2D Computer Graphics

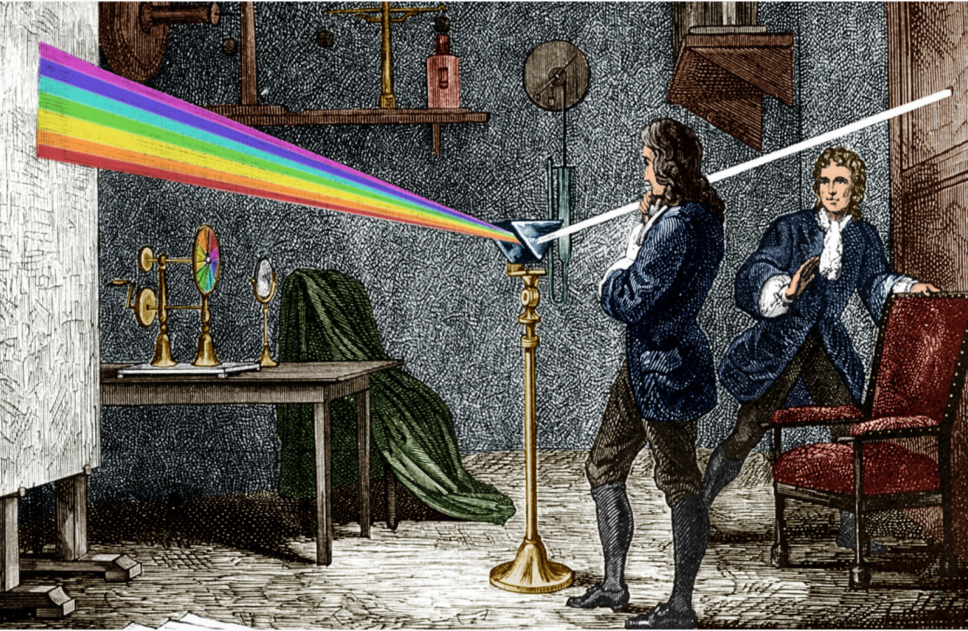
Diego Nehab

Summer 2020

IMPA

Color and compositing

The prism experiment



Visible light: prism experiment (Newton, 1666)

More than visible light

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Infrared light: thermometers (Herschel, 1800)

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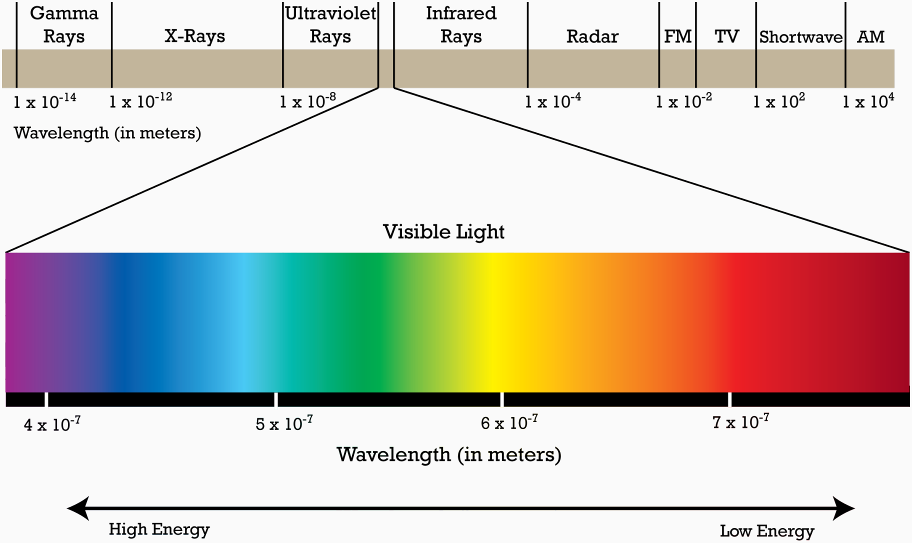
Infrared light: thermometers (Herschel, 1800)

Ultraviolet light: silver chloride (Ritter, 1801)

Full electromagnetic spectrum



Radiometry

Measurement of radiant energy in terms of absolute power

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Wave vs. particle

- Wavelength (λ), frequency ($\nu = \frac{c}{\lambda}$), and amplitude (A)
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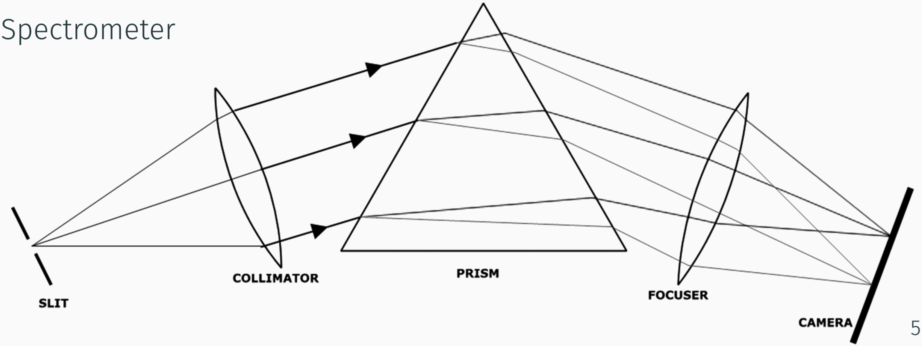
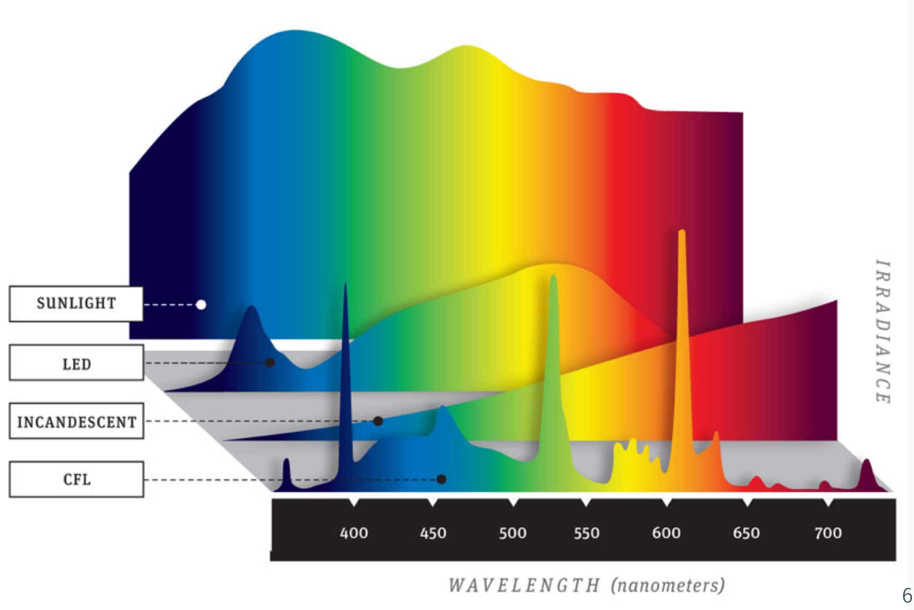
Spectrometer



Colors are spectral distributions



As a continuous function of $c(\lambda)$

$$c : \mathsf{R}_{>0} \to \mathsf{R}_{\geq 0}, \quad \lambda \mapsto A_\lambda$$

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As a discrete set of values $c(\lambda_i)$

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Spectral representation

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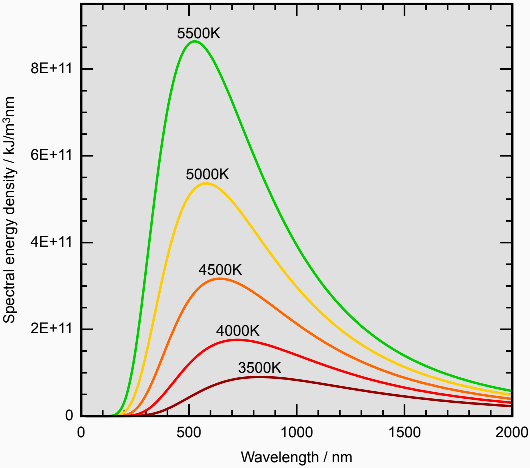
Light emitter has a spectrum, material properties modulate the reflected spectrum (Fluorescence is something else)

Black-body radiation

$$B(\nu, T) = \frac{2h\nu^3}{c^2}\left(e^{\frac{h\nu}{k_B T}} - 1\right)^{-1}, \quad \text{where } k_B \text{ is Boltsmann's constant}$$



Photometry

Measurement light in terms of perceived brightness to human eye

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Visible light $\lambda \in [390\mathrm{nm}, 700\mathrm{nm}]$ approximately

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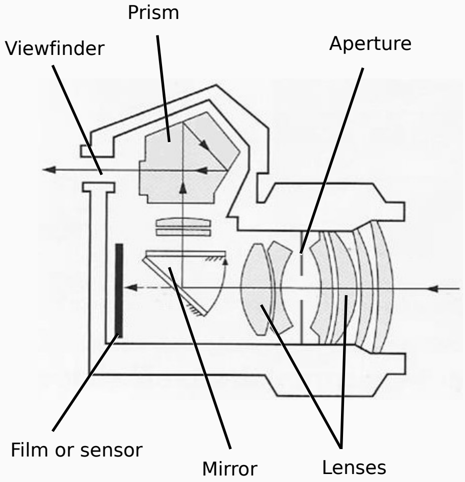

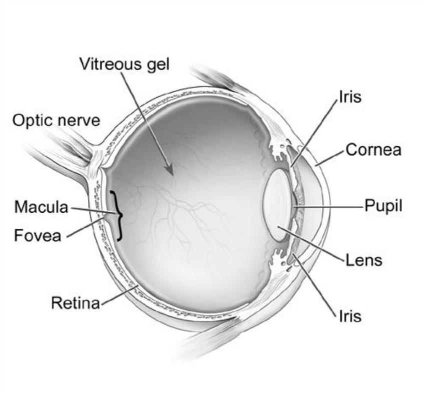

Photopic vision

Well-lit conditions

Photopic vision

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Cones: Three types of retinal cells with distinct spectral responses

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Highly concentrated on fovea

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Response curves S (short λ), M (medium λ), L (long λ)

- Peaks at $\lambda = 420\mathrm{nm}$, $\lambda = 534\mathrm{nm}$, and $\lambda = 564\mathrm{nm}$
- Overlap each other
- Not R, G, and B

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What about the color-blind?

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What about the color-blind?

Are there tetrachromats among us?

Scotopic vision

Low-light conditions

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Rods: One type of retinal cell

Scotopic vision

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Mostly peripheral

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$20\times$ more numerous, $1000\times$ more sensitive than cones

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Response curve

- R: peak at $\lambda = 498\mathrm{nm}$ (between S and M)

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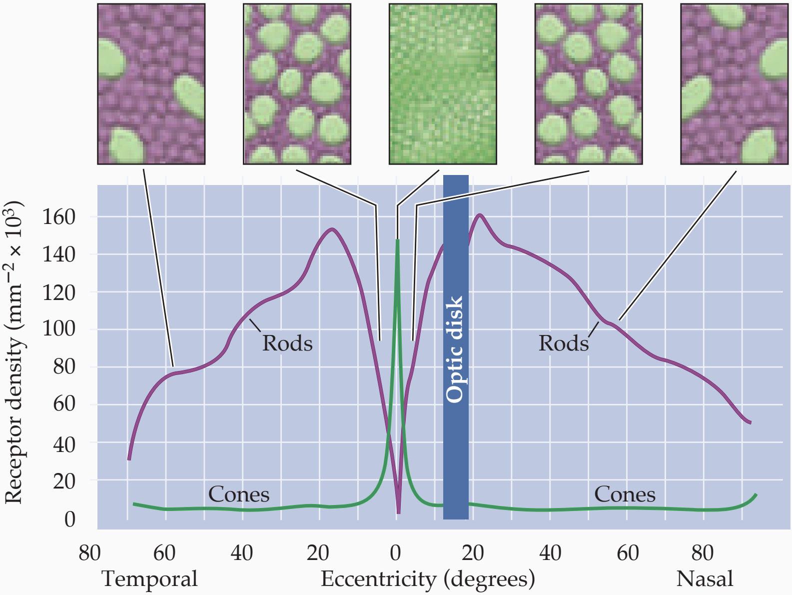
Things look “gray-bluish” at night

Human photoreceptor distribution



Spectral sensitivity $V(\lambda)$ of human perception of *brightness*

Luminous efficiency function

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Different for photopic and scotopic vision

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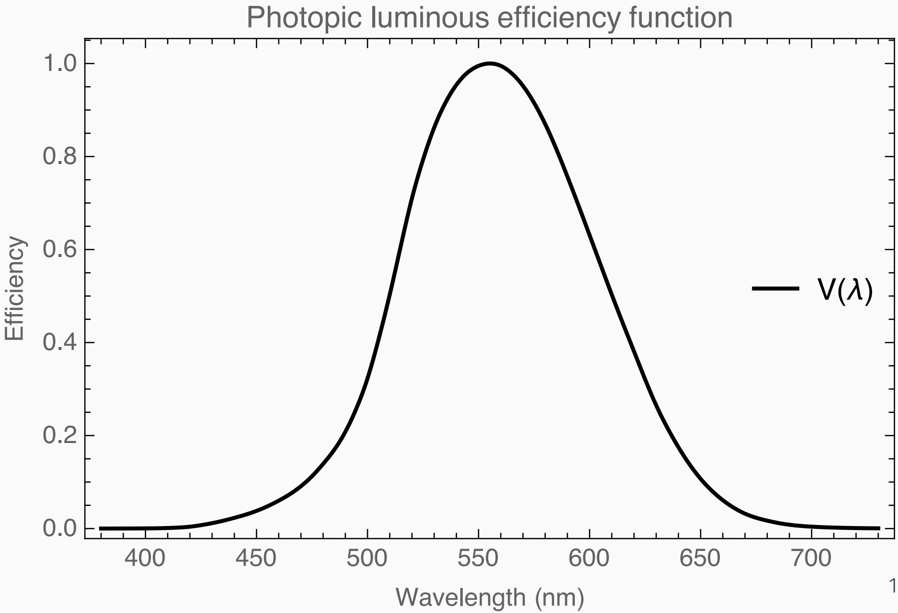
Convert radiant intensity (W/sr) to luminous intensity (cd)

$$v(c) = \int_\lambda c(\lambda)V(\lambda)d\lambda$$


Photopic luminous efficiency function
Efficiency
1.0
0.8
0.6
0.4
0.2
0.0
400
450
500
550
600
650
700
Wavelength (nm)
V(λ)

Nonlinear perceptual response to brightness

Lightness

Nonlinear perceptual response to brightness

Power law

$$L^* \approx \frac{116}{100}\left(\frac{Y}{Y_0}\right)^{\frac{1}{3}} - 0.16$$

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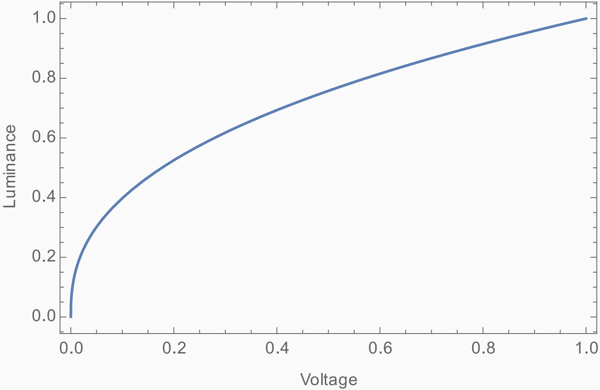
Weber law of just noticeable difference

$$\Delta L^* \approx \frac{Y}{100}$$

Created to compensate for input-output characteristic of CRT displays

$$Y = V^{2.5} = V^{\gamma}$$



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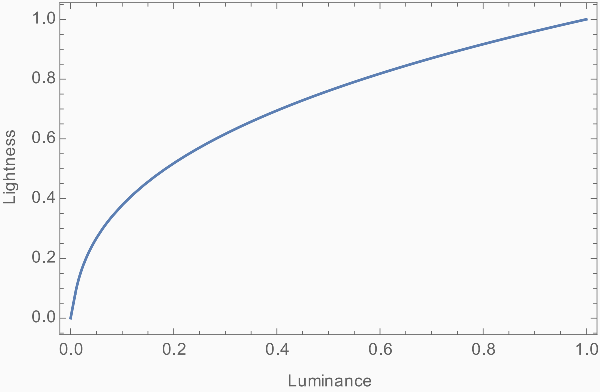
Today, due to remarkable coincidence, it is used for encoding efficiency



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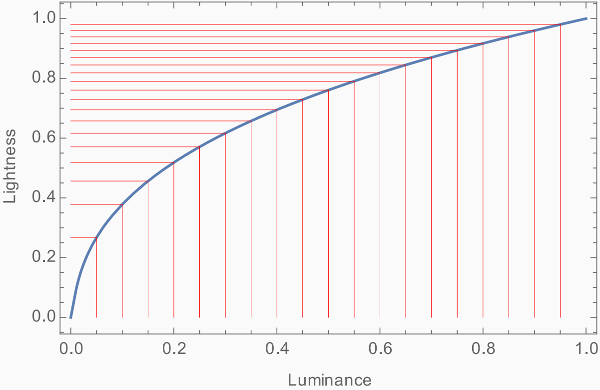

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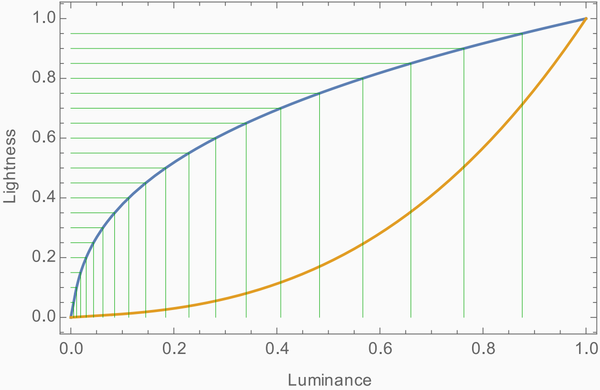
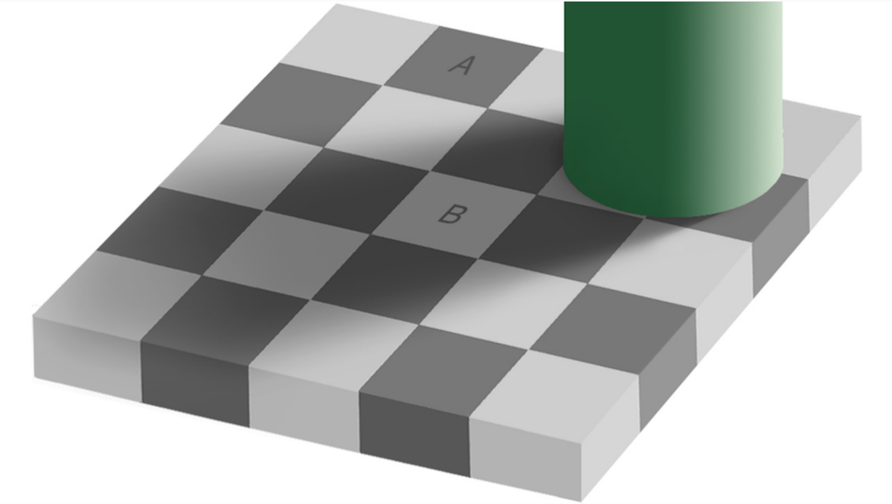


Illusion



Modeling color perception

1st attempt: Measure spectral distribution of stimulus

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- Could use spectrophotometer to measure $c(\lambda)$.
- But how to would you reproduce it?

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2nd attempt: Measure optical nerve response

- Remove eye, attach wires to cones: The Matrix
- Re-inject signal to reproduce
- Painful, but only 3 values per color

3rd attempt: Linear algebra

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Measuring

- $c(\lambda)$ is the target color's spectral distribution

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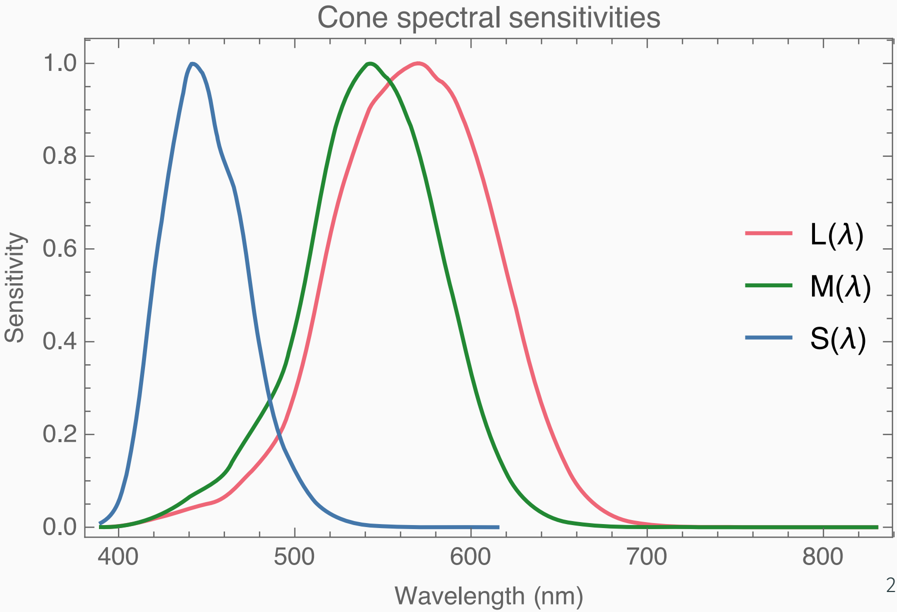
- Inner-product functions f and g is

$$\langle f, g \rangle = \int_{-\infty}^{\infty} f(\lambda) g(\lambda) d\lambda$$

- The cone responses to c must be

$$S_c = \langle c, S \rangle, \qquad M_c = \langle c, M \rangle, \quad \text{and} \qquad L_c = \langle c, L \rangle$$

Cone spectral sensitivities (not to scale)



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- Assume 3 different stimuli colors $r(\lambda)$, $g(\lambda)$, and $b(\lambda)$

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Result R_c, G_c, or B_c could be non-convex

- There is no negative light...

Space of visible colors

All convex combinations of visible monochromatic colors

- Could use entire spectrum

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Obtain R_c, G_c, and B_c directly from c and RGB color matching functions

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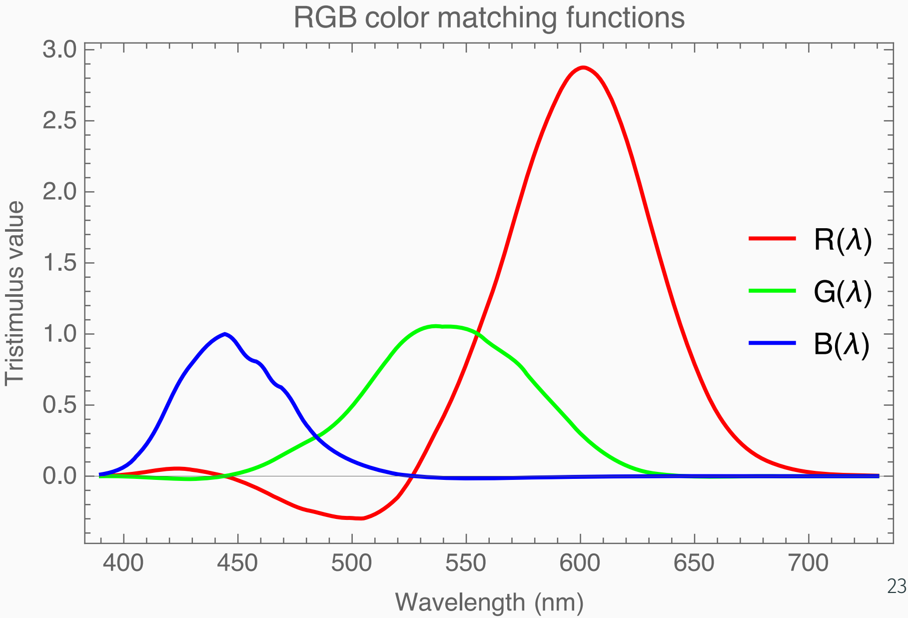
How to measure color matching functions R, G, and B

CIE 1931 RGB color matching functions



XYZ COLOR MATCHING FUNCTIONS

Visible colors always use non-negative coordinates

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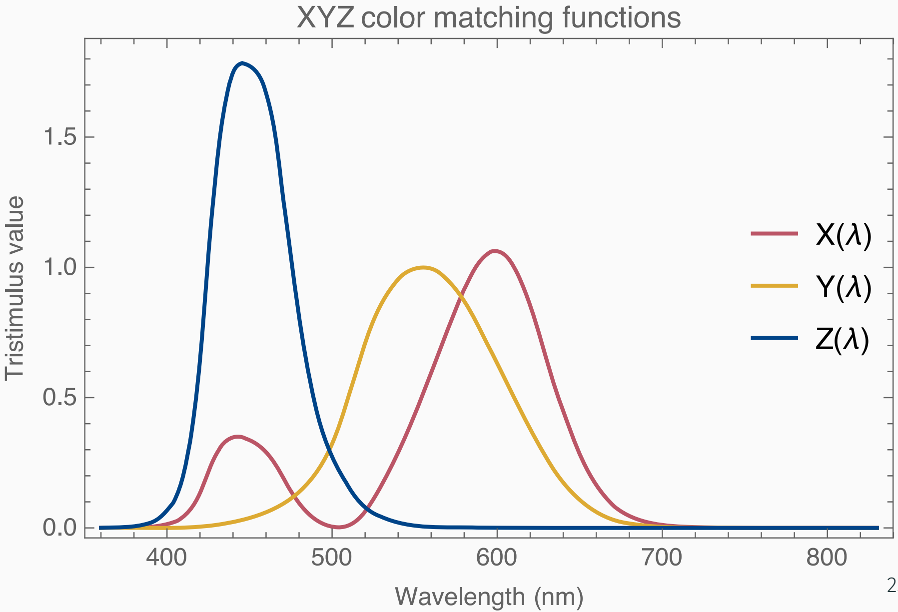
Z ended up almost equal to S

XYZ color matching functions



CIE chromaticity diagram

Similar to $\mathbb{RP}^2$

- Given $\alpha > 0$, $\begin{bmatrix} \alpha X & \alpha Y & \alpha Z \end{bmatrix}$ have same *chromaticity*
- Different *brightness*

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Separation of chromaticity and brightness

$$x = \frac{X}{X + Y + Z} \qquad y = \frac{Y}{X + Y + Z}$$

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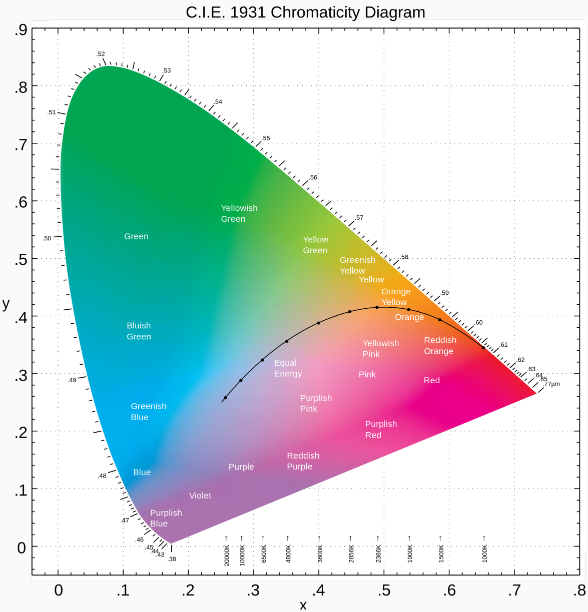

Horseshoe shape

CIE chromaticity diagram

Horseshoe shape

Locus of monochromatic colors

Horseshoe shape

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Locus of black-body colors

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Color gamut

CIE chromaticity diagram

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Color gamut

Color calibration and matching

sRGB [IEC Project Team 61966, 1998]

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} \gamma(R_\ell) \\ \gamma(G_\ell) \\ \gamma(B_\ell) \end{bmatrix}, \quad \begin{bmatrix} R_\ell \\ G_\ell \\ B_\ell \end{bmatrix} = \begin{bmatrix} 3.2406 & -1.5372 & -0.4986 \\ -0.9689 & 1.8758 & 0.0415 \\ 0.0557 & -0.2040 & 1.0570 \end{bmatrix} \begin{bmatrix} X_{\mathrm{D65}} \\ Y_{\mathrm{D65}} \\ Z_{\mathrm{D65}} \end{bmatrix}$$

$$\gamma(u) = \begin{cases} 12.92u & u < 0.0031308 \\ 1.055u^{1/2.4} - 0.055 & \text{otherwise} \end{cases}$$

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Additive (*CMY* and *CMYK*)

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TV (PAL *YUV*, NTSC *YIQ*)

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Perceptual (CIE $L^*a^*b^*$)

sRGB [IEC Project Team 61966, 1998]

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} \gamma(R_\ell) \\ \gamma(G_\ell) \\ \gamma(B_\ell) \end{bmatrix}, \quad \begin{bmatrix} R_\ell \\ G_\ell \\ B_\ell \end{bmatrix} = \begin{bmatrix} 3.2406 & -1.5372 & -0.4986 \\ -0.9689 & 1.8758 & 0.0415 \\ 0.0557 & -0.2040 & 1.0570 \end{bmatrix} \begin{bmatrix} X_{\mathrm{D65}} \\ Y_{\mathrm{D65}} \\ Z_{\mathrm{D65}} \end{bmatrix}$$

$$\gamma(u) = \begin{cases} 12.92u & u < 0.0031308 \\ 1.055u^{1/2.4} - 0.055 & \text{otherwise} \end{cases}$$

Munsel (*HSV* and *HSL*)

Additive (*CMY* and *CMYK*)

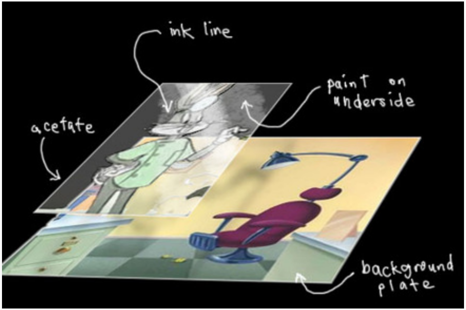
TV (PAL *YUV*, NTSC *YIQ*)

Perceptual (CIE $L^*a^*b^*$)

Opponent color models

Transparency

Semitransparent color f on top of opaque background color b



- Assume probability of light hitting f is α
- Reflected color (integrated over small area) is

$$f, \alpha \oplus b = \alpha f + (1 - \alpha) b$$

- This is what we call *alpha blending* or the *over* operator

Compositing

Now imagine f_1, α_1 on top of f_2, α_2 on top of b

- Reflected color is

$$f_1, \alpha_1 \oplus (f_2, \alpha_2 \oplus b) = \alpha_1 f_1 + (1 - \alpha_1)\big(\alpha_2 f_2 + (1 - \alpha_2) b\big)$$

Now imagine f_1, α_1 on top of f_2, α_2 on top of b

- Reflected color is

$$f_1, \alpha_1 \oplus (f_2, \alpha_2 \oplus b) = \alpha_1 f_1 + (1 - \alpha_1)\big(\alpha_2 f_2 + (1 - \alpha_2) b\big)$$

Can we combine f_1, α_1 and f_2, α_2 into a single material f, α?

$$\begin{aligned} \alpha f + (1 - \alpha) b &= \alpha_1 f_1 + (1 - \alpha_1)\big(\alpha_2 f_2 + (1 - \alpha_2) b\big) \\ &= \alpha_1 f_1 + (1 - \alpha_1)\alpha_2 f_2 + (1 - \alpha_1)(1 - \alpha_2) b \end{aligned}$$

Now imagine f_1, α_1 on top of f_2, α_2 on top of b

- Reflected color is

$$f_1, \alpha_1 \oplus (f_2, \alpha_2 \oplus b) = \alpha_1 f_1 + (1 - \alpha_1)\big(\alpha_2 f_2 + (1 - \alpha_2) b\big)$$

Can we combine f_1, α_1 and f_2, α_2 into a single material f, α?

$$\begin{aligned} \alpha f + (1 - \alpha) b &= \alpha_1 f_1 + (1 - \alpha_1)\big(\alpha_2 f_2 + (1 - \alpha_2) b\big) \\ &= \alpha_1 f_1 + (1 - \alpha_1)\alpha_2 f_2 + (1 - \alpha_1)(1 - \alpha_2) b \end{aligned}$$

So we have

$$\begin{cases} (1 - \alpha) b = (1 - \alpha_1)(1 - \alpha_2) b \\ \alpha f = \alpha_1 f_1 + (1 - \alpha_1)\alpha_2 f_2 \end{cases} \Rightarrow \begin{cases} \alpha = \alpha_1 + (1 - \alpha_1)\alpha_2 \\ \alpha f = \alpha_1 f_1 + (1 - \alpha_1)\alpha_2 f_2 \end{cases}$$

Compositing

$$\begin{cases} (1-\alpha)b = (1-\alpha_1)(1-\alpha_2)b \\ \alpha f = \alpha_1 f_1 + (1-\alpha_1)\alpha_2 f_2 \end{cases} \Rightarrow \begin{cases} \alpha = \alpha_1 + (1-\alpha_1)\alpha_2 \\ \alpha f = \alpha_1 f_1 + (1-\alpha_1)\alpha_2 f_2 \end{cases}$$

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Setting $\tilde{f} = \alpha f, \quad \tilde{f}_1 = \alpha_1 f_1, \quad$ and $\tilde{f}_2 = \alpha_2 f_2$, we obtain

$$\begin{cases} \alpha = \alpha_1 + (1-\alpha_1)\alpha_2 \\ \tilde{f} = \tilde{f}_1 + (1-\alpha_1)\tilde{f}_2 \end{cases}$$

$$\begin{cases}(1-\alpha)b = (1-\alpha_1)(1-\alpha_2)b \\ \alpha f = \alpha_1 f_1 + (1-\alpha_1)\alpha_2 f_2\end{cases} \Rightarrow \begin{cases}\alpha = \alpha_1 + (1-\alpha_1)\alpha_2 \\ \alpha f = \alpha_1 f_1 + (1-\alpha_1)\alpha_2 f_2\end{cases}$$

Setting $\tilde{f} = \alpha f, \quad \tilde{f}_1 = \alpha_1 f_1, \quad$ and $\tilde{f}_2 = \alpha_2 f_2$, we obtain

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This is what we call *pre-multiplied alpha*

Compositing

$$\begin{cases}(1-\alpha)b = (1-\alpha_1)(1-\alpha_2)b \\ \alpha f = \alpha_1 f_1 + (1-\alpha_1)\alpha_2 f_2\end{cases} \Rightarrow \begin{cases}\alpha = \alpha_1 + (1-\alpha_1)\alpha_2 \\ \alpha f = \alpha_1 f_1 + (1-\alpha_1)\alpha_2 f_2\end{cases}$$

Setting $\tilde{f} = \alpha f, \quad \tilde{f}_1 = \alpha_1 f_1, \quad$ and $\tilde{f}_2 = \alpha_2 f_2$, we obtain

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This is what we call *pre-multiplied alpha*

Blending becomes associative

$$\tilde{f}_1, \alpha_1 \oplus (\tilde{f}_2, \alpha_2 \oplus b) = (\tilde{f}_1, \alpha_1 \oplus \tilde{f}_2, \alpha_2) \oplus b$$

$$\begin{cases}(1-\alpha)b = (1-\alpha_1)(1-\alpha_2)b \\ \alpha f = \alpha_1 f_1 + (1-\alpha_1)\alpha_2 f_2\end{cases} \Rightarrow \begin{cases}\alpha = \alpha_1 + (1-\alpha_1)\alpha_2 \\ \alpha f = \alpha_1 f_1 + (1-\alpha_1)\alpha_2 f_2\end{cases}$$

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Blending becomes associative

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Should we blend front-to-back or back-to-front?

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