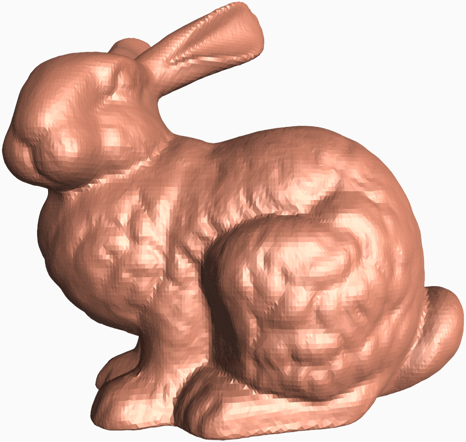

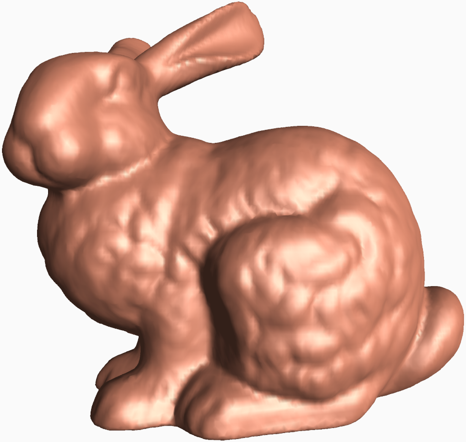

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- Each patch is defined by 4 connected cubic Bézier segments

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- Curves are setup to share endpoints like such

$$\begin{aligned} v_{00} &= v_0(0) = h_0(0) & \qquad v_{01} &= v_0(1) = h_1(0) \\ v_{10} &= v_1(0) = h_0(1) & \qquad v_{11} &= v_1(1) = h_1(1) \end{aligned}$$

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Shading types

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- Note that $v(s,t)$ and $h(s,t)$ interpolate all shared vertices

Type 6: Coons patch mesh (continued)

- Define bilinear map $m : V^4 \times [0,1]^2 \to \mathsf{R}^2$

$$m_{c,d}^{a,b}(s,t) = (1-s)(1-t)\,a + (1-s)\,t\,b + s\,(1-t)\,c + s\,t\,d$$

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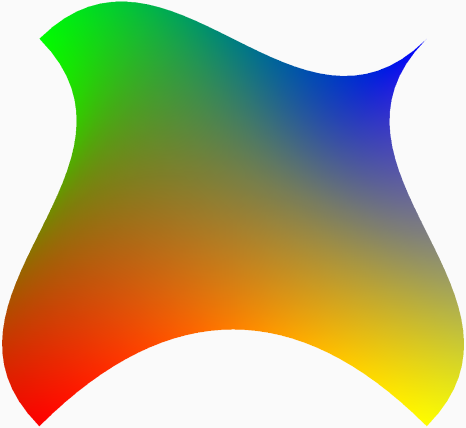

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where $b_{i,3}$,$b_{j,3}$ are the cubic Bernstein polynomials

Shading types

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- (Coons patch is a special case of tensor-product patch)

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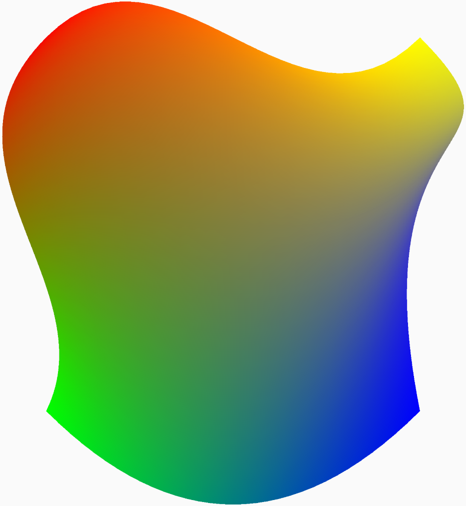
$$p(s,t) \mapsto m_{c_{10},c_{11}}^{c_{00},c_{01}}(s,t)$$

- Patches can be defined independently or connected by strips
- (Coons patch is a special case of tensor-product patch)
- Show EPS file



Examples

References

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