A few exercises

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Please try solving these exercises without looking their solutions up. The ones marked with one skull (☠) are harder.

1. How do you compute the intersection between a line $ax + by + c = 0$ and circle $(x - x_c)^2 + (y - y_c)^2 - r^2 = 0$. Use transformations to simplify the problem.
2. How do you compute the intersection between two circles $(x - a)^2 + (y - b)^2 - r^2 = 0$ and $(x - c)^2 + (y - d)^2 + s^2 = 0$. Use transformations to simplify the problem.
3. Is there an *elegant* way to unify these two problems?
4. Let $\gamma(t),\ t \in [0, 1]$ be an integral quadratic Bézier segment and q a point. How do you find the point p in the segment that minimizes $\|p - q\|$? (Assume you have a function that finds roots of polynomials of any degree.)
5. What about for an integral cubic Bézier segment?
6. What about for a rational quadratic Bézier segment?
7. If the control points for a rational quadratic Bézier curve $r(t)$ in canonic form (i.e., $w_0 = w_2 = 1$) represent an elliptical arc (i.e., $|w_1| < 1$), show that the Bézier control points of any affine reparameterization $r(at + b)$ of this curve can also be put in canonic form.
8. If, in the item above, $w_1 < -1$, show that the segment has an ideal point for some $t \in [0, 1]$ unless the curve is degenerate.
9. ☠ How do you convert from the SVG representation for elliptical arc segments to the control points for a rational quadratic Bézier that corresponds the same segment?
10. ☠ How do you perform the opposite operation?
11. Let $\begin{bmatrix} p_0 & p_1 & p_2 & p_3 \end{bmatrix}$ be the control points for an integral cubic Bézier segment, with $p_0 = p_1$ and $p_2 \neq p_1$. Show that the segment $p_0 p_2$ is tangent to the curve.
12. Consider a circle centered at c with radius r. Let f be a point in the interior of the circle. Let p be an arbitrary point distinct from f. Let q be the intersection between the circle and the ray from f through p. Find an expression for the ratio $|p - f|/|q - f|$.
13. Show that the "radial shading" of the PDF and PostScript standards is powerful enough to represent the "radial gradients" of SVG.