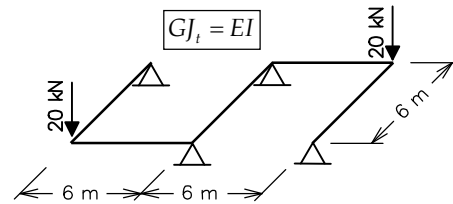


# ENG 1204 - ANÁLISE DE ESTRUTURAS II - 2º Semestre - 2016

## Segunda Prova - Parte 2 - 28/11/2016 - Duração: 1:45 hs - Sem Consulta

### 2ª Questão (3,5 pontos)

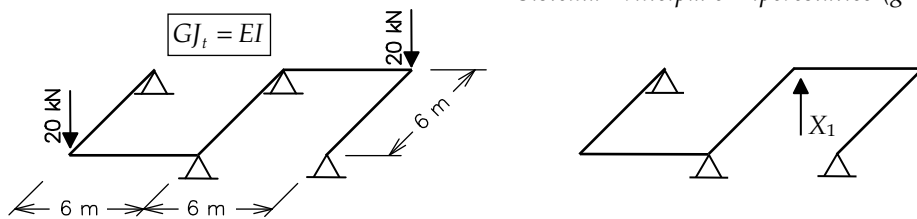
Empregando-se o Método das Forças, obter os diagramas de momentos fletores e momentos torçores para a grelha ao lado. Todas as barras têm a relação indicada entre a rigidez à torção  $GJ_t$  e a rigidez à flexão  $EI$ .



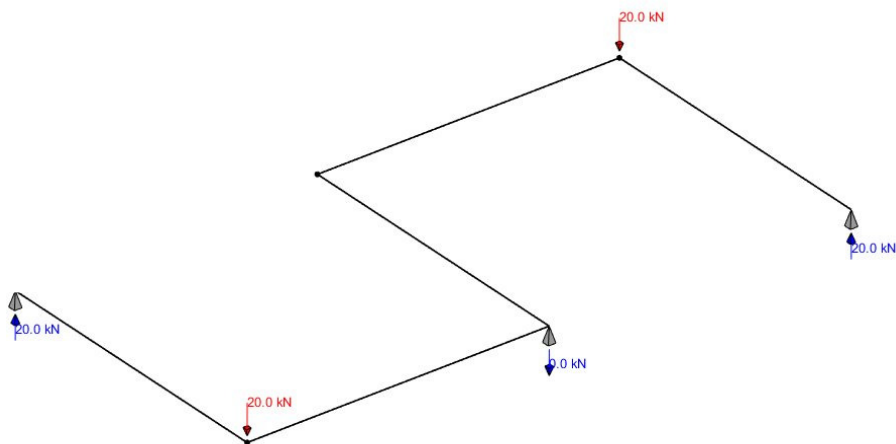
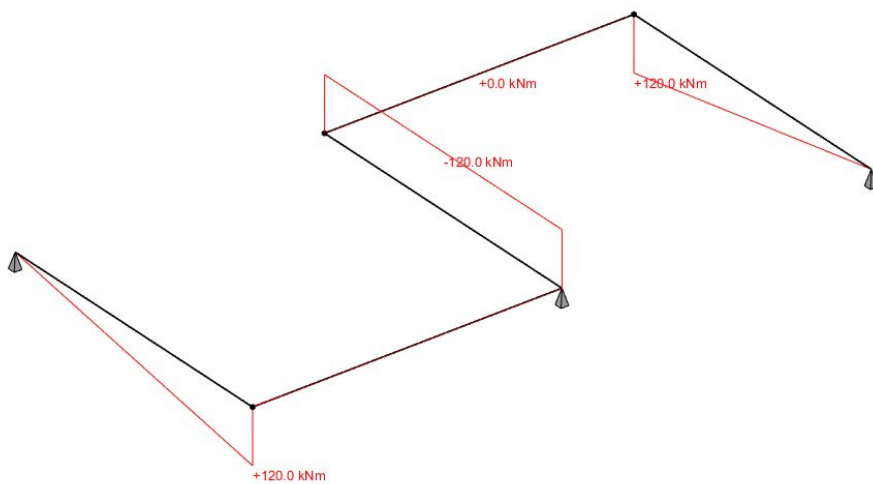
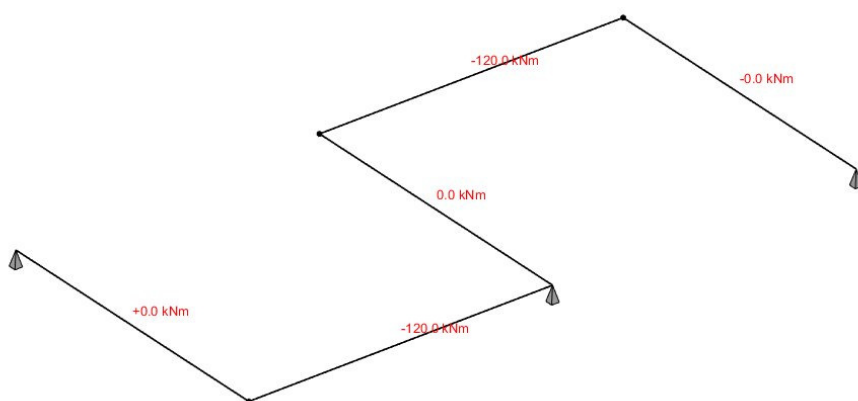
Solução de um sistema de 2 equações a 2 incógnitas:

$$\begin{Bmatrix} e \\ f \end{Bmatrix} + \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \Rightarrow \begin{cases} X_1 = \frac{bf - de}{ad - bc} \\ X_2 = \frac{ce - af}{ad - bc} \end{cases}$$

## 2ª Questão

Sistema Principal e Hiperestático ( $g = 1$ )

Caso (0) – Solicitação externa isolada no Sistema Principal

Diagrama de momentos fletores  $M_0$ Diagrama de momentos torçores  $T_0$ 

Caso (1) – Hiperestático  $X_1$  isolado no Sistema Principal

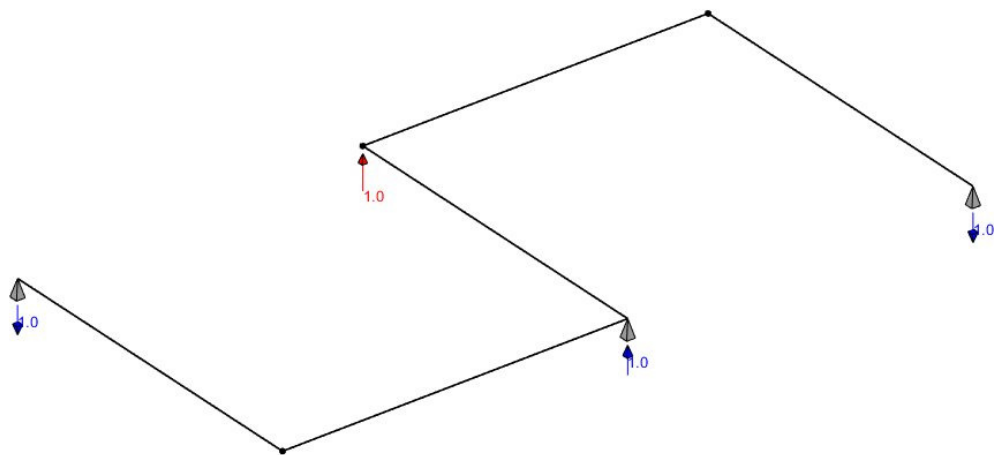


Diagrama de momentos fletores  $M_1$

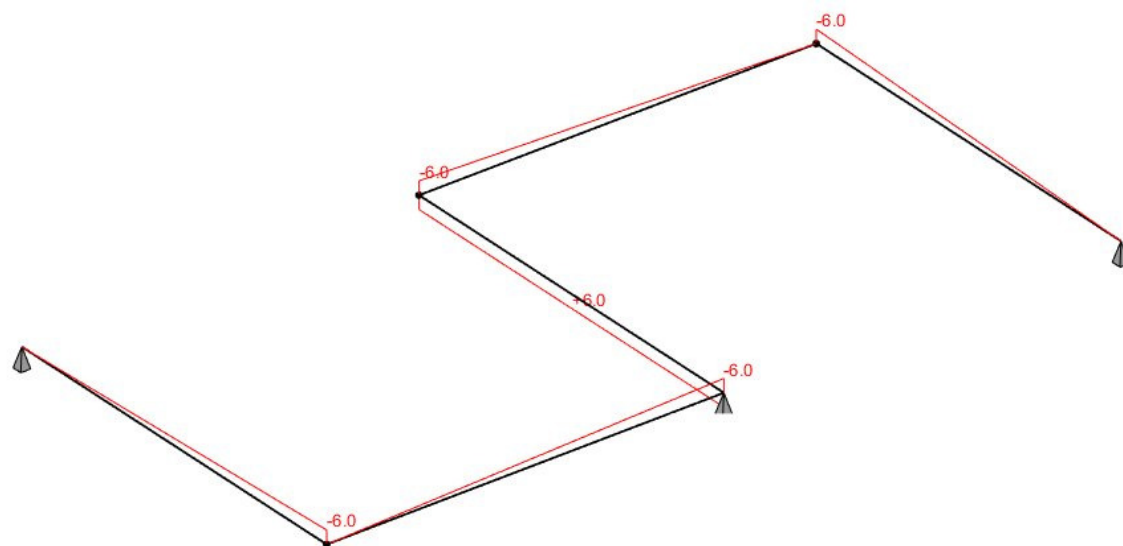
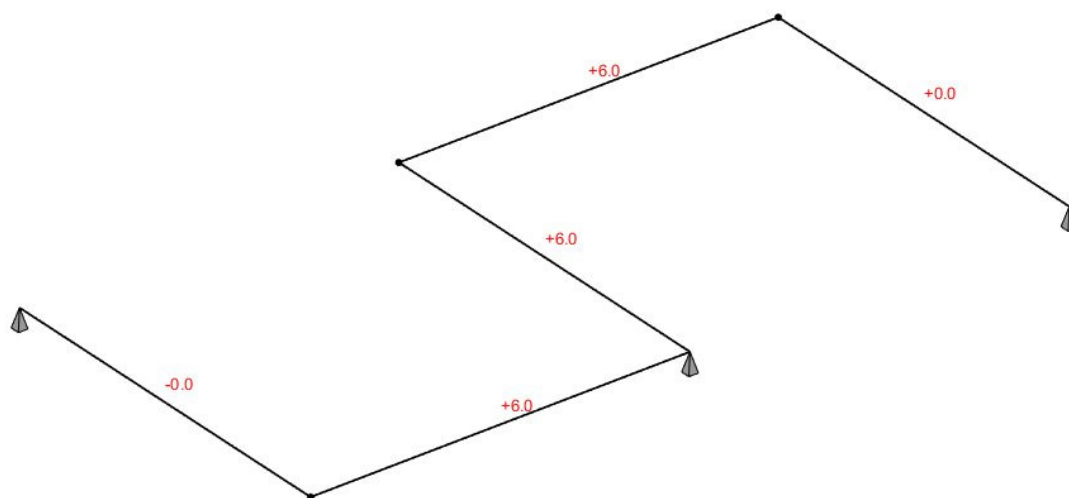


Diagrama de momentos torçores  $T_1$

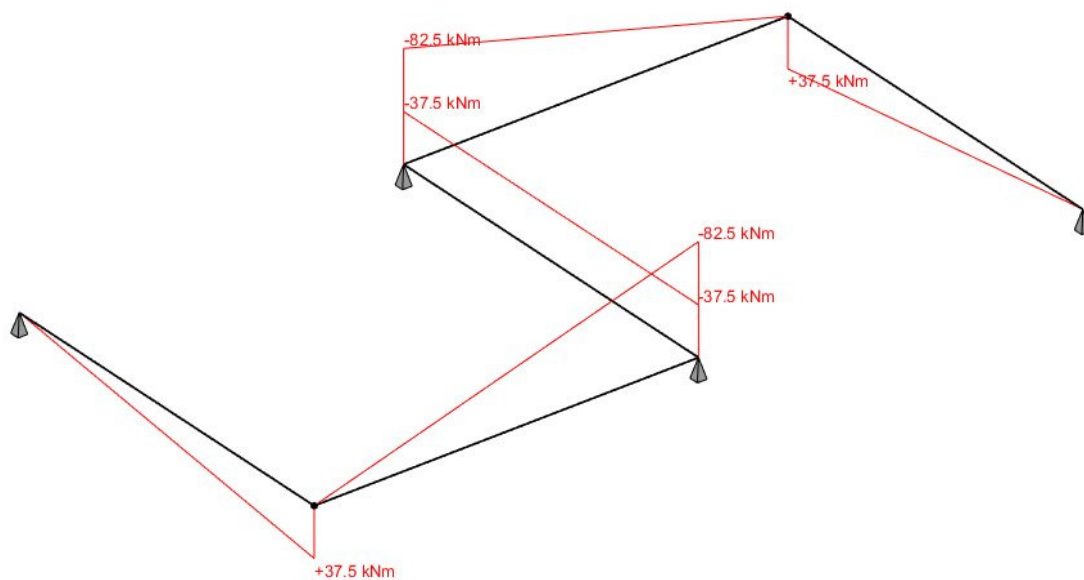


Equação de compatibilidade

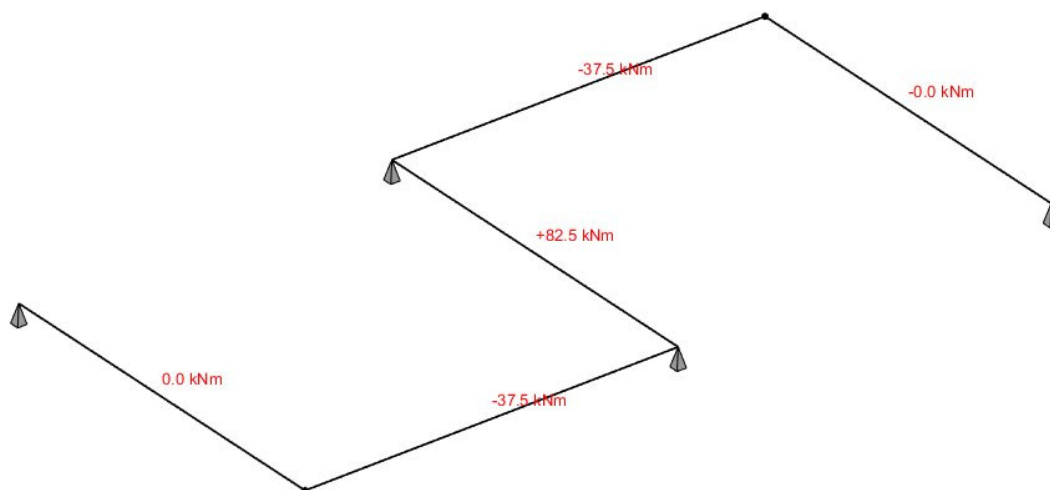
$$\delta_{10} + \delta_{11}X_1 = 0 \quad \delta_{10} = \left[ -\frac{1}{3} \cdot 6 \cdot 120 \cdot 6 - 6 \cdot 120 \cdot 6 - \frac{1}{3} \cdot 6 \cdot 120 \cdot 6 \right] \cdot \frac{1}{EI} + \left[ -6 \cdot 120 \cdot 6 - 6 \cdot 120 \cdot 6 \right] \cdot \frac{1}{GJ_t} = -\frac{15840}{EI}$$

$$\delta_{11} = \left[ 4 \cdot \left( +\frac{1}{3} \cdot 6 \cdot 6 \cdot 6 \right) \right] \cdot \frac{1}{EI} + \left[ 3 \cdot (-6) \cdot (-6) \cdot 6 \right] \cdot \frac{1}{GJ_t} = +\frac{1152}{EI} \Rightarrow -\frac{15840}{EI} + \frac{1152}{EI} \cdot X_1 = 0 \quad \therefore X_1 = +13.75 \text{ kN}$$

Momentos fletores finais:  $M = M_0 + M_1 \cdot X_1$  (kNm)



Momentos torçores finais:  $T = T_0 + T_1 \cdot X_1$  (kNm)



2ª Questão – Outra opção (2) de Sistema Principal:

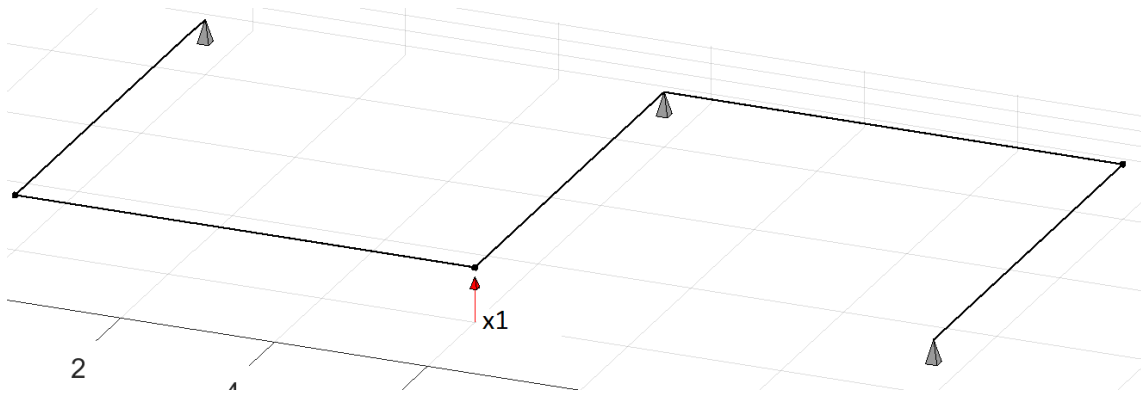


Diagrama de Momentos Fletores  $M_0$

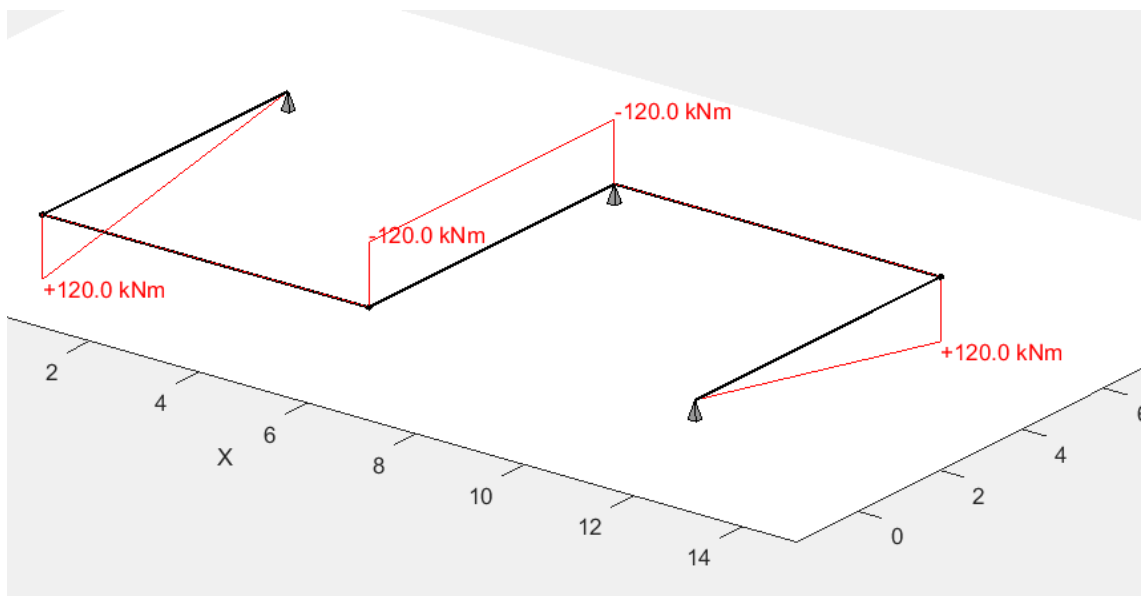
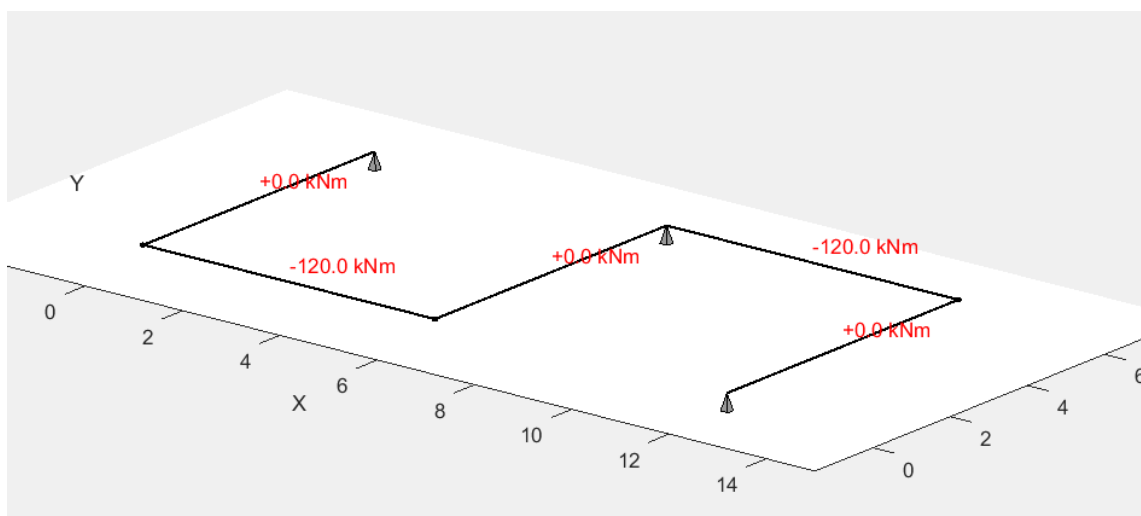


Diagrama de Momentos Torsores  $T_0$



Caso (1) – Hiperestático  $X_1$  isolado no Sistema Principal

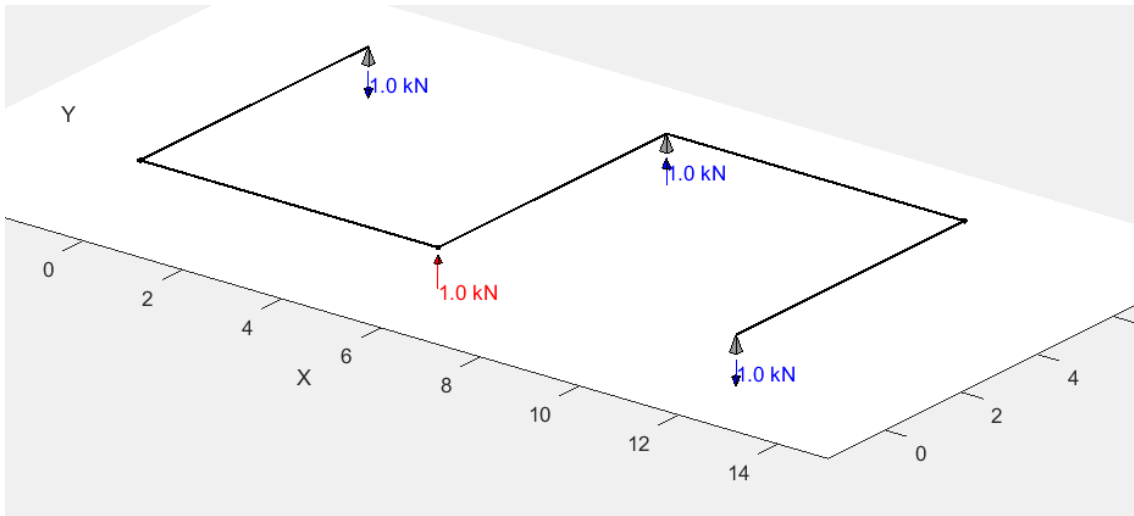


Diagrama de Momentos Fletores  $M_1$

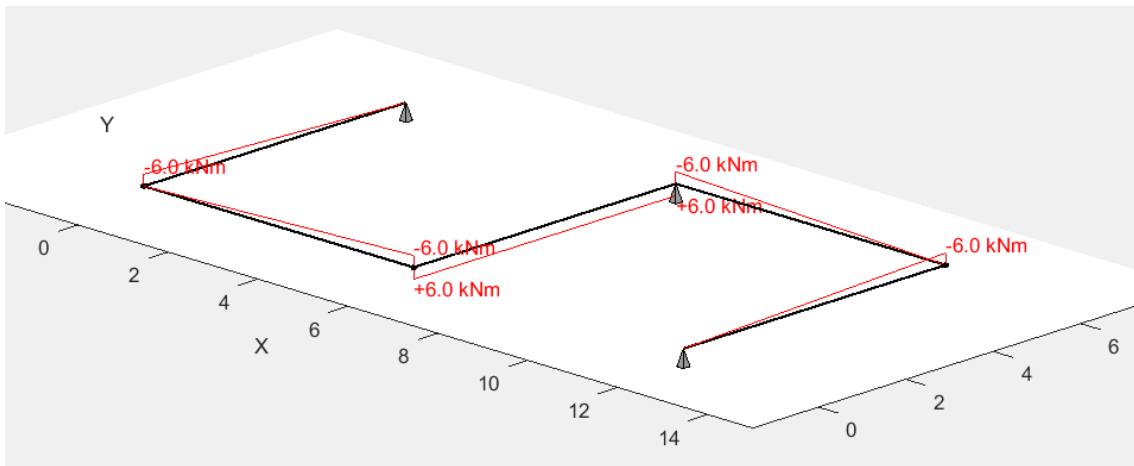
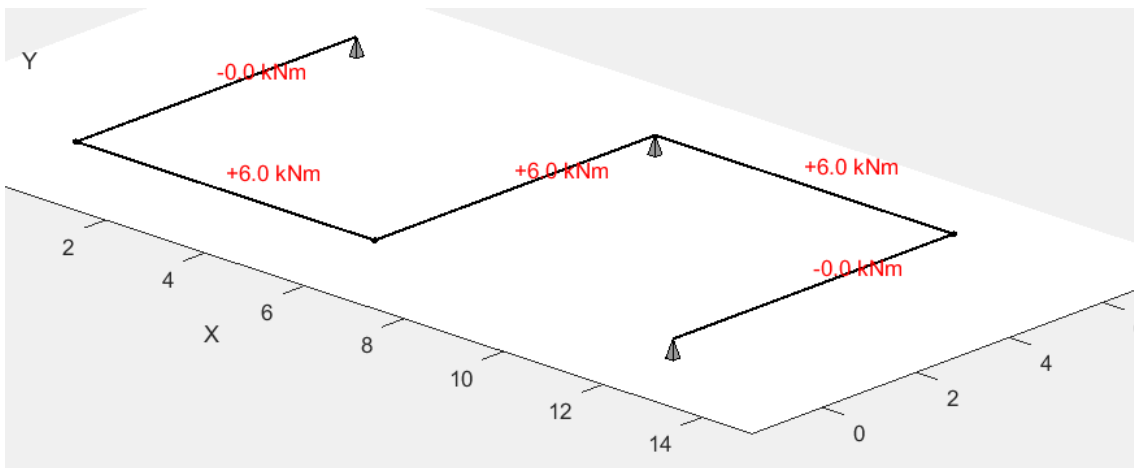
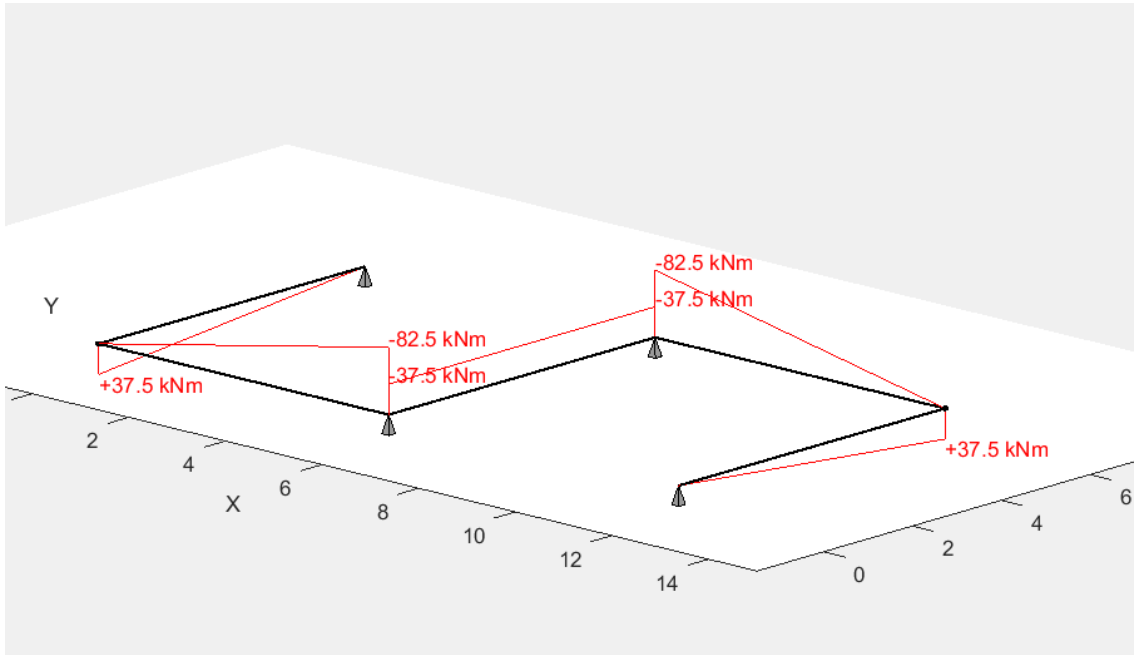


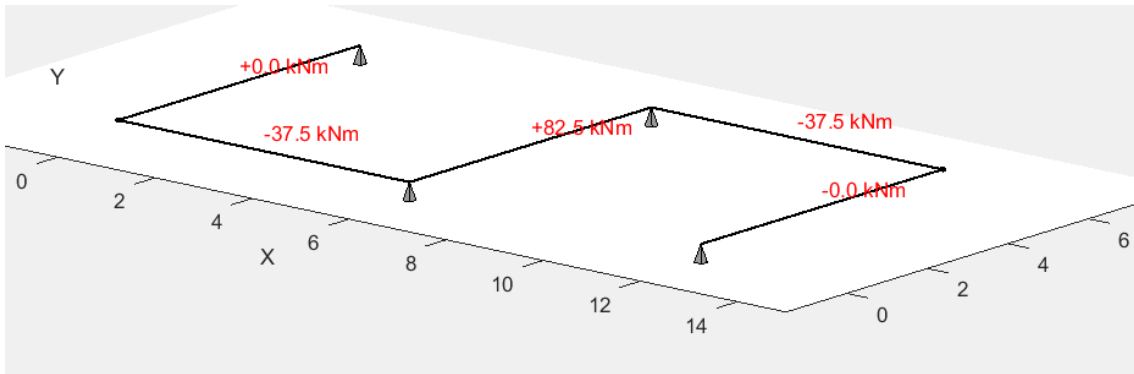
Diagrama de Momentos Torsores  $T_1$



Momentos Fletores Finais:  $M = M_0 + M_1X_1$  (kNm)



Momentos Torsores Finais:  $T = T_0 + T_1X_1$  (kNm)



2ª Questão – Outra opção (3) de Sistema Principal:

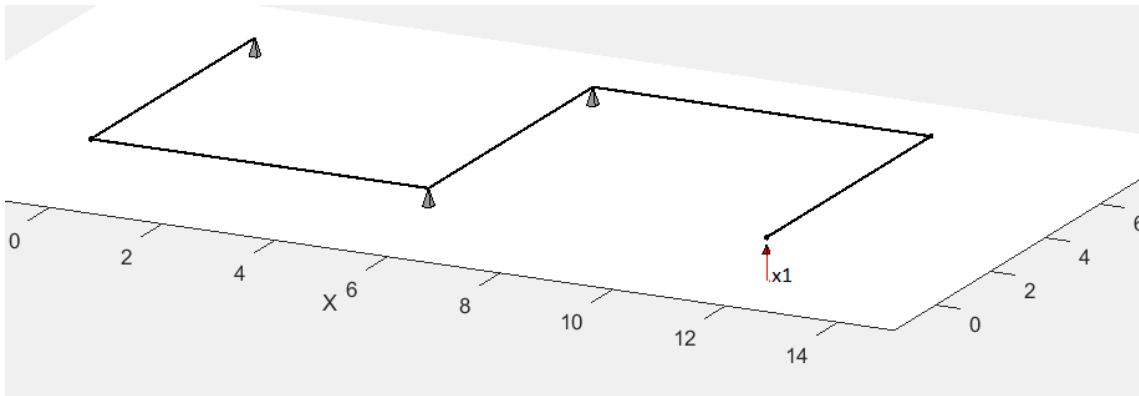


Diagrama de Momentos Fletores  $M_0$

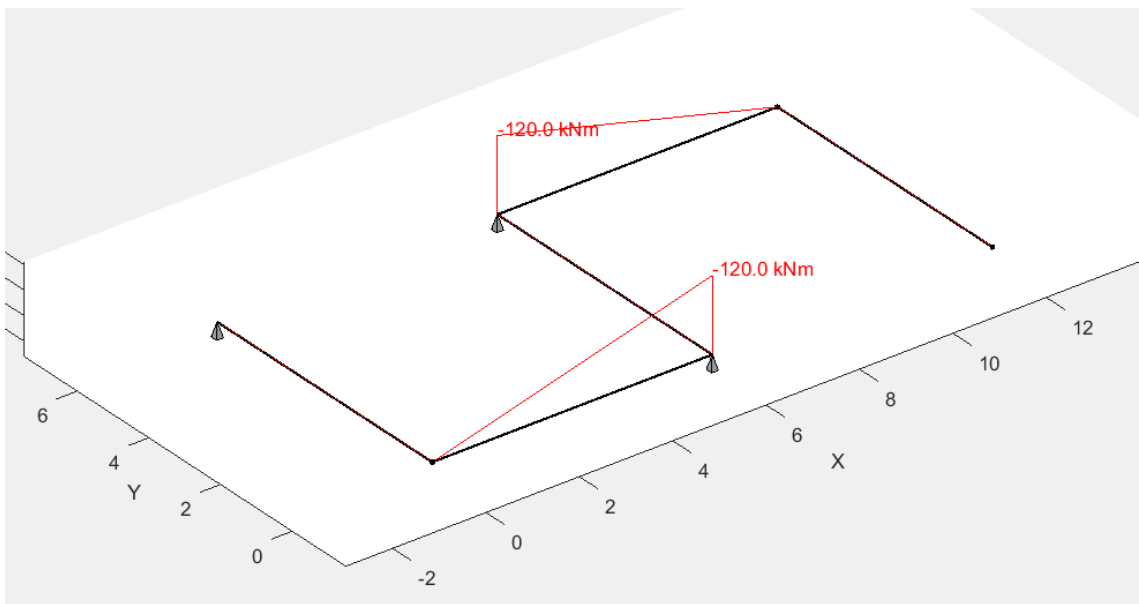
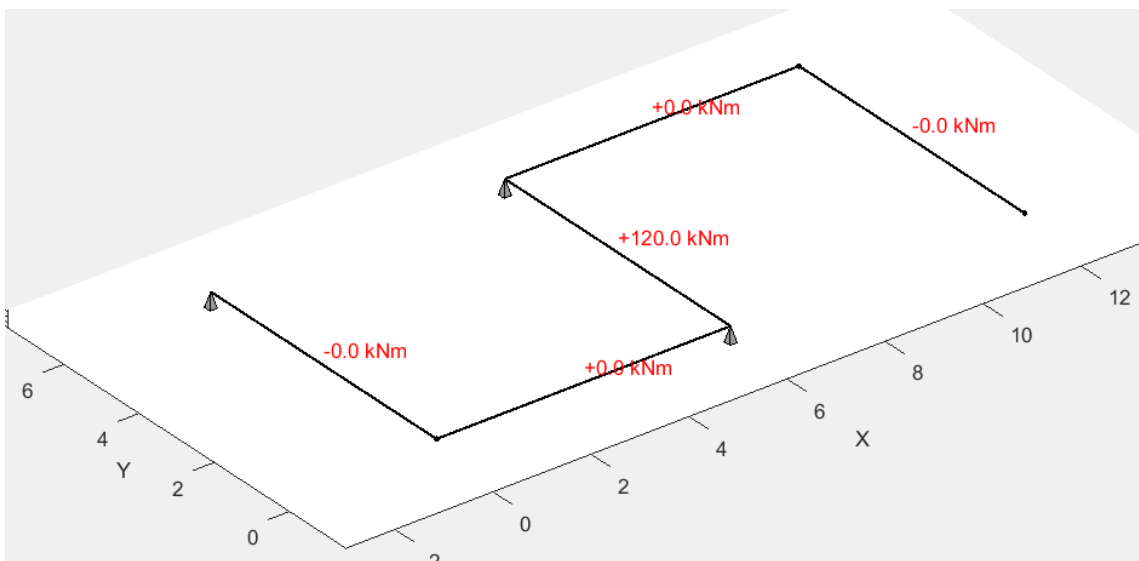


Diagrama de Momentos Torsores  $T_0$





Caso (1) – Hiperestático  $X_1$  isolado no Sistema Principal

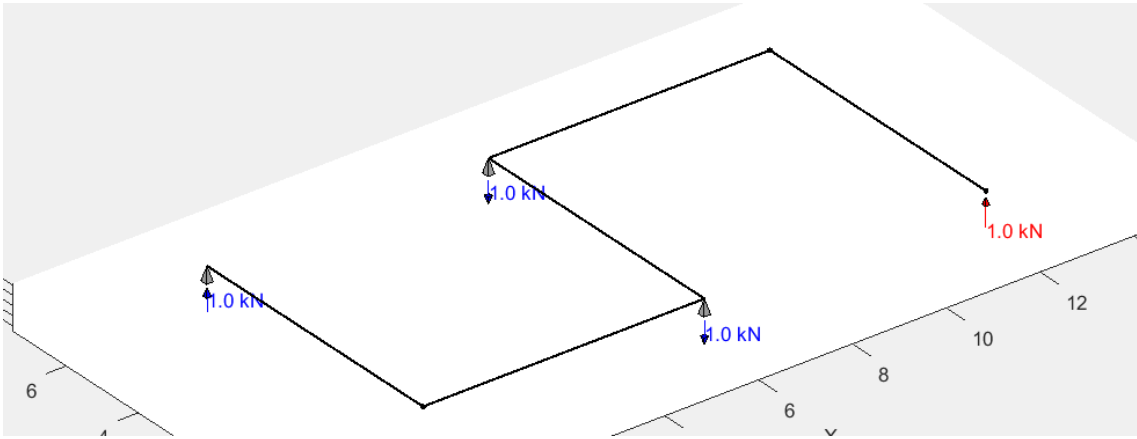


Diagrama de Momentos Fletores  $M_1$

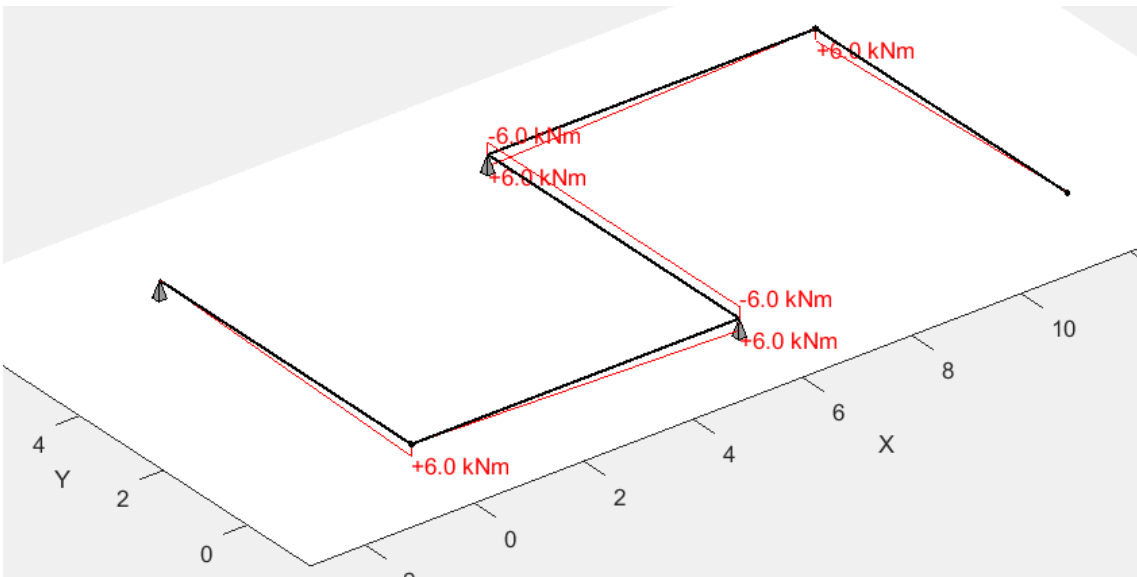
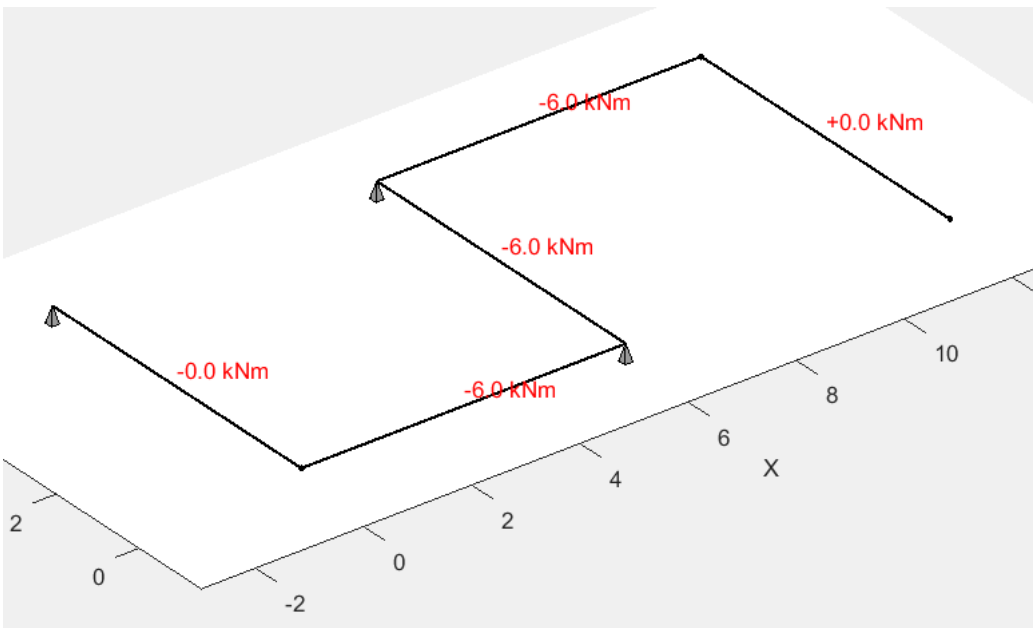
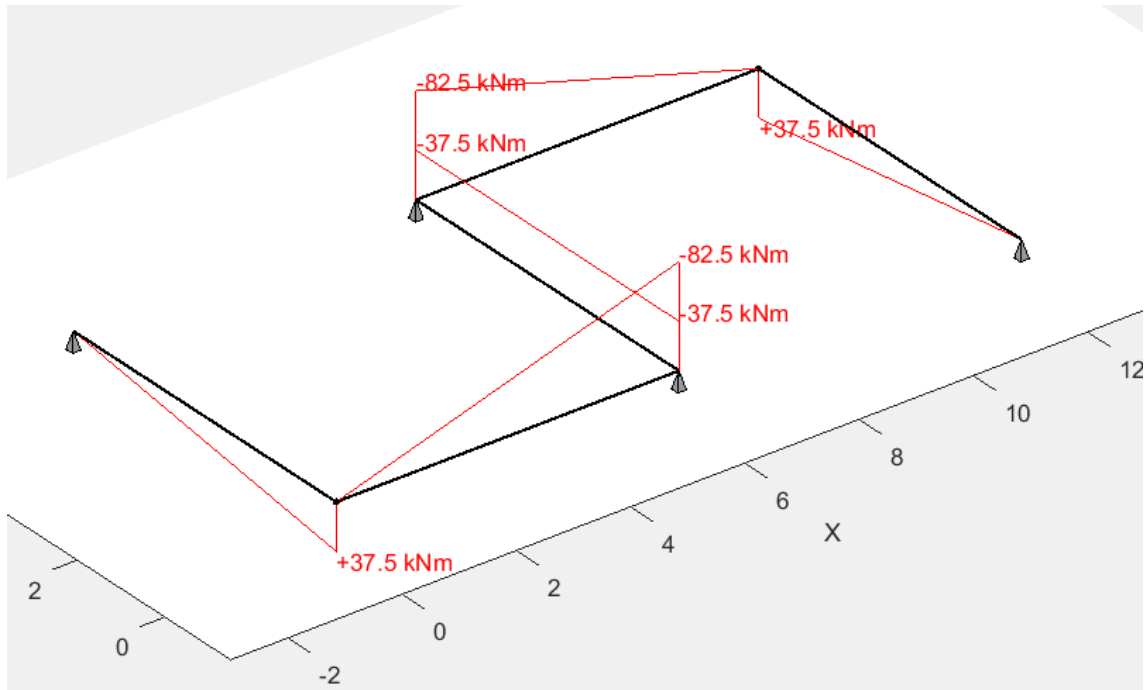


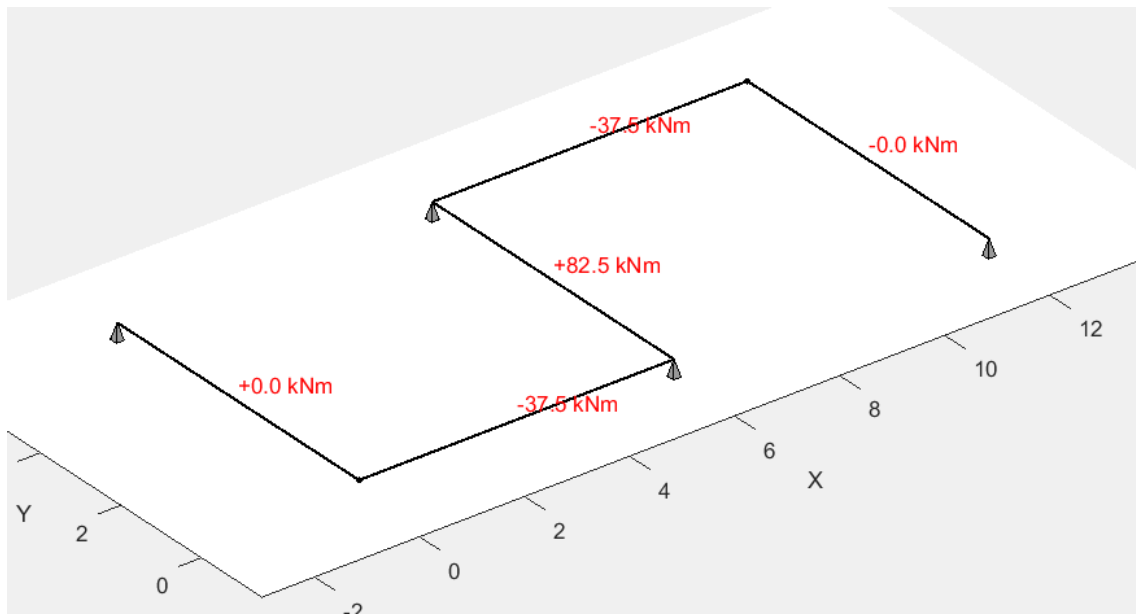
Diagrama de Momentos Torsores  $T_1$



Momentos Fletores Finais:  $M = M_0 + M_1X_1$  (kNm)



Momentos Torsores Finais:  $T = T_0 + T_1X_1$  (kNm)



2ª Questão – Outra opção (2) de Sistema Principal:

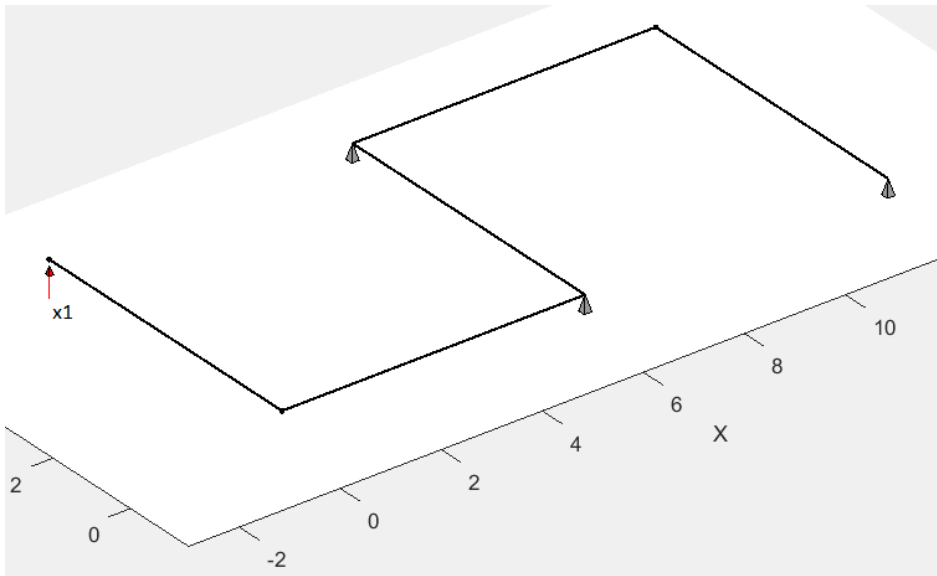


Diagrama de Momentos Fletores  $M_0$

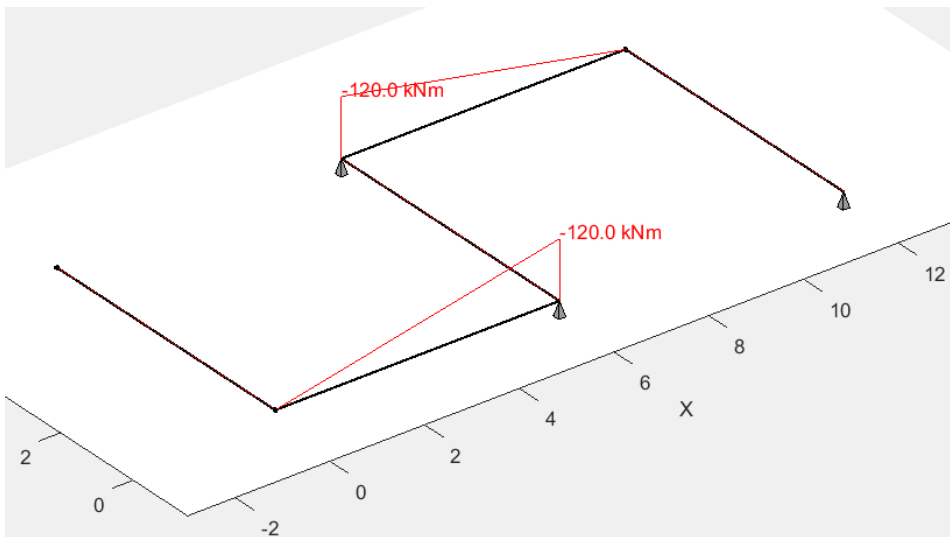
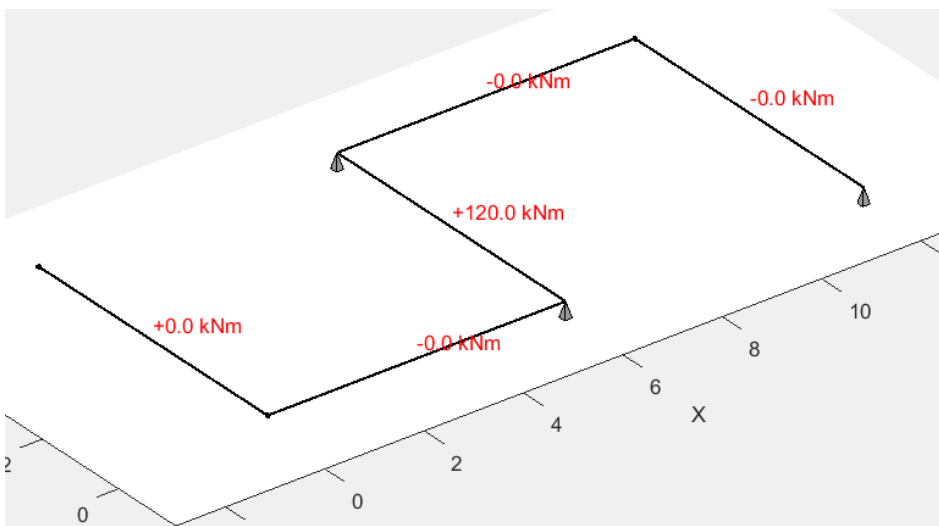


Diagrama de Momentos Torsores  $T_0$



Caso (1) – Hiperestático  $X_1$  isolado no Sistema Principal

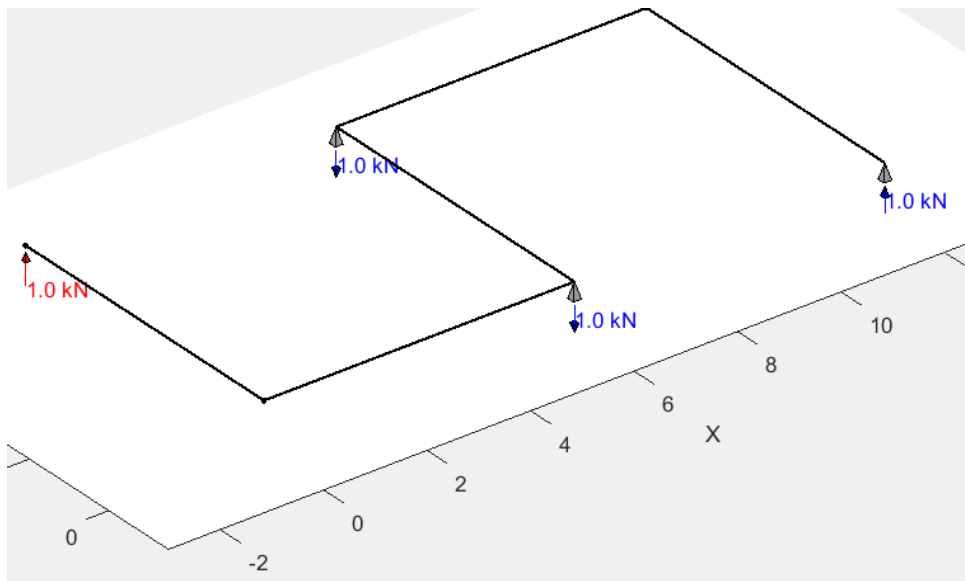


Diagrama de Momentos Fletores  $M_1$

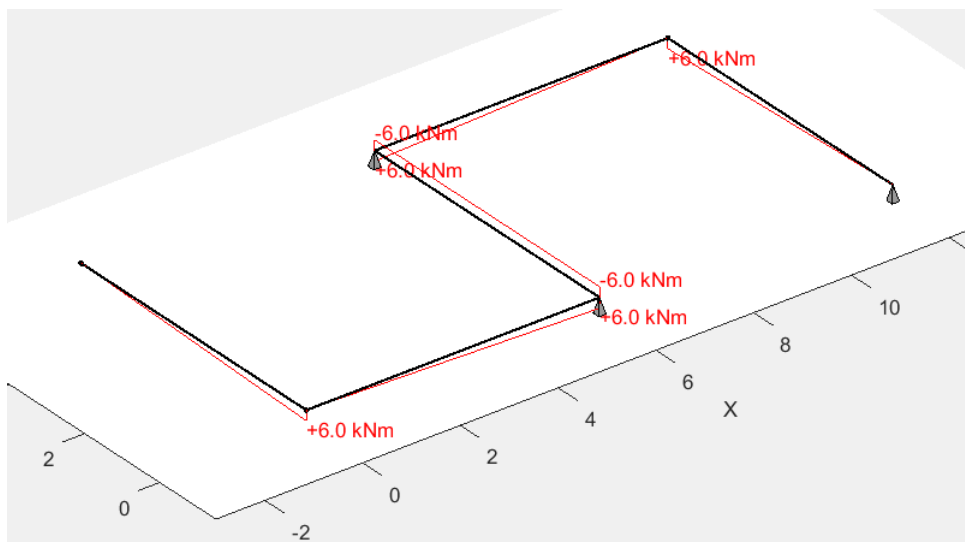
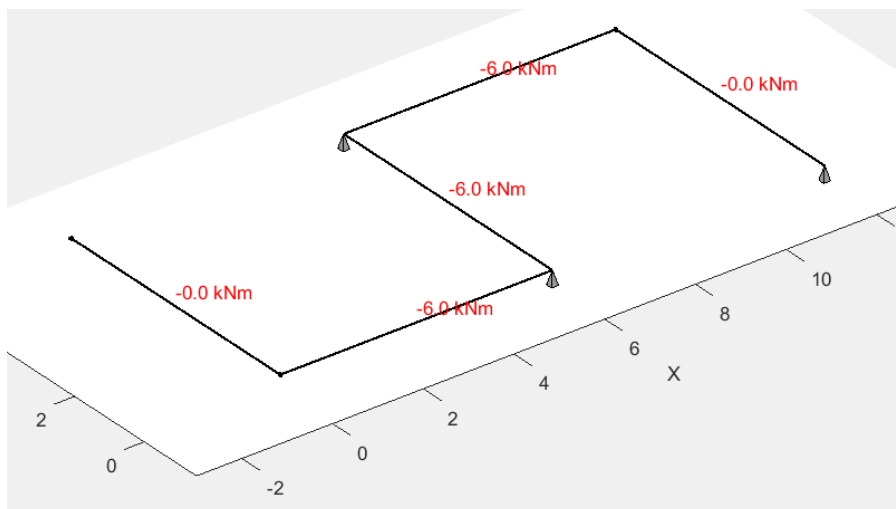
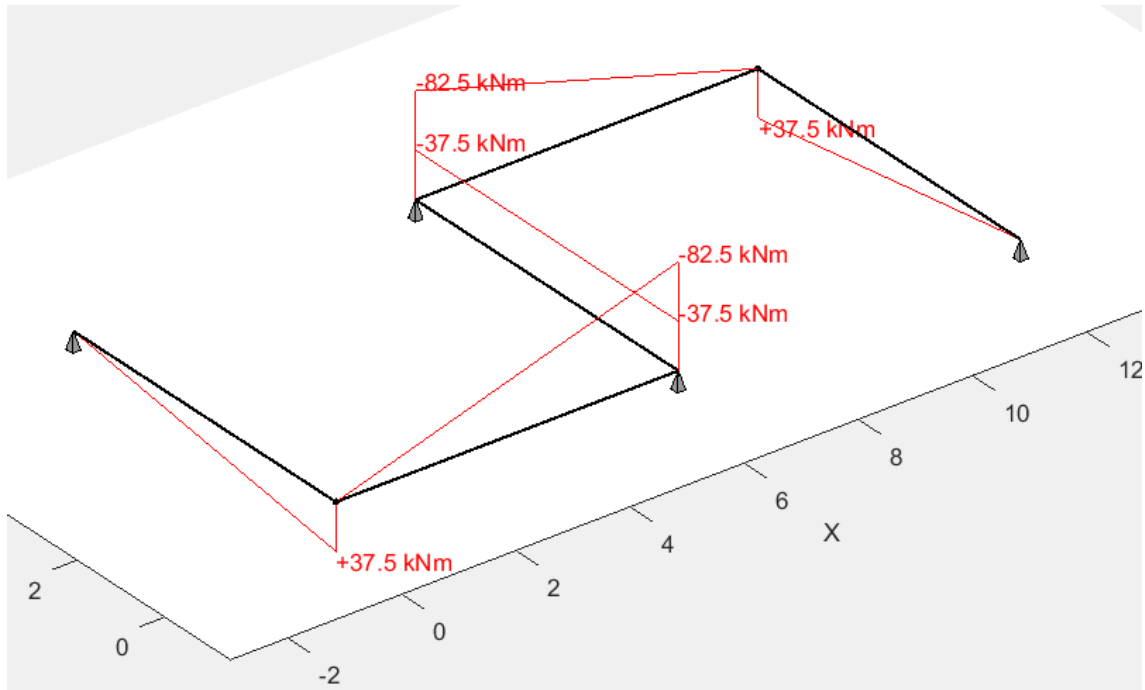


Diagrama de Momentos Torsores  $T_1$



Momentos Fletores Finais:  $M = M_0 + M_1X_1$  (kNm)



Momentos Torsores Finais:  $T = T_0 + T_1X_1$  (kNm)

