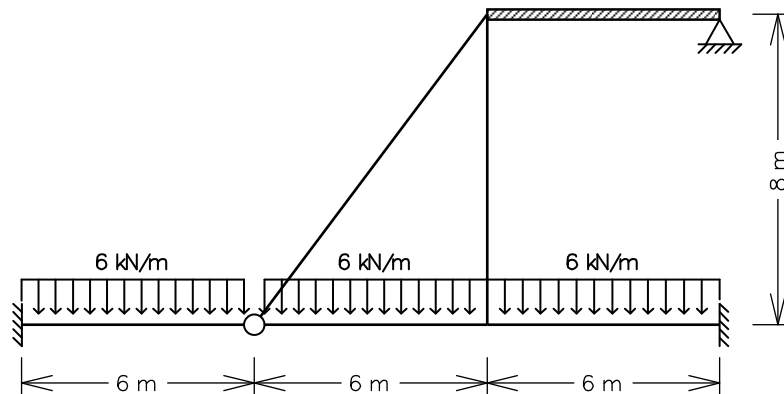


ENG 1204 - ANÁLISE DE ESTRUTURAS II - 2º Semestre - 2017

Segunda Prova - 1ª Parte - 30/10/2017 - Duração: 1:45 hs - Sem Consulta

1ª Questão (5,5 pontos)

Empregando-se o Método dos Deslocamentos, obter o diagrama de momentos fletores para o quadro abaixo (barras inextensíveis). Todas as barras têm a mesma inércia à flexão $EI = 2.4 \times 10^4$ kNm², com exceção da barra horizontal superior, que é infinitamente rígida à flexão.



Solução de um sistema de 2 equações a 2 incógnitas:

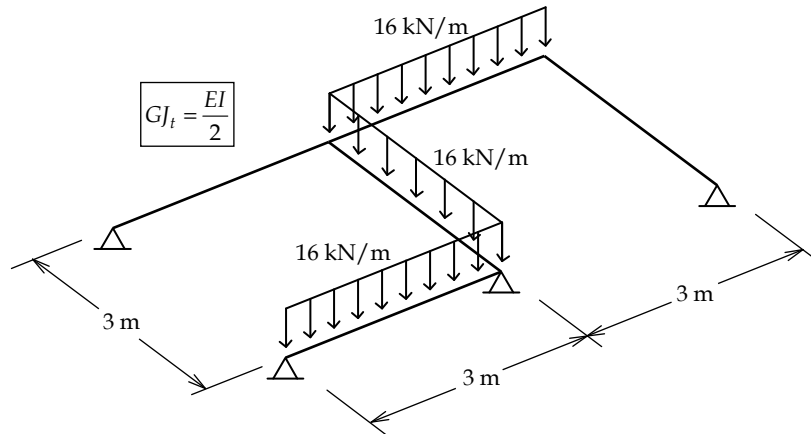
$$\begin{Bmatrix} e \\ f \end{Bmatrix} + \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{Bmatrix} D_1 \\ D_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \Rightarrow \begin{cases} D_1 = \frac{bf - de}{ad - bc} \\ D_2 = \frac{ce - af}{ad - bc} \end{cases}$$

ENG 1204 - ANÁLISE DE ESTRUTURAS II - 2º Semestre - 2017

Segunda Prova - Parte 2 - 01/11/2017 - Duração: 1:15 hs - Sem Consulta

2ª Questão (3,5 pontos)

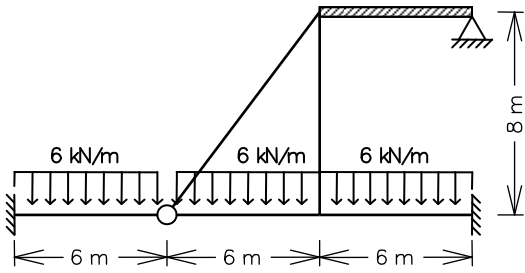
Empregando-se o Método das Forças, obter os diagramas de momentos fletores e momentos torçores para a grelha abaixo. Todas as barras têm a relação indicada entre a rigidez à torção GJ_t e a rigidez à flexão EI .



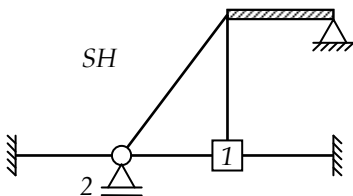
Solução de um sistema de 2 equações a 2 incógnitas:

$$\begin{Bmatrix} e \\ f \end{Bmatrix} + \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \Rightarrow \begin{cases} X_1 = \frac{bf - de}{ad - bc} \\ X_2 = \frac{ce - af}{ad - bc} \end{cases}$$

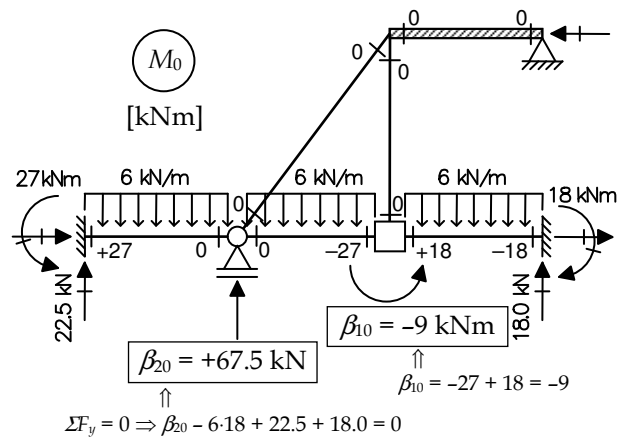
1ª Questão



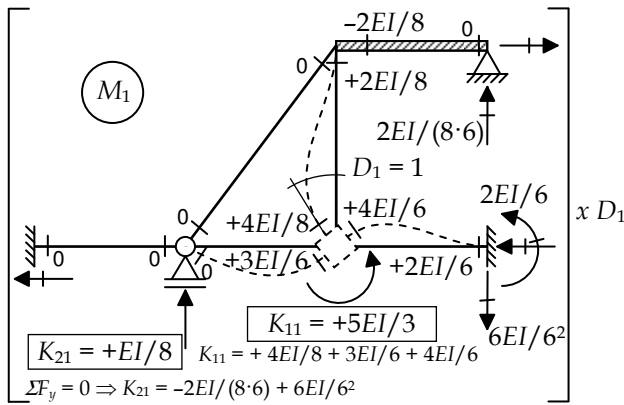
Sistema Hipergeométrico



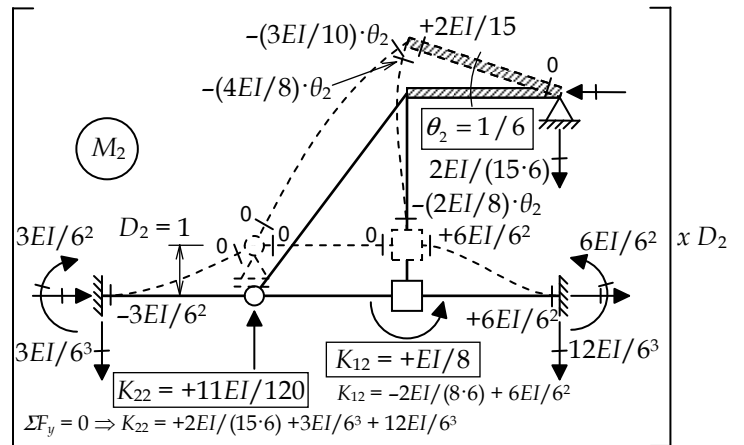
Caso (0) - Solicitação externa isolada no SH



Caso (1) - Deslocabilidade D1 isolada no SH



Caso (2) - Deslocabilidade D2 isolada no SH

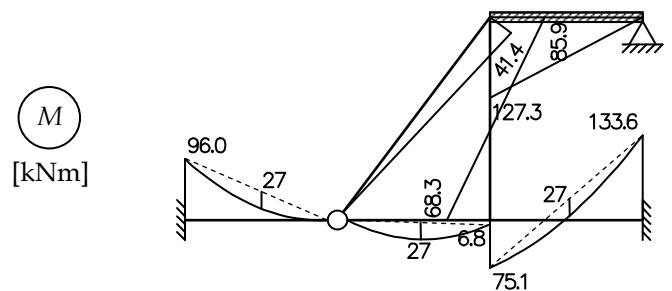
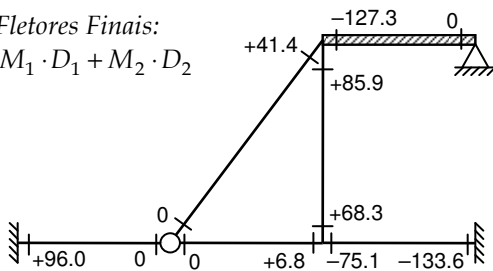


Equações de equilíbrio:

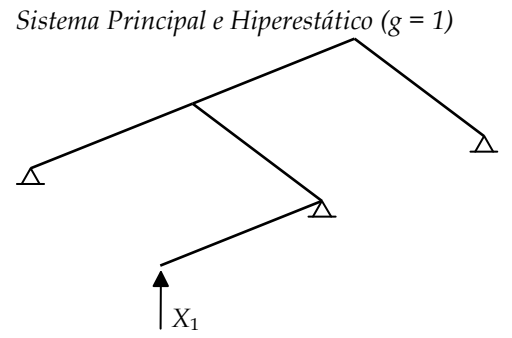
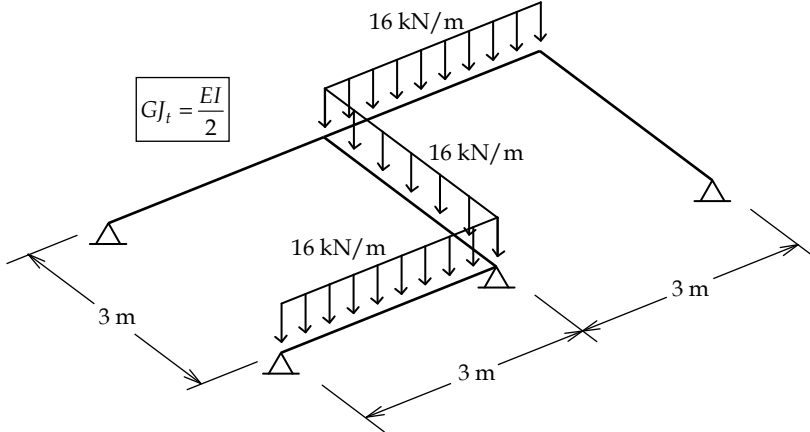
$$\begin{cases} \beta_{10} + K_{11}D_1 + K_{12}D_2 = 0 \\ \beta_{20} + K_{21}D_1 + K_{22}D_2 = 0 \end{cases} \Rightarrow \begin{Bmatrix} -9 \\ +67.5 \end{Bmatrix} + EI \cdot \begin{bmatrix} +5/3 & +1/8 \\ +1/8 & +11/120 \end{bmatrix} \cdot \begin{Bmatrix} D_1 \\ D_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \Rightarrow \begin{cases} D_1 = + \frac{67.534}{EI} \\ D_2 = - \frac{828.46}{EI} \end{cases}$$

Momentos Fletores Finais:

$$M = M_0 + M_1 \cdot D_1 + M_2 \cdot D_2$$



2ª Questão - 1ª opção para Sistema Principal



Caso (0) - Solicitação externa isolada no Sistema Principal

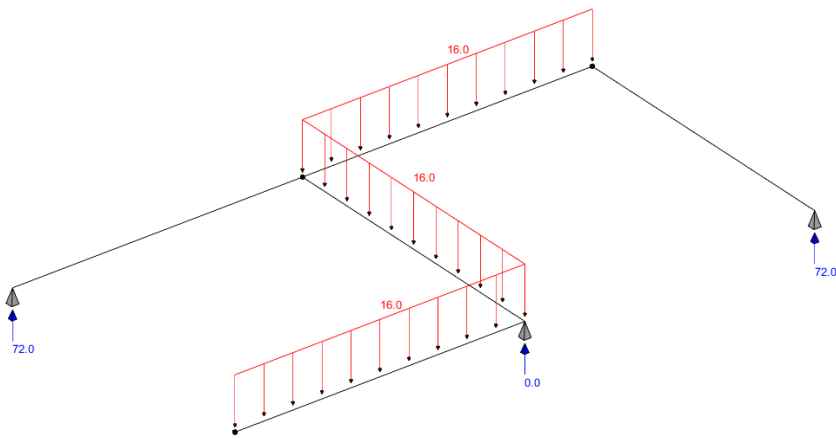


Diagrama de momentos fletores M_0

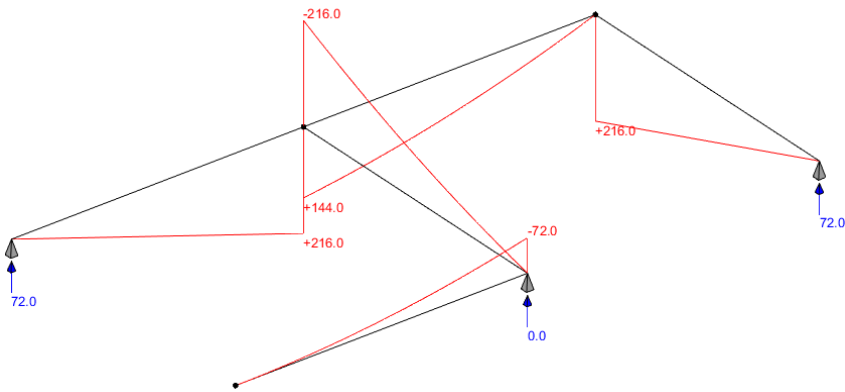
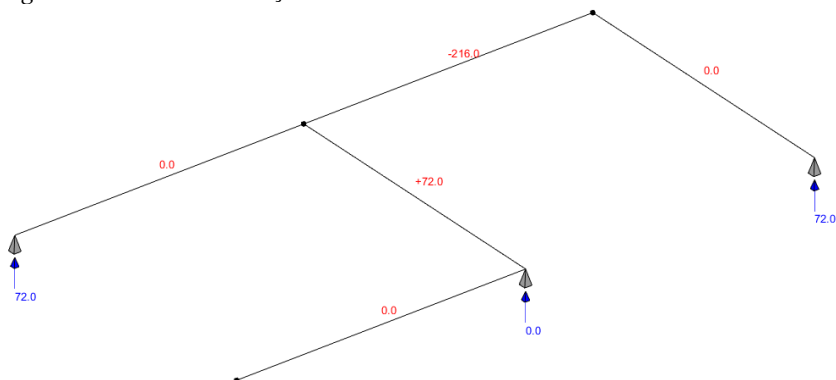


Diagrama de momentos torçores T_0



Caso (1) – Hiperestático X_1 isolado no Sistema Principal

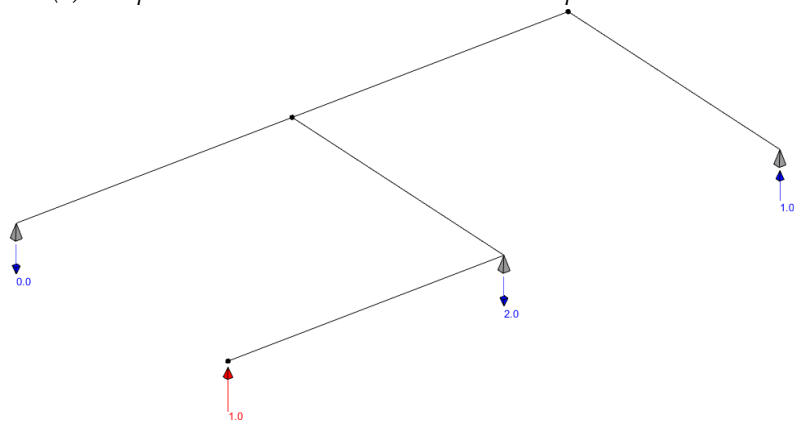


Diagrama de momentos fletores M_1

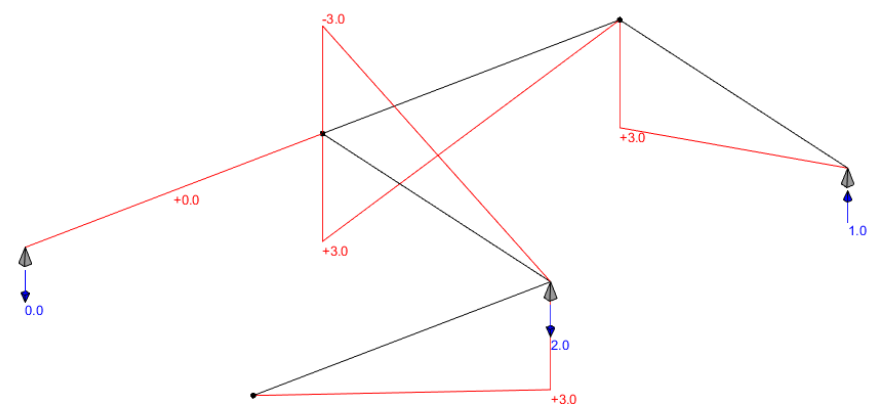
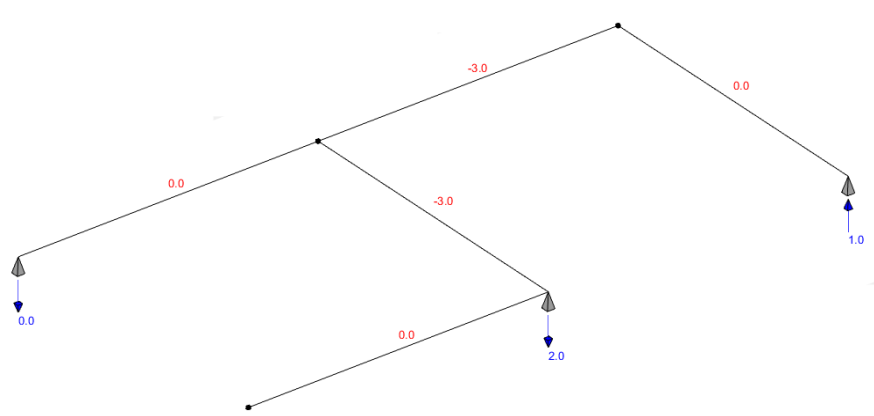


Diagrama de momentos torçores T_1



Equação de compatibilidade

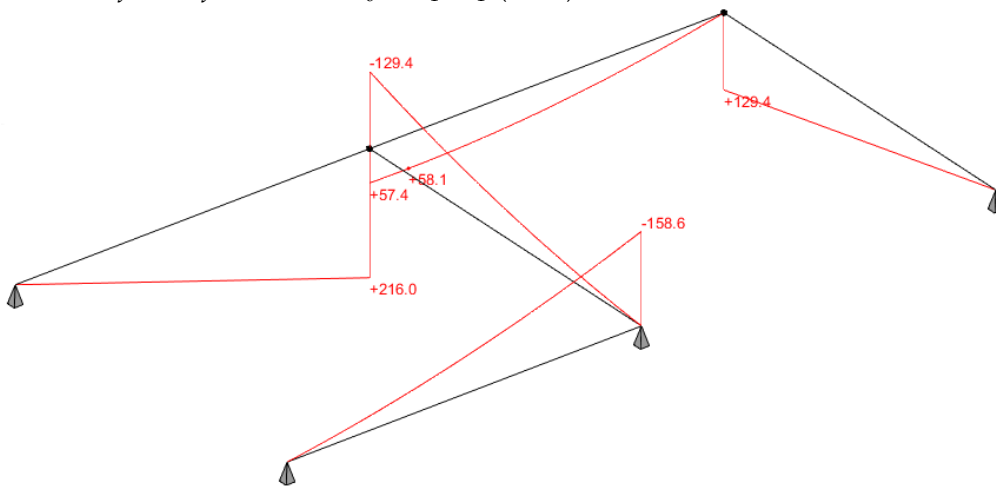
$$\delta_{10} + \delta_{11} X_1 = 0$$

$$\delta_{10} = \left[-\frac{1}{3} \cdot 3 \cdot 72 \cdot 3 + \frac{1}{3} \cdot 3 \cdot 18 \cdot 3 + \frac{1}{3} \cdot 3 \cdot 216 \cdot 3 - \frac{1}{3} \cdot 3 \cdot 18 \cdot 3 + \frac{1}{3} \cdot 3 \cdot 144 \cdot 3 + \frac{1}{3} \cdot 3 \cdot 18 \cdot 3 + \frac{1}{3} \cdot 3 \cdot 216 \cdot 3 \right] \cdot \frac{1}{EI} + \left[-3 \cdot 72 \cdot 3 + 3 \cdot 216 \cdot 3 \right] \cdot \frac{1}{GJ_t} = +\frac{1566}{EI} + \frac{1296}{GJ_t} = +\frac{4158}{EI}$$

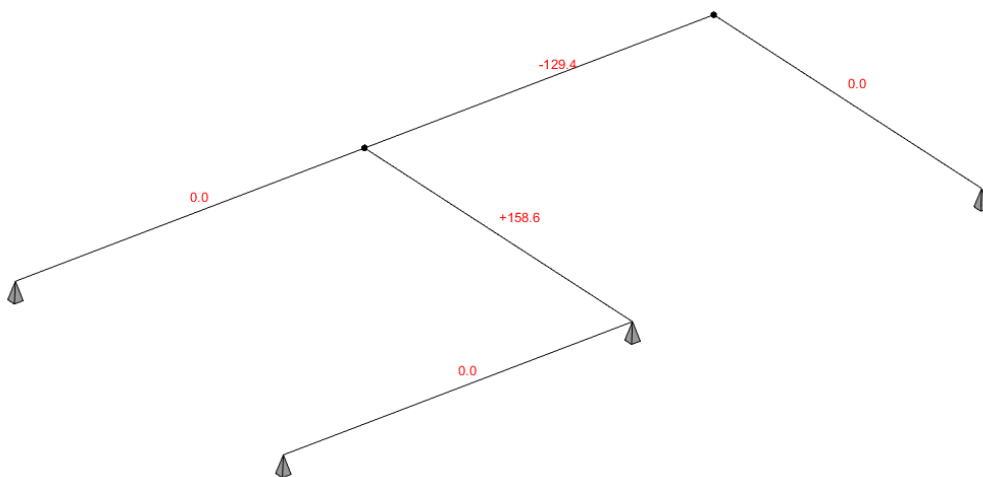
$$\delta_{11} = \left[4 \cdot \left(+\frac{1}{3} \cdot 3 \cdot 3 \cdot 3 \right) \right] \cdot \frac{1}{EI} + \left[2 \cdot (-3) \cdot (-3) \cdot 3 \right] \cdot \frac{1}{GJ_t} = +\frac{36}{EI} + \frac{54}{GJ_t} = +\frac{144}{EI}$$

$$\Rightarrow +\frac{4158}{EI} + \frac{144}{EI} \cdot X_1 = 0 \quad \therefore X_1 = -28.875 \text{ kN}$$

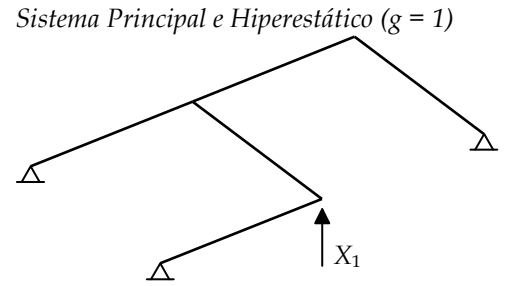
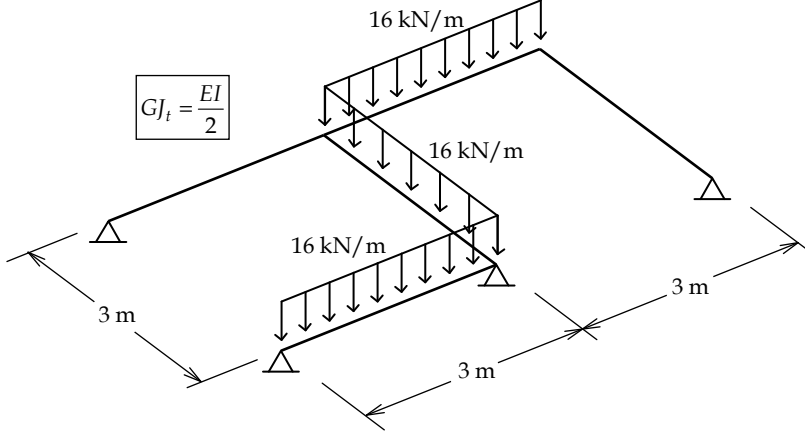
Momentos fletores finais: $M = M_0 + M_1 \cdot X_1$ (kNm)



Momentos torçores finais: $T = T_0 + T_1 \cdot X_1$ (kNm)



2ª Questão - 2ª opção para Sistema Principal



Caso (0) - Solicitação externa isolada no Sistema Principal

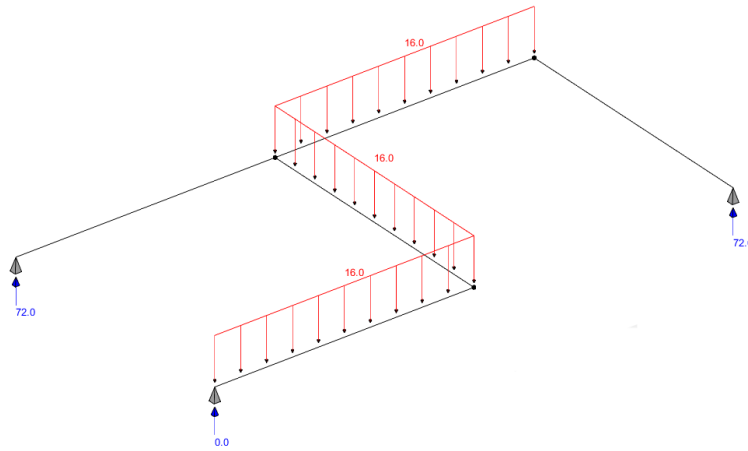


Diagrama de momentos fletores M_0

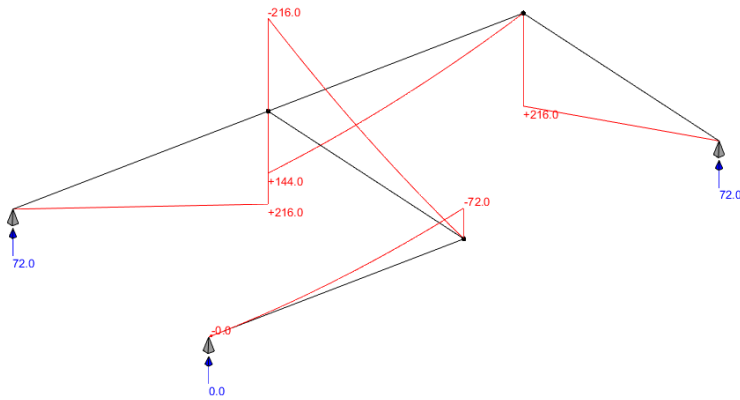
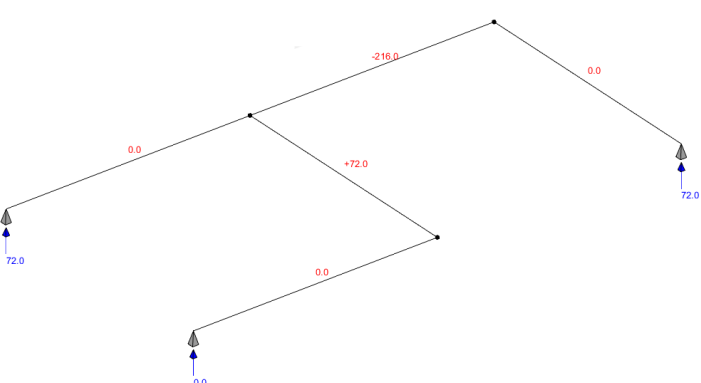


Diagrama de momentos torçores T_0



Caso (1) – Hiperestático X_1 isolado no Sistema Principal

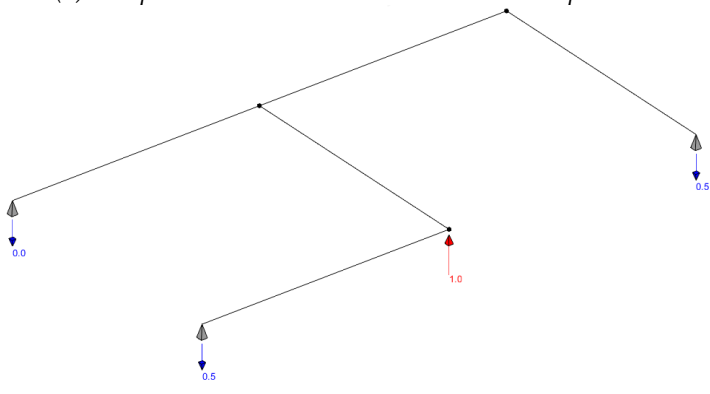


Diagrama de momentos fletores M_1

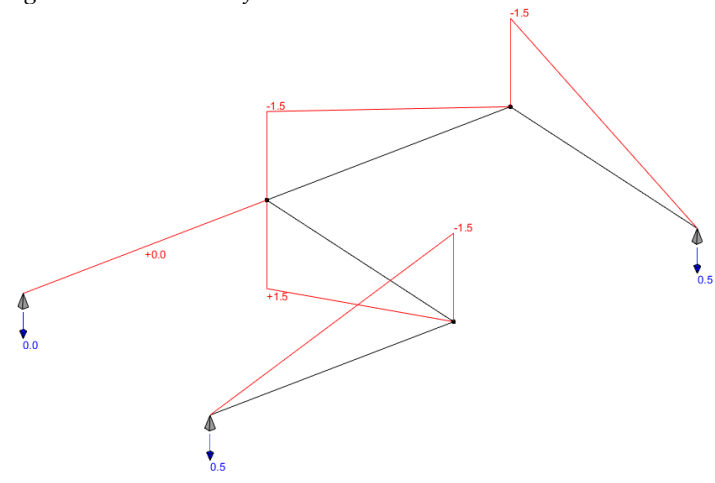
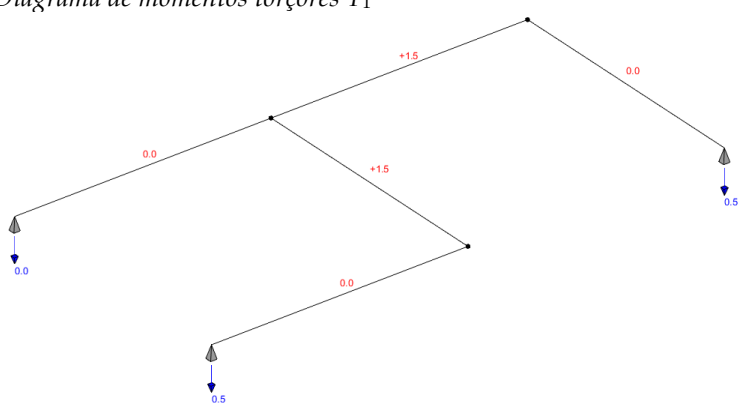


Diagrama de momentos torçores T_1



Equação de compatibilidade

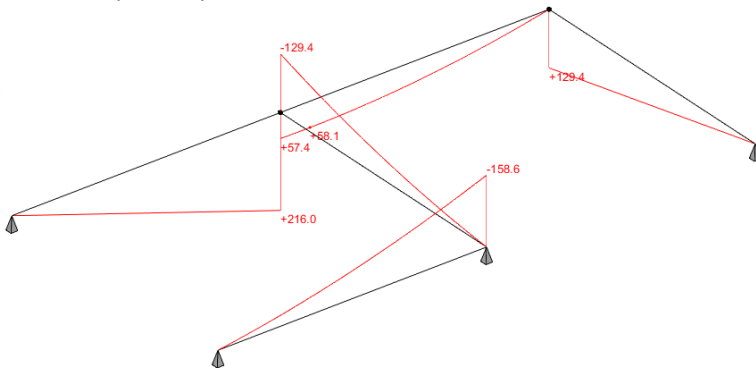
$$\delta_{10} + \delta_{11} X_1 = 0$$

$$\delta_{10} = \left[+\frac{1}{3} \cdot \frac{3}{2} \cdot 72 \cdot 3 - \frac{1}{3} \cdot \frac{3}{2} \cdot 18 \cdot 3 - \frac{1}{3} \cdot \frac{3}{2} \cdot 216 \cdot 3 + \frac{1}{3} \cdot \frac{3}{2} \cdot 18 \cdot 3 - \frac{1}{3} \cdot \frac{3}{2} \cdot 144 \cdot 3 - \frac{1}{3} \cdot \frac{3}{2} \cdot 18 \cdot 3 - \frac{1}{3} \cdot \frac{3}{2} \cdot 216 \cdot 3 \right] \cdot \frac{1}{EI} + \left[+\frac{3}{2} \cdot 72 \cdot 3 - \frac{3}{2} \cdot 216 \cdot 3 \right] \cdot \frac{1}{GJ_t} = -\frac{783}{EI} - \frac{648}{GJ_t} = -\frac{2079}{EI}$$

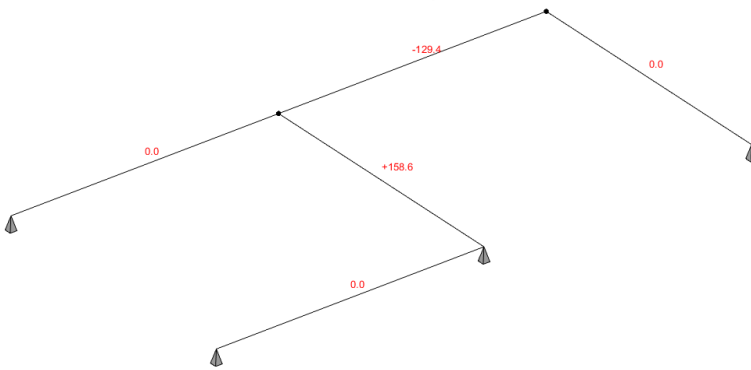
$$\delta_{11} = \left[4 \cdot \left(+\frac{1}{3} \cdot \frac{3}{2} \cdot \frac{3}{2} \cdot 3 \right) \right] \cdot \frac{1}{EI} + \left[2 \cdot \left(\frac{3}{2} \cdot \frac{3}{2} \cdot 3 \right) \right] \cdot \frac{1}{GJ_t} = +\frac{9}{EI} + \frac{27}{2GJ_t} = +\frac{36}{EI}$$

$$\Rightarrow -\frac{2079}{EI} + \frac{36}{EI} \cdot X_1 = 0 \quad \boxed{\therefore X_1 = +57.750 \text{ kN}}$$

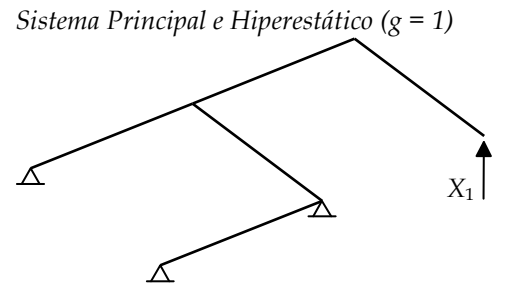
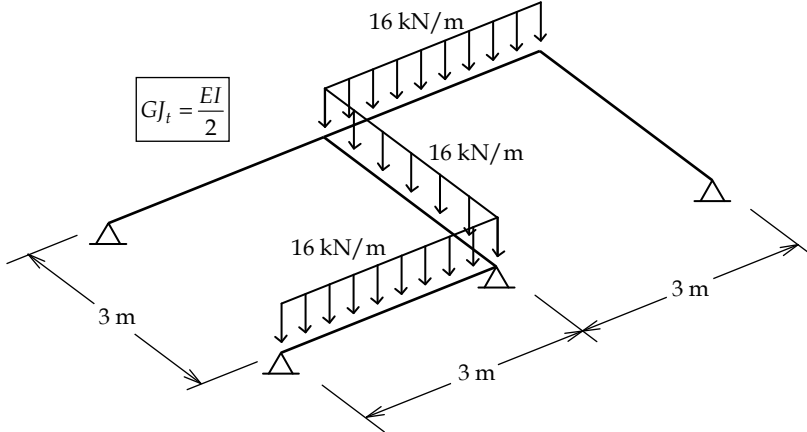
Momentos fletores finais: $M = M_0 + M_1 \cdot X_1$ (kNm)



Momentos torçores finais: $T = T_0 + T_1 \cdot X_1$ (kNm)



2ª Questão - 3ª opção para Sistema Principal



Caso (0) - Solicitação externa isolada no Sistema Principal

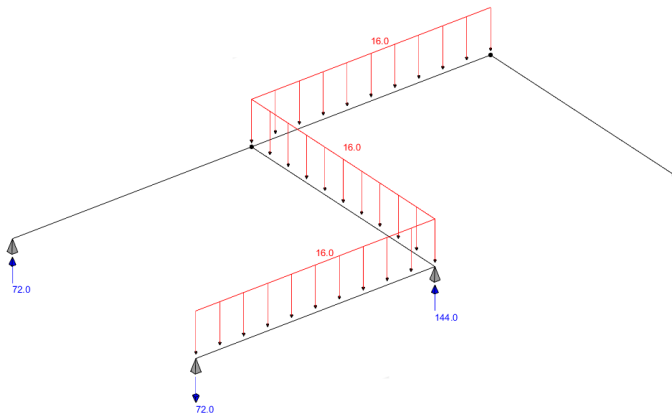


Diagrama de momentos fletores M_0

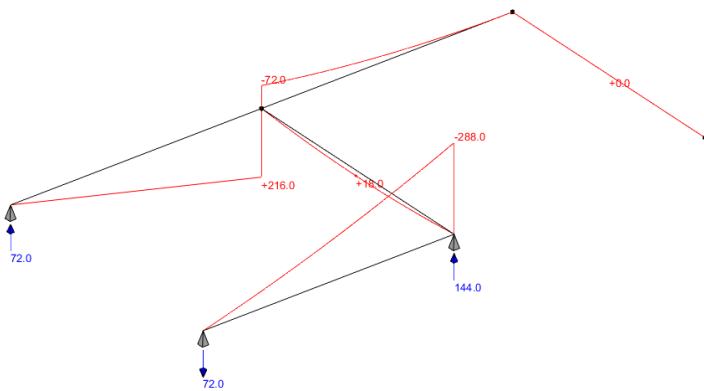
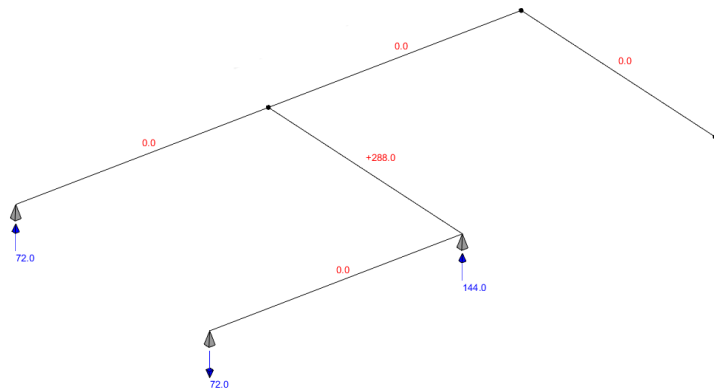


Diagrama de momentos torçores T_0



Caso (1) – Hiperestático X_1 isolado no Sistema Principal

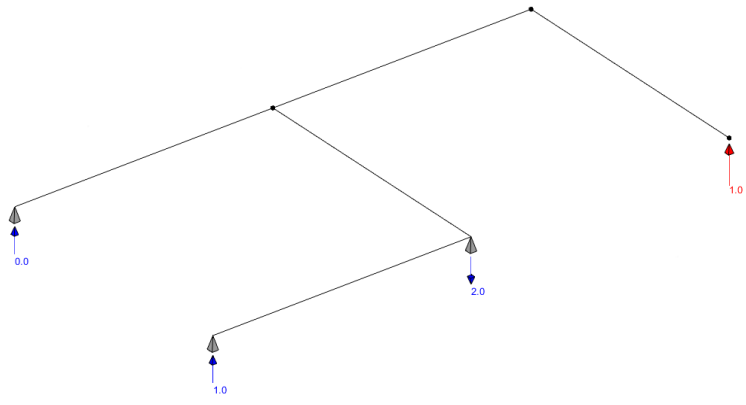


Diagrama de momentos fletores M_1

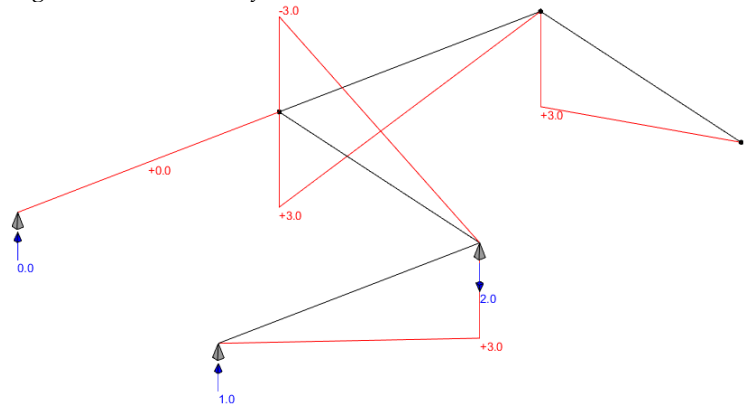
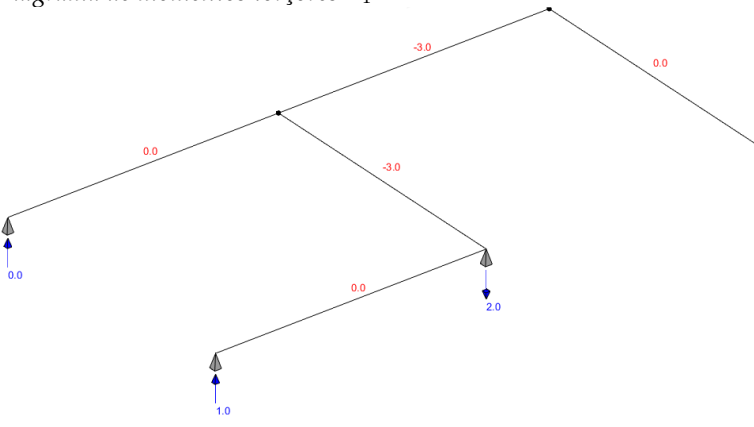


Diagrama de momentos torçores T_1



Equação de compatibilidade

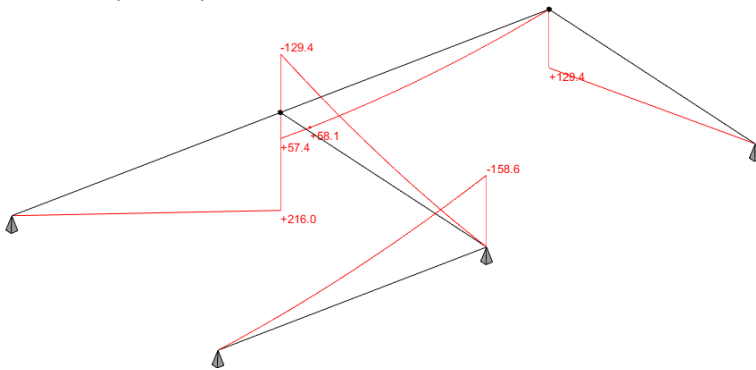
$$\delta_{10} + \delta_{11} X_1 = 0$$

$$\delta_{10} = \left[-\frac{1}{3} \cdot 3 \cdot 288 \cdot 3 + \frac{1}{3} \cdot 3 \cdot 18 \cdot 3 - \frac{1}{3} \cdot 3 \cdot 18 \cdot 3 - \frac{1}{3} \cdot 3 \cdot 72 \cdot 3 + \frac{1}{3} \cdot 3 \cdot 18 \cdot 3 \right] \cdot \frac{1}{EI} + \left[-3 \cdot 288 \cdot 3 \right] \cdot \frac{1}{GJ_t} = -\frac{1026}{EI} - \frac{2592}{GJ_t} = -\frac{6210}{EI}$$

$$\delta_{11} = \left[4 \cdot \left(+\frac{1}{3} \cdot 3 \cdot 3 \cdot 3 \right) \right] \cdot \frac{1}{EI} + [2 \cdot (-3) \cdot (-3) \cdot 3] \cdot \frac{1}{GJ_t} = +\frac{36}{EI} + \frac{54}{GJ_t} = +\frac{144}{EI}$$

$$\Rightarrow -\frac{6210}{EI} + \frac{144}{EI} \cdot X_1 = 0 \quad \boxed{\therefore X_1 = +43.125 \text{ kN}}$$

Momentos fletores finais: $M = M_0 + M_1 \cdot X_1$ (kNm)



Momentos torçores finais: $T = T_0 + T_1 \cdot X_1$ (kNm)

